Department of Economics **Boğaziçi University**

EC 205 Macroeconomics I

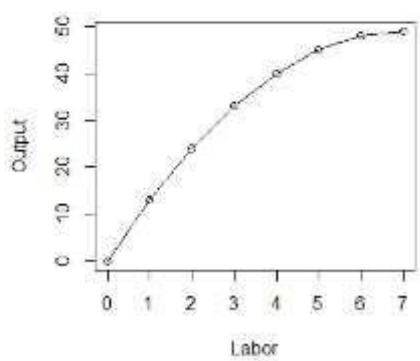
Fall 2022

Problem Session 2 Suggested Solutions

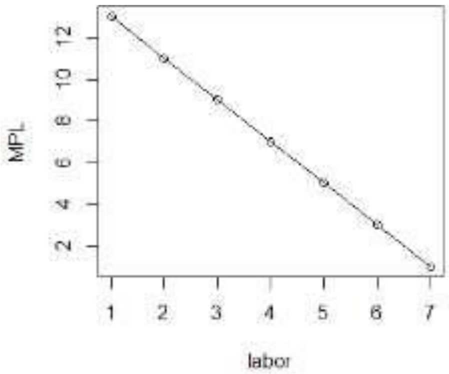
Q1.

Y	L	MPL
0	0	0
13	1	**13**
24	2	11
33	3	**9**
40	4	**7**
45	5	5
48	6	3
49	7	**1**

a)



b)



c) When $L = 5$, if we want to hire additional worker, then
MPL = marginal benefit from additional worker, W/P = marginal cost of hiring labor.
As 3 (marginal benefit) < 9 (marginal cost), the employer should not hire.

d) Labor supply increases whereas labor demand is fixed >>> Nominal wages fall >>> W/P also falls as P is constant.

e) Technology advances >>> Marginal productivity increases >>> MPL=W/P
by the theory, we expect real wages to rise.

f) Prices triple whereas all other factors remain constant (short run) >>> W/P falls.

Q2.

a) Plugging $A = 2$, $K = 10000$, and $L = 81$ into $Y = AK^{0.75}L^{0.25}$, one can calculate that output equals $Y = 6000$

b) If factors of production are paid their marginal production, then real wage (w) = $W/P = MPL = \partial Y/\partial L = (1-a)\ AK^{a}L^{-a}$
Plugging the values for $a = 0.75$, $A = 2$, $K = 10000$, and $L = 81$, $(1-a)\ AK^{a}L^{-a}$ = 18.51

c) Again, if factors of production are paid their marginal production, then real rental price (k) = $K/P = MPK = \partial Y/\partial K = (a)\ AK^{a-1}L^{1-a}$
Plugging the values for $a = 0.75$, $A = 2$, $K = 10000$, and $L = 81$, $(a)\ AK^{a-1}L^{1-a}$ = 0.45

d) Total income of labor owners is $MPL \times L = (1-a)\ AK^{a}L^{-a}\ x\ L = (1-a)\ Y$ =1500.
Total income of capital owners is $MPK \times K = (a)AK^{a-1}L^{1-a}\ x\ K = (a)Y$ =4500.
Then labor income share is ($1-a$) = 25% and capital income share is a = 75%

e) By the fundamental equilibrium definition of the demand side, we know that $Y=C+I+G$ (NX=0, hence excluded since we work under a closed economy).
Plugging in the functional forms, we can rewrite the above equation as:
$Y = 900 + 0.6(Y-T) + 950 - 1000r + G$
Rearranging yields $0.4Y = 1850 - 0.6T - 1000r + G$
Plugging in the values for Y=6000 (found in part a) T=1500 and G=1650, the above equation implies 2400 = 1850 - 900 - 1000r + 1650 or $1000r=200$, $r = 0.2$ (in percentage expression)
Then $C=900+0.6(Y-T) = 3600$, $I=950-1000r = 750$, and $r=0.2\%$

f) $S_{private}$ = (Y-T) – C (Disposable Income – Consumption) = 900
S_{public} = T – G = -150 (i.e. Government deficit)
$S_{national}$ = Y – C – G = $S_{private}$ + S_{public} = 750 (note that $S=I$ must hold in equilibrium)

g) This time Y=C+I+G can be written as
$Y = 900 + 0.6(Y\text{-}T) + 1200 - 1000r + G$ and rearranging yields
$0.4Y = 2100 - 0.6T - 1000r + G$ and plugging the values for Y=6000, T=1500, and G=1650, the above equation implies 2400=2850-1000r or 1000r=450, $r=0.45$ (in percentage expression)
Then $C = 900 + 0.6(Y\text{-}T) = 3600$ (does not change)
$I = 1200 - 1000r = 750$ (does not change since saving is still fixed as Y, C, G and T are fixed)
and finally, $r=0.45\%$

h) $S_{private}$ = (Y-T) – C = 900 (same as before)
S_{public} = T – G = -150 (Government deficit, same as before)
$S_{national}$ = Y – C – G = $S_{private}$ + S_{public} = 750 ($S=I$ must hold in equilibrium)

Q3.

Diminishing marginal returns properties: f '(x) > 0 and f "(x) < 0

a. $F(K, L) = K^{0.5}L^{0.5}$

$F(\lambda K, \lambda L) = (K\lambda)^{0.5}(L\lambda)^{0.5} = \lambda K^{0.5} L^{0.5} = \lambda F(K, L)$

Constant returns to scale property is satisfied.

$F_K (K, L) = 0.5K^{-0.5}L^{0.5} > 0$ since both K and L are positive.

$F_L (K, L) = 0.5K^{0.5}L^{-0.5} > 0$ since both K and L are positive.

$F_{KK} (K, L) = -0.25K^{-1.5}L^{0.5} < 0$ since both K and L are positive.

$F_{LL} (K, L) = -0.25K^{0.5}L^{-1.5} < 0$ since both K and L are positive.

Positive and diminishing marginal products of labor and capital is satisfied.

b. $F(K, L) = AK^{\beta}L^{1\text{-}\beta}$

$F(\lambda K, \lambda L) = A(\lambda K)^{\beta}(\lambda L)^{1\text{-}\beta} = A\lambda^{\beta}K^{\beta}\lambda^{1\text{-}\beta}L^{1\text{-}\beta} = \lambda AK^{\beta}L^{1\text{-}\beta} = \lambda F(K, L)$

Constant returns to scale property is satisfied.

$F_K (K, L) = (\beta)K^{-\beta} L^{1\text{-} \beta} > 0$ since β, K and L are positive.

$F_L (K, L) = (1\text{-}\beta)K^{\beta} L^{\text{-}\beta} > 0$ since (1-β), K and L are positive.

$F_{KK} (K, L) = \beta(\beta -1)K^{\beta\text{-}2}L^{1\text{-} \beta} < 0$ since β, K and L are positive, and (β-1) is negative

$F_{LL} (K, L) = -\beta(1\text{-}\beta)K^{\beta} L^{\text{-}\beta\text{-}1} < 0$ since 1-β, K and L are positive, and -β is negative

Positive and diminishing marginal products of labor and capital is satisfied.

c. $F(K,L)=L$

$F(\lambda K, \lambda L)=(L\lambda)= \lambda L=\lambda F(K,L)$

Constant returns to scale property is satisfied.

$F_L(K,L)=1>0$ but $F_{LL}(K,L) = 0$

Since $F_{LL}(K, L)=0$ is non-negative, diminishing marginal returns property is **violated**, although positive marginal product of labor is satisfied since $F_L(K,L)=1>0$.

Note that K does not appear as an input for the production function so positive and diminish marginal returns of capital is **violated**.

Although, obviously, one could show that, $F_K(K, L)=0$ and $F_{KK}(K,L)=0$

d. $F(K,L)=K^{0.5}L^{0.5}+L$

$F(\lambda K, \lambda L) = (K\lambda)^{0.5}(L\lambda)^{0.5} +L\lambda= \lambda K^{0.5}L^{0.5}+\lambda L = \lambda(K^{0.5}L^{0.5}+L) =\lambda F(K,L)$

Constant returns to scale property is satisfied.

$F_K(K, L)=0.5K^{-0.5}L^{0.5} >0$ since K and L are all positive.

$F_L(K,L)=0.5K^{0.5}L^{-0.5}+1 >0$ since K and L are positive.

$F_{KK}(K,L)=(-0.5)0.5K^{-1.5}L^{0.5} < 0$ since K and L are positive.

$F_{LL}(K,L)=(-0.5)0.5K^{0.5}L^{-1.5} < 0$ since both since K and L are positive.

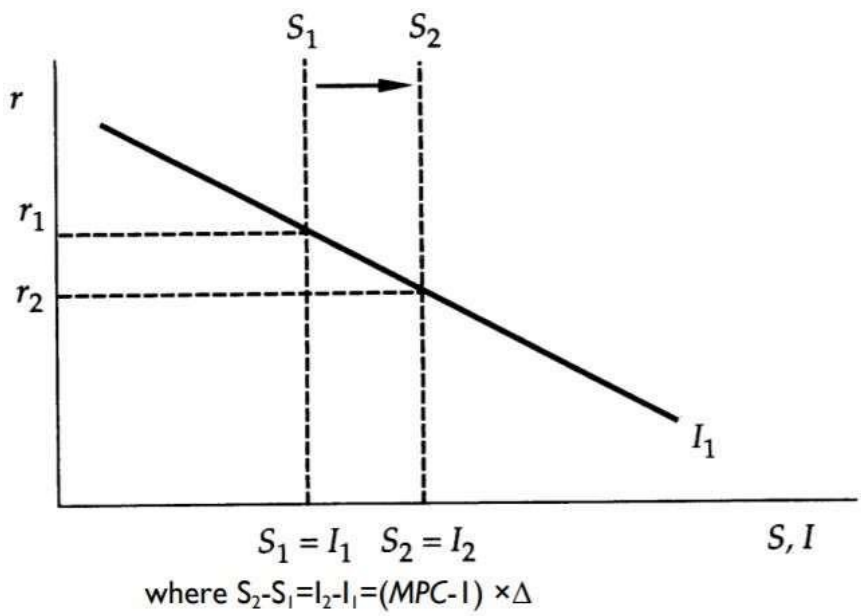
Positive and diminishing marginal products of labor and capital property is satisfied.

Q4.

From the Neoclassical Theory Model that we studied in Chapter 3, we know that national savings in an economy is given by $S=Y–C–G$. Further, we saw that consumption has to follow a functional form of $C=A+MPC(Y-T)$ where A is just a constant, and MPC denotes the marginal propensity to consume which is strictly less than 1 and greater than 0. Then we can rewrite the saving equation as:

S=Y-A-MPC(Y-T)-G

Since the government wants to reduce both G and T by the same amount, call it Δ (where Δ is a NEGATIVE number since G and T are lower after reduction) , the change in savings will be $-MPC\times\Delta+\Delta=(MPC-1)\times\Delta$ as other variables, Y and A do not change, at all. Since $0<MPC<1$, it must be that (MPC-1) is negative (to be precise, it must be that -1<(MPC-1)<0) hence (MPC-1) ×Δ, the change in savings is positive, by the multiplication of two negative numbers. This then means that savings, S, increases in this economy. Since in equilibrium, it must be that savings equal investment, i.e. S=I(r), it has to be that change in savings is equal to change in investment, as well. Hence, investment also increases by (MPC-1) ×Δ. Finally, since investment demand is inversely related with real interest rate r, higher investment requires lower real interest rate, therefore, real interest rate falls. Graphically, the change can be shown as follows:



We have already shown that real interest rate falls, and (national) saving and investment rises

i. real interest rate decreases
ii. national saving increases
iii. investment increases

We know that output, Y, in this economy is determined only by fixed amount of labor and capital, neither of which changes. Therefore, output would not change, at all.

iv. output is unchanged, fixed because it is determined by the factors of production.

Since Y is fixed, and taxes T are lower now, consumption C=A+MPC(Y-T) increases due to decrease in T. In other words, higher disposable income requires greater consumption.

v. consumption increases.