24 November 2023 Friday 00:58

1.

a) CRTS condition $\rightarrow F(\lambda K, \lambda L) = \lambda F(K, L)$

$F(\lambda K, \lambda L) = \lambda^{0.4} K^{0.4} \lambda^{0.6} L^{0.6} = \lambda K^{0.4} L^{0.6} \rightarrow$ satisfies

b) $F(\frac{K}{L}, 1) = \left(\frac{K}{L}\right)^{0.4}$

c) First, let's define $k = \frac{K}{L}$, $y = \frac{Y}{L} = f(k)$

Then, we get;

s	δk	k^*	$f(k)$	$(1-s)f(k)$
0	0	0	0	0
0.10	0.15k	$(2/3)^{5/3}$	$(2/3)^{2/3}$	$0.9(2/3)^{2/3}$
0.20	0.15k	$(4/3)^{5/3}$	$(4/3)^{2/3}$	$0.8(4/3)^{2/3}$
⋮	⋮	⋮	⋮	⋮
1	0.15k	$(20/3)^{5/3}$	$(20/3)^{2/3}$	$0.(20/3)^{2/3}$

d) As graph shows, output per worker increases as saving rate increases.

To find maximized consumption, we should use Golden Rule

$$MPK - \delta = 0 \quad MPK = f'(k) \rightarrow (0.4)(k)^{-0.6} = 0.15$$

Then, we get;

$$k = \left(\frac{8}{3}\right)^{5/3}, \quad sk^{0.4} = \delta k \rightarrow s\left(\frac{8}{3}\right)^{2/3} = 0.15\left(\frac{8}{3}\right)$$

$$s = 0.15\left(\frac{8}{3}\right)^{1/3} \quad C = (1-s)f(k) = \left(1 - 0.15\left(\frac{8}{3}\right)^{1/3}\right)\left(\frac{8}{3}\right)^{2/3}$$

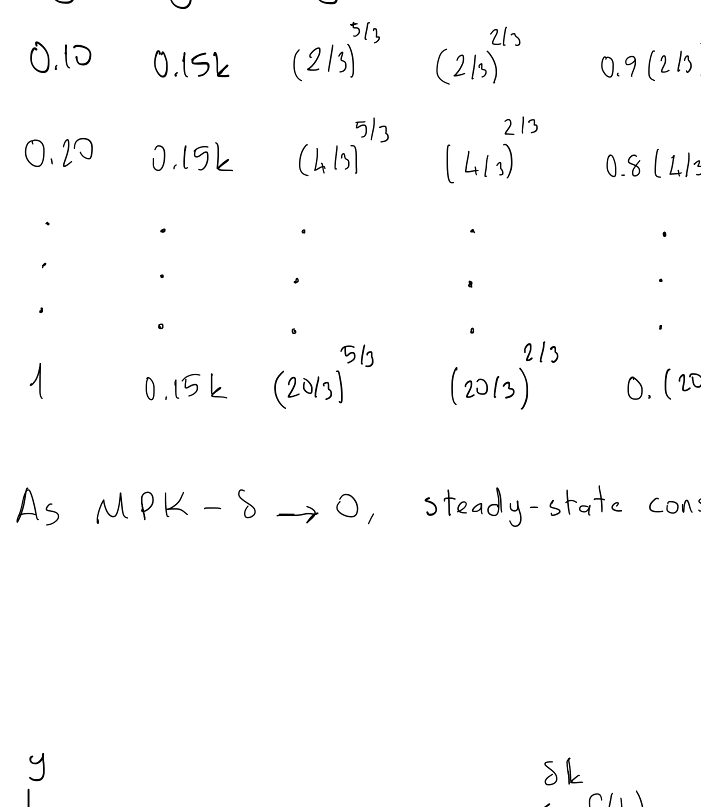
e)

s	δk	k^*	$f(k)$	$(1-s)f(k)$	MPK
0	0	0	0	0	0
0.10	0.15k	$(2/3)^{5/3}$	$(2/3)^{2/3}$	$0.9(2/3)^{2/3}$	0.4 (3/2)
0.20	0.15k	$(4/3)^{5/3}$	$(4/3)^{2/3}$	$0.8(4/3)^{2/3}$	0.4 (3/4)
⋮	⋮	⋮	⋮	⋮	⋮
1	0.15k	$(20/3)^{5/3}$	$(20/3)^{2/3}$	$0.(20/3)^{2/3}$	0.4 (3/20)

As $MPK - \delta \rightarrow 0$, steady-state consumption increases.

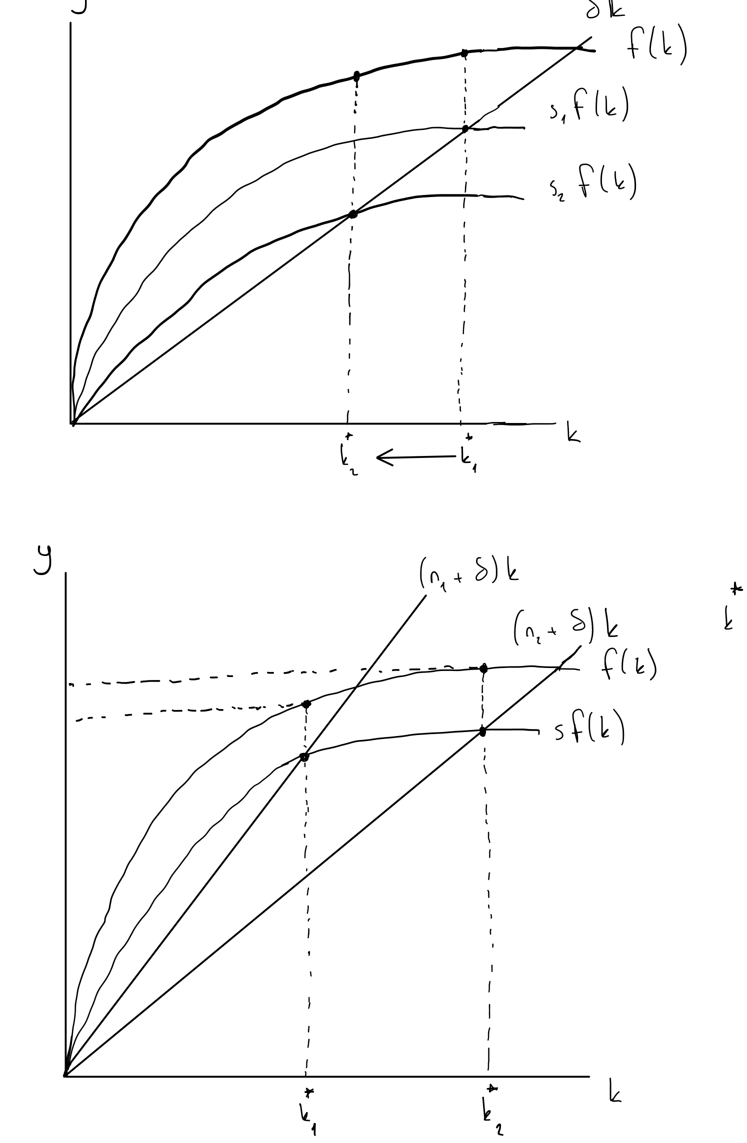
2.

a)



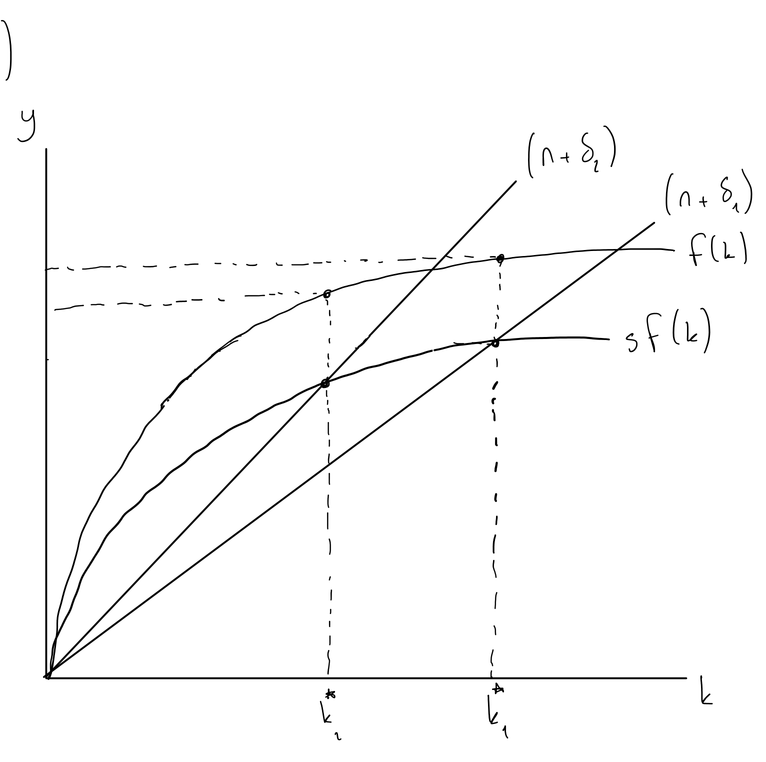
$k^* \downarrow \quad f(k^*) \downarrow$

b)



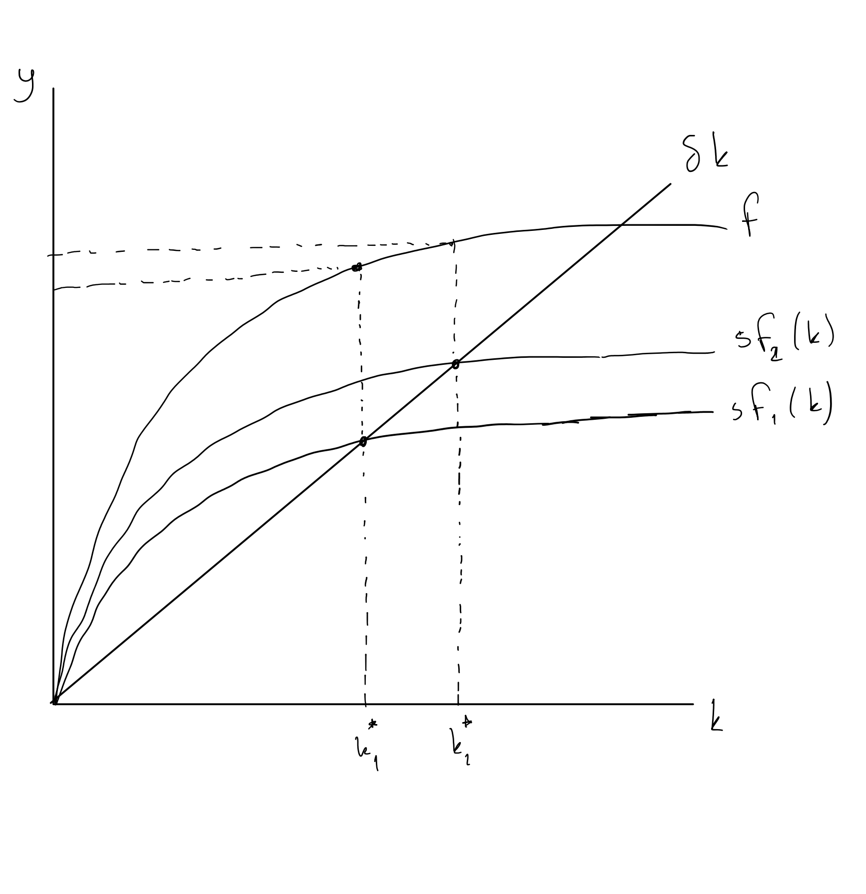
$k^* \uparrow \quad f(k^*) \uparrow$

c)



$k^* \downarrow \quad f(k^*) \downarrow$

d)



$f(k) = A.k^{\alpha}$

$k^* \uparrow \quad f(k^*) \uparrow$

3.

a) Let $f(k) = Ak^{\alpha}$

Law of Motion

$$k_{t+1} = sf(k_t) + (1-\delta)k_t$$

At steady state, $k_{t+1} = k_t$. Then we get;

$$\delta k = s.Ak^{\alpha} \rightarrow k^{1-\alpha} = \frac{sA}{\delta} \rightarrow k = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}}$$

To get relationship between δ and k, we take derivative wrt δ

$$\frac{\partial k}{\partial \delta} = \underbrace{\left(\frac{-1}{1-\alpha}\right)\left(\frac{\delta}{sA}\right)^{\frac{-2+\alpha}{1-\alpha}}}$$

This is always negative.
That means it is decreasing function. Thus, when $\delta \uparrow$, $k^* \downarrow$

b) Same thing, but this time we take derivative wrt s.

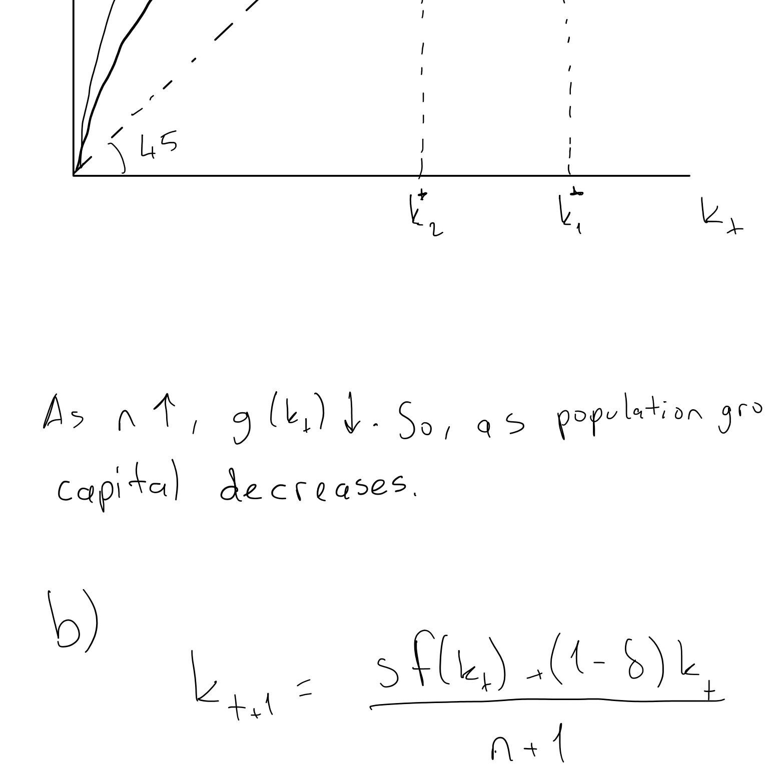
$$\frac{\partial k}{\partial s} = \underbrace{\left(\frac{1}{1-\alpha}\right)\left(\frac{sA}{\delta}\right)^{\frac{\alpha}{1-\alpha}}}$$

This is always >0
thus it is increasing

4.

a) Let's define a function g that describes law of motion with population growth

$$k_{t+1} \equiv g(k_t) = \frac{sf(k_t) + (1-\delta)k_t}{1+n}$$



As $n \uparrow$, $g(k_t) \downarrow$. So, as population growth rate increases, then per capita capital decreases.

b)

$$k_{t+1} = \frac{sf(k_t) + (1-\delta)k_t}{n+1}$$

At steady state, $k_{t+1} = k_t$

$$(n+1)k = sA.k^{\alpha} + (1-\delta)k \rightarrow (n+\delta)k = sAk^{\alpha} \rightarrow k^{1-\alpha} = \frac{sA}{n+\delta}$$

$$k = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}} \qquad y = f(k) = A\left(\frac{sA}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$(1-s)f(k) = (1-s)A\left(\frac{sA}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}} \qquad sf(k) = sA\left(\frac{sA}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

c) We just plug in

$$k^* = \left(\frac{0.20.5}{0.10+0.15}\right)^{\frac{1}{1-0.4}} = 4^{5/3} \qquad f(k) = 5.4^{2/3}$$

$$sf(k) = 0.20.5.4^{2/3} = 4^{2/3} \qquad (1-s)f(k) = 3.4^{2/3}$$