Department of Economics **Boğaziçi University**

EC 205 Macroeconomics I

Fall 2023

Problem Session 3

1.

a. According to the neoclassical theory of distribution, the real wage equals the marginal product of labor. Because of diminishing returns to labor, an increase in the labor force causes the marginal product of labor to fall. Hence, the real wage falls.
b. The real rental price equals the marginal product of capital. If an earthquake destroys some of the capital stock (yet miraculously does not kill anyone and lower the labor force), the marginal product of capital rises and, hence, the real rental price rises.
c. If a technological advance improves the production function, this is likely to increase the marginal products of both capital and labor. Hence, the real wage and the real rental price both increases.

2. Consider the formula for the natural rate of unemployment,

$$U/L = s / (s + f)$$

If the new law lowers the chance of separations but has no effect on the rate of job finding *f,* then the natural rate of unemployment falls.

For several reasons, however, the new law might tend to reduce *f.* First, raising the cost of firing might make firms more careful about hiring workers, since firms have a harder time firing workers who turn out to be a poor match. Second, if searchers think that the new legislation will lead them to spend a longer period of time on a particular job, then they might weigh more carefully whether or not to take that job. If the reduction in *f* is large enough, then the new policy may even increase the natural rate of unemployment.

3.

a. The demand for labor is determined by the amount of labor that a profit-maximiz ing firm wants to hire at a given real wage. The profit-maximizing condition is that the firm hire labor until the marginal product of labor equals the real wage,

$$MPL = W/P$$

The marginal product of labor is found by differentiating the production function with respect to labor

$$MPL = dY/dL$$

$$= d(K^{1/3}L^{2/3})/dL$$

$$= 2/3\ K^{1/3}L^{-1/3}$$

In order to solve for labor demand, we set the MPL equal to the real wage and solve for L:

$$2/3\ K^{1/3}L^{-1/3} = W/P$$

$$L = 8/27\ K\ (W/P)^{-3}$$

Notice that this expression has the intuitively desirable feature that increases in the real wage reduce the demand for labor.

b. We asume that the 1,000 units of capital and the 1,000 units of labor are supplied inelastically (i.e., they will work at any price). In this case we know that all 1,000 units of each will be used in equilibrium, so we can substitute them into the above labor demand function and solve for W/P

$$1000 = 8/27\ 1000\ (W/P)^{-3}$$

$$W/P = 2/3$$

In equilibrium, employment will be 1,000, and multiplying this by 2/3 we find that the workers earn 667 units of output. The total output is given by the production function:

$$Y = K^{1/3}L^{2/3}$$

$$1000^{1/3}1000^{2/3}$$

$$=1000$$

Notice that workers get two-thirds of output, which is consistent with what we know about the Cobb-Douglas production function from Chapter 3.

c. The congressionally mandated wage of 1 unit of output is above the equilibrium wage of 2/3 units of output.

d. Firms will use their labor demand function to decide how many workers to hire at the given real wage of 1 and capital stock of 1,000:

$$L = 8/27\ 1000\ (1)^{-3}$$

$$=296$$

so 296 workers will be hired for a total compensation of 296 units of output. To find the new level of output, plug the new value for labor and the value for capital into the production function and you will find Y= 444.

e. The policy redistributes output from the 704 workers who become involuntarily unemployed to the 296 workers who get paid more than before. The lucky workers benefit less than the losers lose as the total compensation to the working class falls from 667 to 296 units of output.

f. This problem does focus the analysis of minimum-wage laws on the two effects of these laws: they raise the wage for some workers while downward-sloping labor demand reduces the total number of jobs. Note, however, that if labor demand is less elastic than in this example, then the loss of employment may be smaller, and the change in worker income might be positive.

4.

To show that the unemployment rate evolves over time to the steady-state rate, let's begin by defining how the number of people unemployed changes over time. The change in the number of unemployed equals the number of people losing jobs (*sE*) minus the number finding jobs (*fU*). In equation form, we can express this as:

$$U_{t+1} - U_t = \Delta U_{t+1} = sE_t - fU_t = s(L - U_t) - fU_t$$

Dividing by *L*, we get an expression for the change in the unemployment rate from t to $t+1$:

$$\Delta\left(\frac{U}{L}\right)_{t+1} = s\left(1 - \frac{U_t}{L}\right) - \frac{fU_t}{L} = s - s\frac{U_t}{L} - f\frac{U_t}{L} = s - \frac{(s+f)U_t}{L}$$

Multiply the right-hand side by (s + f)/(s + f) and rearrange terms to end up with

$$\Delta\left(\frac{U}{L}\right)_{t+1} = (s+f)\left(\frac{s}{s+f} - \frac{U_t}{L}\right) = (s+f)(\left(\frac{U}{L}\right)^n - \frac{U_t}{L})$$

Then you can see that, when the unemployment rate is lower than the natural unemployment rate, $\left(\frac{U}{L}\right)^n$,the change of the unemployment rate will be positive which in turn increases the unemployment. If the unemployment is higher than the natural rate, then the change of the unemployment rate will be negative thus the unemployment rate will decrease. Therefore, wherever the unemployment locates, it will move to its steady state in long run.