

04/10/2024 week 1

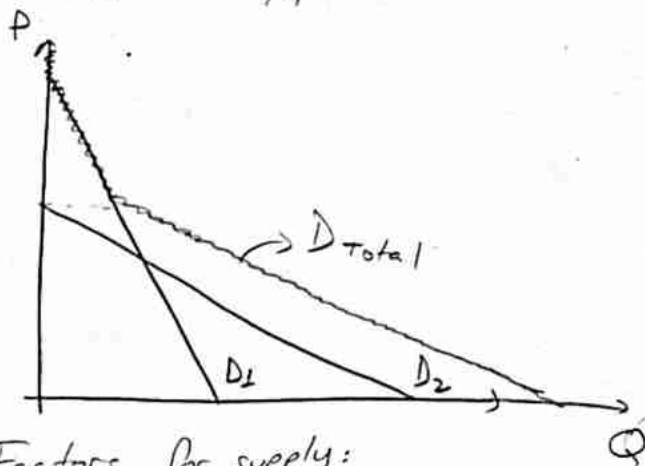
Factors determine the demand for a good:

- 1) Price
- 2) Tastes
- 3) Information
- 4) Complements and substitutes
- 5) Income
- 6) Regulations
- 7) Expectations

Aggregating the demand:

n consumers each with $Q_i = D_i(p)$ then

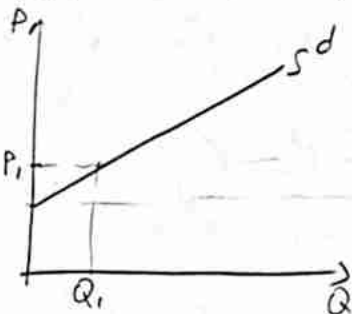
$$Q = \sum_{i=1}^n Q_i = \sum_{i=1}^n D_i(p)$$



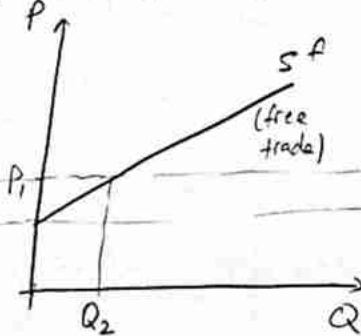
Factors for supply:

- 1) Price
- 2) Cost
- 3) Regulations

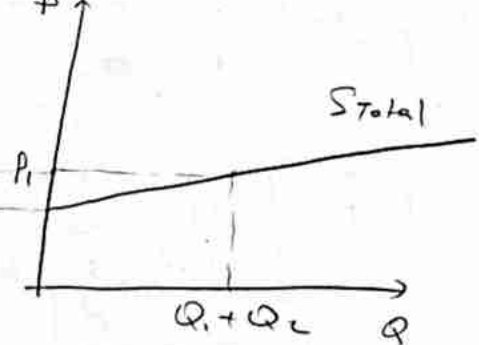
Turkish domestic supply



Foreign supply



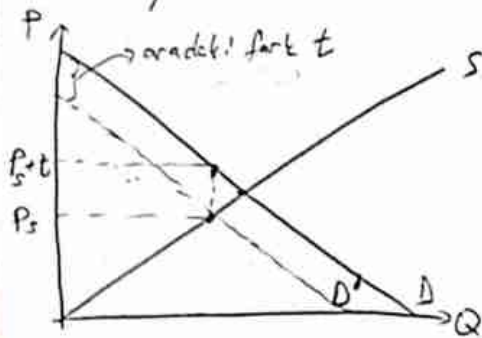
Total supply



Quantity tax: t on each unit traded

- * How does quantity tax affect the equilibrium?
- * Who actually pays the tax?

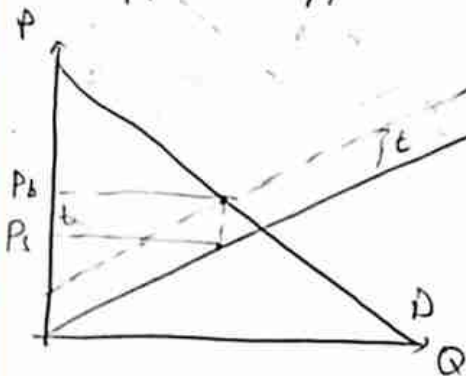
→ Suppose the government required consumers to keep track of their purchases and send a tax payment to the government.



Tax ödeyeceğim için daha düşük fiyatla almak mı diye pazarlık yapıyorum ve verginin bir kısmını supplier'a yansıtıyorum.

- Producer gets $\rightarrow P_s$
- Consumer pays $\rightarrow P_s + t \rightarrow$ to gov.

→ Suppose suppliers include the tax into the price



- Producer gets $\rightarrow P_s + t \rightarrow$ to gov.
- Consumer pays $\rightarrow P_b$

* The economic effect of a unit tax is independent of which party makes the payment to the government

Ex: $D(p) = a - bP^b$ $S(P) = c + dP^s$ $P^b = P^s + t$

buyers pay: $P_b' = \frac{a}{b} - \frac{Q^d}{b} - t$ (new demand curve) $P_b' = P_s$
 $P_s = \frac{Q^s}{d} - \frac{c}{d}$ (no change) $Q^s = Q^d$

Sellers pay: $P_b = \frac{a}{b} - \frac{Q^d}{b}$ (no change) $P_b = P_s'$
 $P_s' = \frac{Q^s}{d} - \frac{c}{d} + t$ (new supply curve) $Q_s = Q^d$

same effect

* Who suffer how much depends on elasticity of the curves

Elasticity: The percentage change in a variable in response to a given percentage change in another variable

Price elasticity of demand: $E = \frac{\% \Delta Q}{\% \Delta P} = \frac{\Delta Q / Q}{\Delta P / P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$

$$\frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = \frac{1}{\text{slope}} \cdot \frac{P}{Q}$$

$D(P) = S(P-t)$, $D(P(t)) = S((P(t))-t)$ differentiating with respect to tax

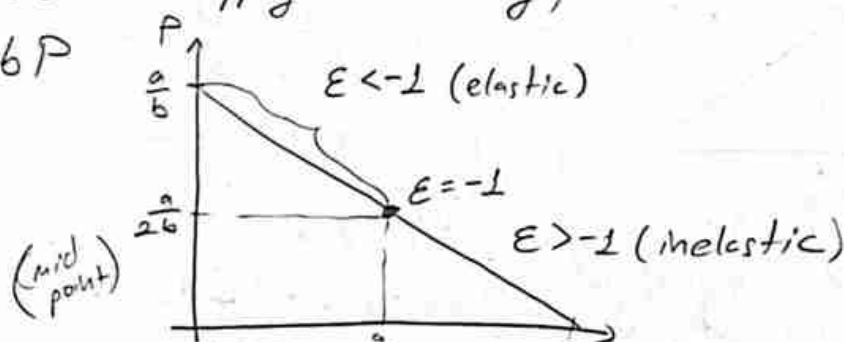
$$\frac{\partial D}{\partial P} \cdot \frac{\partial P}{\partial t} = \frac{\partial S}{\partial P} (\frac{\partial P}{\partial t} - 1)$$

$$\frac{\partial P}{\partial t} = \frac{\frac{\partial S / \partial P}{\frac{\partial S}{\partial P} - \frac{\partial D}{\partial P}} \text{ multiply } \frac{P}{Q} \quad \frac{\frac{\partial S / \partial P}{\frac{\partial S}{\partial P} - \frac{\partial D}{\partial P}}}{\frac{E_S}{E_S - E_D}} = 1 - \frac{\partial P}{\partial t}$$

Given E_S , the more elastic the demand, the less rise in price
given E_D , the smaller supply elasticity, the smaller rise in price

Ex: $Q^d = a - bP$

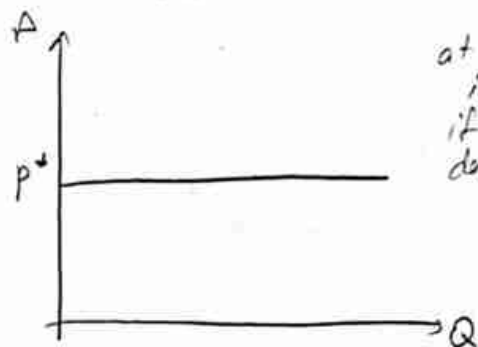
$$E_D = -b \cdot \frac{P}{Q}$$



Ex: $Q = kP^a$

$$E = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = k \cdot a \cdot P^{a-1} \cdot \frac{P}{k \cdot P^a} = a \text{ (constant elasticity)}$$

Perfectly elastic demand:



$$E_D = \infty$$

Perfectly inelastic demand



$$E = 0$$

Income elasticity: $E_d = \frac{\% \Delta Q}{\% \Delta \text{Income}}$

Crossprice elasticity: $E_c = \frac{\% \Delta Q_1}{\% \Delta P_2} = \frac{\partial Q_1}{\partial P_2} \cdot \frac{P_2}{Q_1}$

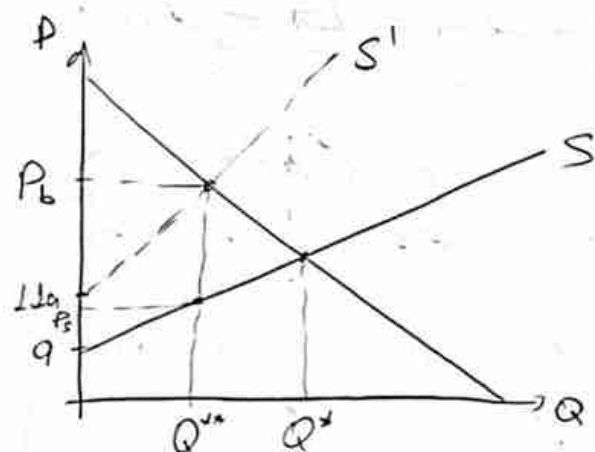
$E_c > 0$ substitute goods

$E_c < 0$ complementary goods

→ Elasticities tend to be higher in the long run. However, in the short run, people don't change their habits

Ad valorem tax

→ Suppose 10% VAT imposed on sellers $P_s = a + b Q_s$
 $P_d = c + d Q_d$
 $t \rightarrow 1.1$ multiplier $P_s' = 1.1(a + b Q_s)$



Slope increases

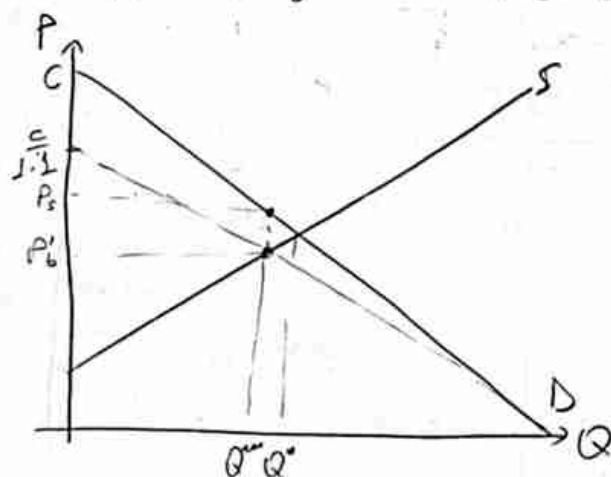
$$P_s' = P_d \quad Q_s = Q_d$$

$$Q^{**} = \frac{c - a t}{b t - d}$$

P_b : Price that buyers pay

P_s : Price that sellers receive

→ Suppose buyers are paying 10% tax



$$P_d = c + d Q_d \quad P_d' = \frac{c + d Q_d}{1.1}$$

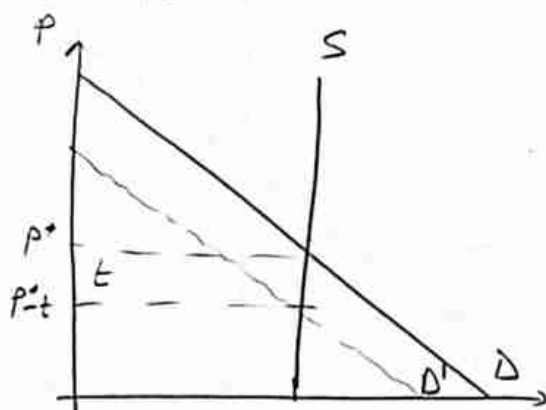
$$P_s = a + b Q_s$$

$$Q^{**} = \frac{\frac{c}{1.1} - a}{b - \frac{d}{1.1}}$$

linear demand curve
 less rise

$$Q^{**} = Q^{***} \text{ multiplier}$$

→ Suppose inelastic S curve, buyers pay unit tax t



Buyers are paying $P^* + t$ to the seller
They pay t to government

$$\frac{dP}{dt} = \frac{E_S}{E_S - E_D} = 0 \quad \text{all of share the tax burden on seller}$$

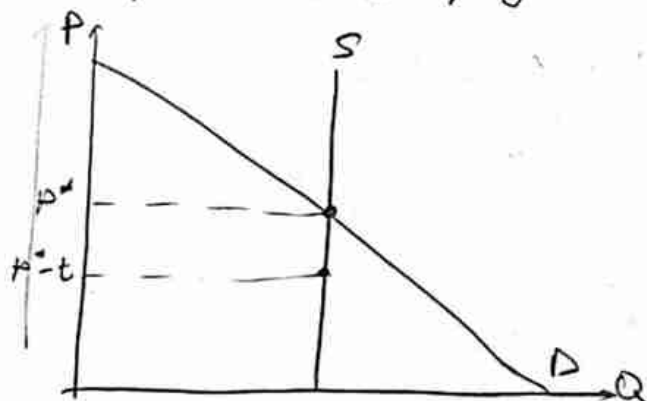
$$P_b^i = P^* \rightarrow P_b^f = P^* \quad \text{buyer price didn't change}$$

$$P_s^i = P^* \rightarrow P_s^f = P^* - t \quad \text{seller price decreased}$$

$$\Delta CS = 0$$

$$\Delta PS = -tQ$$

→ Suppose sellers pay unit tax t



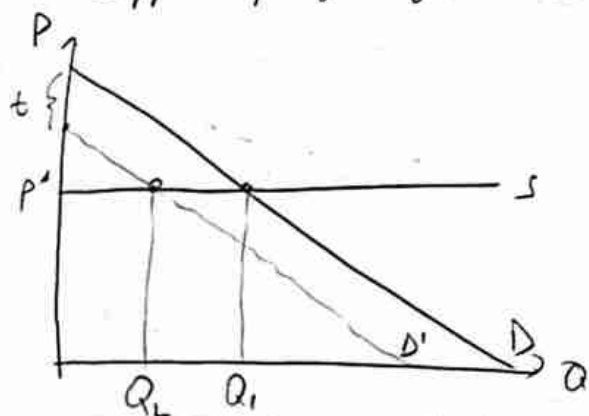
Buyers pay P^* to seller
Sellers receive P^* , send t to gov
Finally, sellers get $P^* - t$

$$P_b^i = P^* \rightarrow P_b^f = P^* \quad \text{buyer price didn't change}$$

$$P_s^i = P^* \rightarrow P_s^f = P^* - t \quad \text{seller price decreased}$$

$$\Delta CS = 0 \quad \Delta PS = -tQ$$

→ Suppose perfectly elastic S curve, buyers pay unit tax t



Buyers pay $P^* + t$ to seller, t to gov
Sellers receive P^*

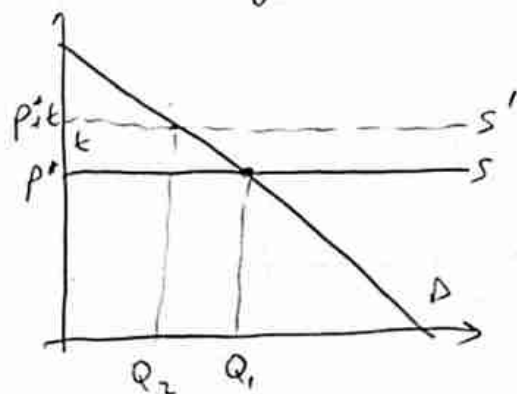
$$P_b^i = P^* \rightarrow P_b^f = P^* + t \quad \text{buyer price increased}$$

$$P_s^i = P^* \rightarrow P_s^f = P^* \quad \text{seller price unchanged}$$

$$\text{all tax burden on payers} \quad \frac{dP}{dt} = 1$$

$$\Delta CS = -tQ \quad \Delta PS = 0$$

→ Sellers pay unit tax



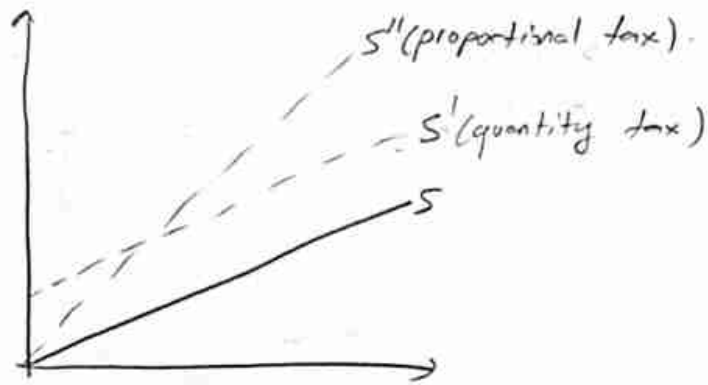
Buyers pay $P^* + t$ to seller
Sellers receive $P^* + t$, send t to gov.

$$P_b^i = P^* \rightarrow P_b^f = P^* + t$$

$$P_s^i = P^* \rightarrow P_s^f = P^*$$

$$\Delta CS = -tQ \quad \Delta PS = 0$$

11/10/2024 week 2



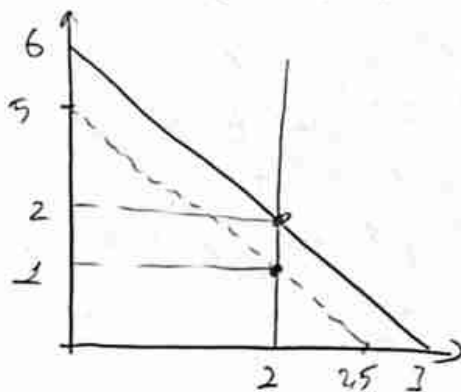
Ex: $Q^s = 2$ $P^d = 6 - 2Q^d$

1) $t = 1\text{€}$ buyers pay

New $P^d = 6 - 2Q^d - 1 = 5 - 2Q^d$

Buyers: 1€ to seller + 1€ to government

Sellers receive 1€ $\frac{dP}{dt} = 0$

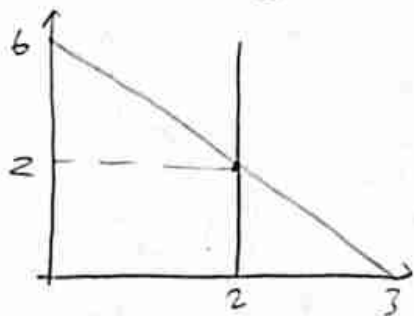


2) Sellers pay 1€

Buyers pay 2€ to seller

Sellers receive 2€ from buyer, pay 1€ to gov.

$\frac{dP}{dt} = 0$

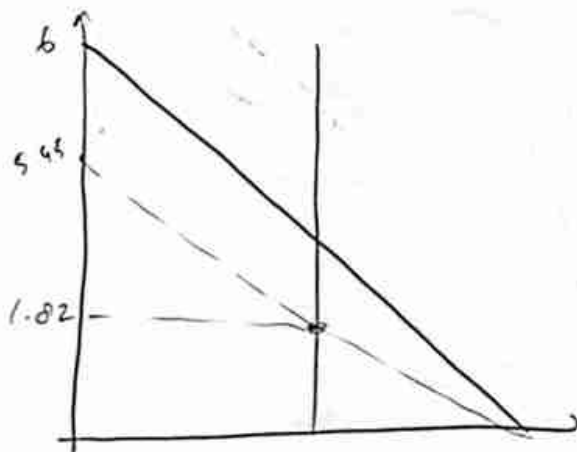


3) VAT 10% imposed on buyers

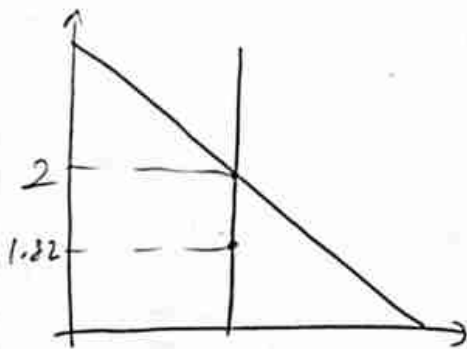
New $P^d = \frac{6 - 2Q^d}{1.1} \approx 5.45 - 1.82Q^d$
 $P^d = 1.82\text{€}$

Buyers: 1.82€ to sellers
 $+ 0.182\text{€}$ to gov
 2€

Sellers receive 1.82€



4) VAT 10% on sellers



Buyers: 2€ to seller
Sellers receive 2€, pay 0.18€ to gov.

$$Q^d = 286 - 20P, \quad Q^s = 88 + 40P \rightarrow P = 3.3$$

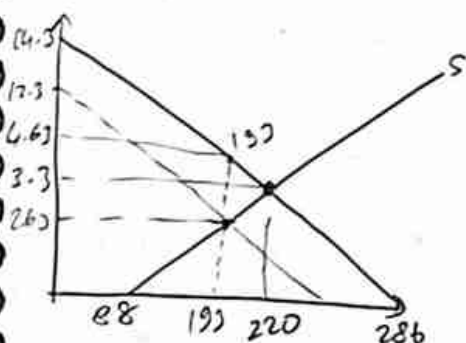
$$P_b = 14.3 - \frac{Q^d}{20}, \quad P_s = \frac{Q^s}{40} - 2.2 \rightarrow P = 3.3$$

1) Buyers pay 2€ tax

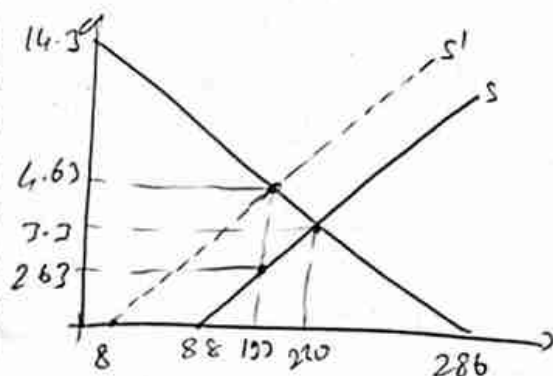
$$P_b' = 14.3 - \frac{Q^d}{20} - 2 = \frac{Q^s}{40} - 2.2 \rightarrow Q' = 193$$

$$P' = 2.63$$

Buyers: Pay 2.63€ to seller 2€ to gov $L = 4.63$ €
Sellers: Receive 2.63€



2) Sellers pay 2€ tax



$$P_s'' = \frac{Q^s}{40} - 2.2 + 2 = 14.3 - \frac{Q^d}{20}$$

$$Q'' = 193 \quad P'' = 4.63$$

Buyers pay 4.63€ to sellers
Sellers receive 4.63€, pay 2€ to gov

17/10/2024 week 3

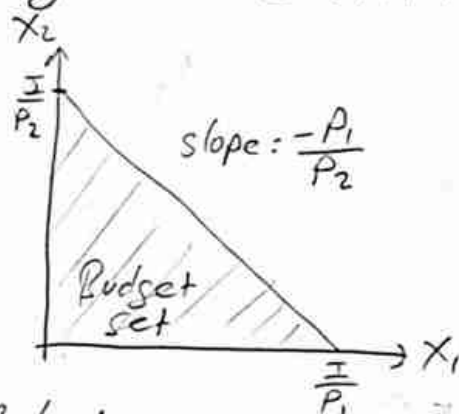
2 goods consumption bundle (x_1, x_2)

Prices: (p_1, p_2) Income: I or m or w

$$p_1 x_1 + p_2 x_2 \leq I \quad B_{p,I} = \{x \in \mathbb{R}^+ \mid p^t x \leq I\}$$

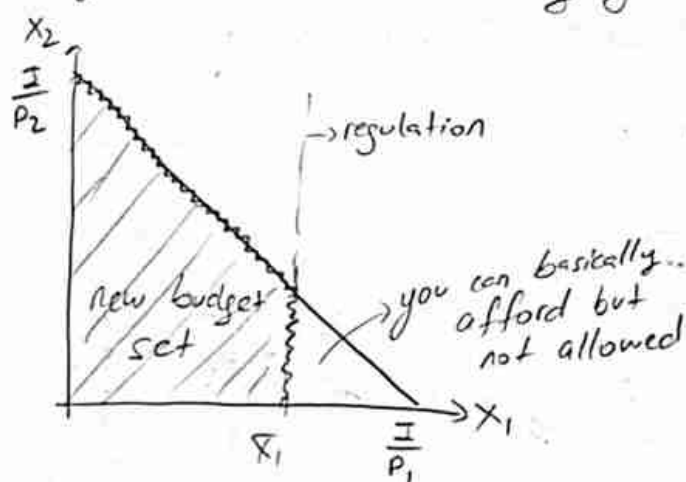
$$\hookrightarrow p_1 x_1 + p_2 x_2 + \dots + p_n x_n \leq I$$

Budget line: $I = p_1 x_1 + p_2 x_2$

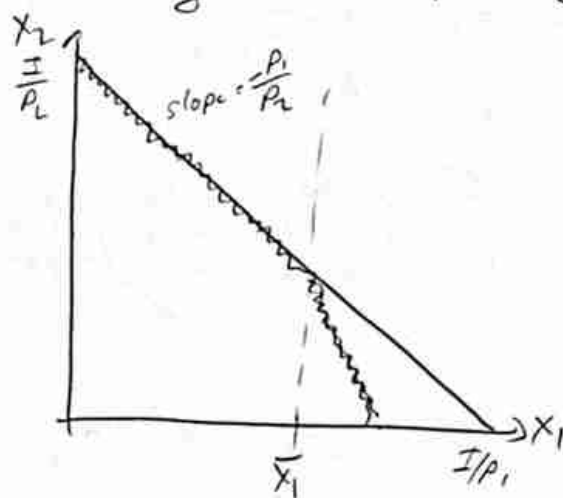


* A set is open if every point in it is an interior point. A set is closed if it contains all of its boundary points.
* A set is compact if every open cover of A contains a subcover of A .

Budget set with rationing you can buy max unit of $\bar{x}_1$
 $\bar{x}_1 < I/p_1$

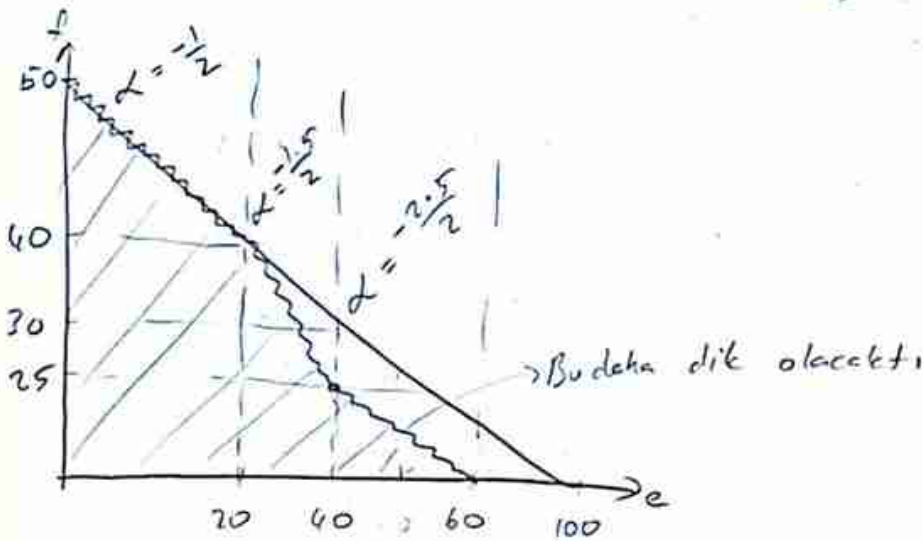


→ Taxing the consumption greater than $\bar{x}_1$. After $\bar{x}_1$, price: $p_1 + T$



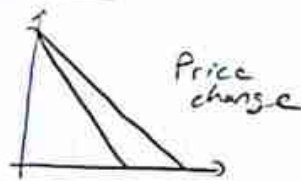
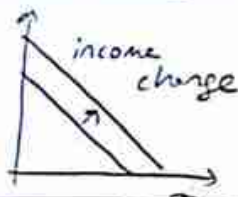
Ex: $I=100$ $P_f=2$ $P_e=2$

If the agent consumes more than 20 units of e , there is a tax of $t_1=0.5$ for each additional unit and if the agent consumes more than 40 units of e , additional tax $t_2=1$

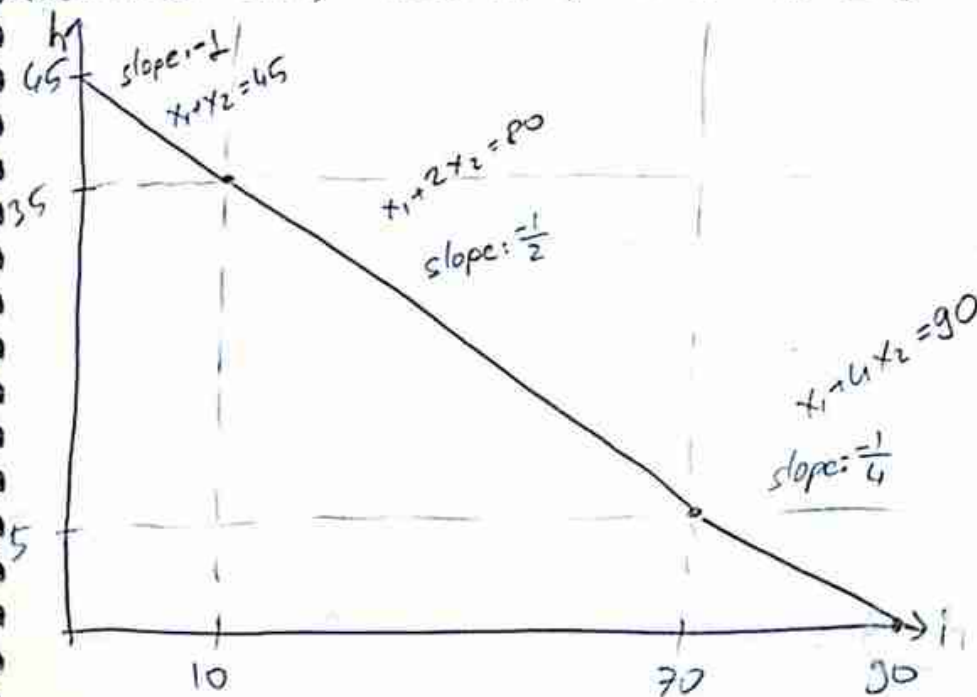


* Aynı anda 2 oranı vergi veya subsidy gelirse?
 * Proportional tax olursa? falan

Budget Shifts:



Ex: Rengin's income is 100 ₺ . She can buy first 10 i with $P_i=2\text{₺}$ each. For each additional i , there is a discount and the price is $P_i=1\text{₺}$ each. Moreover, she can buy first 5 hamburgers at $P_h=4\text{₺}$ each. For each additional unit, discounted price is $P_h=2\text{₺}$



Problem Set 1

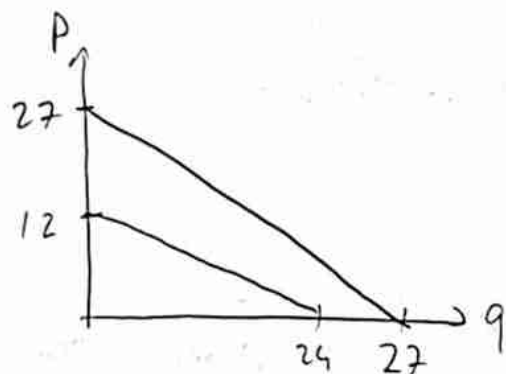
Q1

$$X_1 = 10 - P_x + 0.01 I_1 + 0.4 P_y \quad (\text{demand of person 1})$$

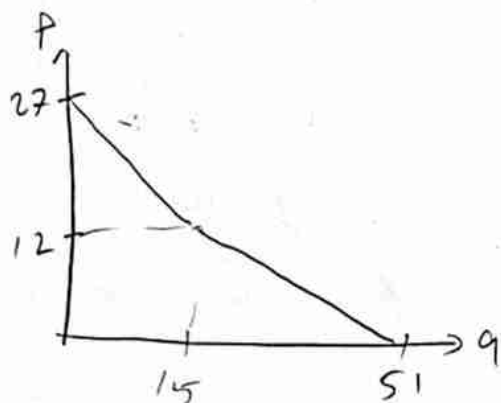
$$X_2 = 5 - P_x + 0.02 I_2 + 0.2 P_y \quad (\text{" " " 2})$$

a) Graph demand curves ($I_1 = 1000$, $I_2 = 1000$, $P_y = 10$)

$$X_1 = 24 - 2P_x \quad X_2 = 27 - P_x$$



b) Graph total market demand



$$X = \begin{cases} 51 - 3P_x & \text{if } 0 \leq P_x \leq 12 \\ 27 - P_x & \text{if } 12 < P_x \leq 27 \\ 0 & \text{if } P_x > 27 \end{cases}$$

Q2

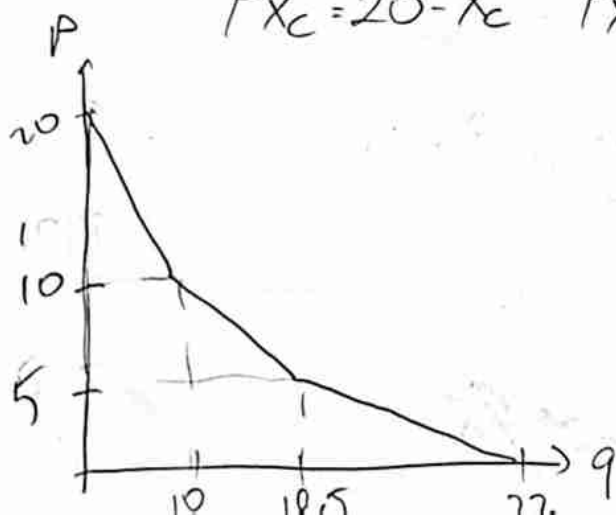
$$P_{X_A} = 5 - X_A$$

$$P_{X_B} = 10 - 5X_B$$

a) Graph market demand

$$P_{X_C} = 20 - X_C$$

$$P_{X_D} = 10 - 2X_D$$



$$X = \begin{cases} 32 - 2.7P & 0 \leq P \leq 5 \\ 27 - 1.7P & 5 < P \leq 10 \\ 20 - P & 10 < P \leq 20 \\ 0 & 20 < P \end{cases}$$

b) Compare elasticity

$0 \leq P \leq 5$	$5 < P \leq 10$	$10 < P \leq 20$
$18.5 \leq Q \leq 32$	$10 \leq Q < 18.5$	$0 \leq Q \leq 10$
$0 \leq \frac{P}{Q} \leq \frac{5}{18.5}$	$\frac{5}{18.5} < \frac{P}{Q} < \frac{10}{10}$	$\frac{10}{10} < \frac{P}{Q} < \frac{20}{0}$
$\frac{dQ}{dP} = -2.7$	$\frac{dQ}{dP} = -1.7$	$\frac{dQ}{dP} = -1$
$0 \leq \epsilon \leq \frac{17.5}{18.5}$	$\frac{8.5}{18.5} \leq \epsilon < 1.7$	$1 < \epsilon < \infty$

c) $S = -6 + \frac{3}{2}P$ find equilibrium

try $(0 \leq P \leq 5)$ $32 - 2.7P = -6 + \frac{3}{2}P \rightarrow P \approx 7.61$
not in the range

try $(5 < P \leq 10)$ $27 - 1.7P = -6 + \frac{3}{2}P \rightarrow P \approx 10.3$
not in range

try $(10 < P \leq 20)$ $20 - P = -6 + \frac{3}{2}P \rightarrow P \approx 10.4$ ✓

d) Not possible as long as $dQ/dP \leq 0$ for all individuals

Q3 $Q = \beta P^\alpha$

a) $\beta > 0, \alpha < 0$

b) $\epsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = \beta \alpha \cdot \frac{P^{\alpha-1}}{\beta P^\alpha} = \alpha$

c) $Q = Y P^\alpha I^\beta$ $Y > 0, \alpha < 0, \beta > 0$
Income

d) $\epsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = Y \alpha P^{\alpha-1} I^\beta \cdot \frac{P}{Y P^\alpha I^\beta} = \alpha$

$\epsilon_I = \frac{dQ}{dI} \cdot \frac{I}{Q} = Y P^\alpha \beta I^{\beta-1} \cdot \frac{I}{Y P^\alpha I^\beta} = \beta$

Q4)

$$E_d(\text{general}) = -0.45$$

$$E_d(\text{teen}) = -0.70$$

$$E_s = 7.00$$

$$P = 5.51\$$$

$$\text{Sales} = 17.4B$$

a) Estimate supply and demand functions

$$Q_d = a + bP$$

$$Q_s = c + dP$$

$$E_d = \frac{dQ}{dP} \cdot \frac{P}{Q} = b \cdot \frac{P}{Q_d} = b \cdot \frac{5.51}{17.4} = -0.45$$

$$b = -1.42$$

$$E_s = \frac{dQ}{dP} \cdot \frac{P}{Q} = d \cdot \frac{5.51}{17.4} = 7.00$$

$$d = 22.1$$

$$a = Q - bP = 25.2$$

$$c = Q - dP = -104.4$$

$$Q_d = 25.2 - 1.42P$$

$$Q_s = -104.4 + 22.1P$$

b) $E_q = ?$ $Q_s = Q_d \rightarrow P = 5.51\$$ $Q = 17.4B$

c) Find supply curve with 1\$ unit tax

$$P = \frac{Q_s + 104.4}{22.1} \quad P^{tax} = \frac{Q_s + 104.4}{22.1} + 1$$

$$Q_s^t = -126.5 + 22.1P$$

d) $Q_s^t = Q_d \rightarrow P = 6.45\$$ $Q = 16.4B$ %6 fall

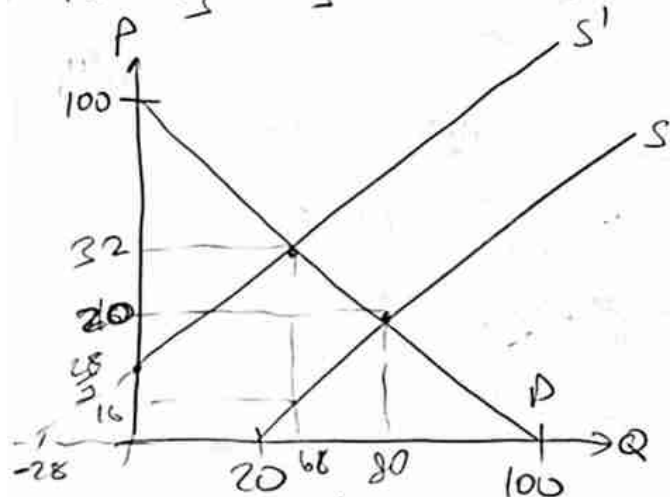
1.6 price increase

Q5)

$$Q_d = 100 - P \quad Q_s = 20 + 3P$$

a) Unit tax 16\$ imposed on the supplier

$$P_s = \frac{Q_s}{3} - \frac{20}{3} \quad P_s^* = \frac{Q_s}{3} - \frac{20}{3} + 16 = \frac{Q_s}{3} + \frac{28}{3}$$



$$P_b = 20 \quad P_b^* = 32$$

$$P_s = 20 \quad P_s^* = 16 + 16 \text{ to gov}$$

$$\text{tax revenue} = 16\$ \cdot 68 = 1088\$$$

tax burdens:

$$\text{buyer} \rightarrow (32 - 20) \cdot 68 = 816\$$$

$$\text{seller} \rightarrow (20 - 16) \cdot 68 = 272\$$$

b) 16\$ unit tax imposed on buyers

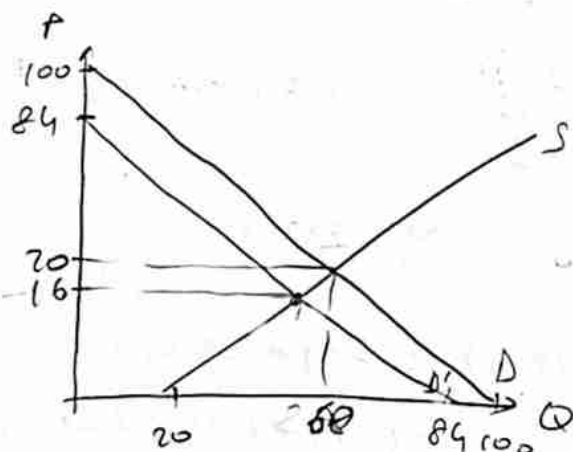
$$Q_d = 100 - P \quad P_d = 100 - Q_d \quad P_d^* = 100 - Q_d - 16 = 84 - Q_d$$

$$Q_d^* = 84 - P$$

$$Q_s = 20 + 3P$$

$$84 - P = 20 + 3P$$

$$4P = 64 \quad P = 16 \quad Q = 68$$



$$P_b = 20 \quad P_b^* = 16 + 16\$ \text{ to gov}$$

$$P_s = 20 \quad P_s^* = 16$$

$$\text{Tax revenue} = 68 \cdot 16 = 1088\$$$

Tax burden

$$\text{buyers} = 12 \cdot 68$$

$$\text{sellers} = 4 \cdot 68$$

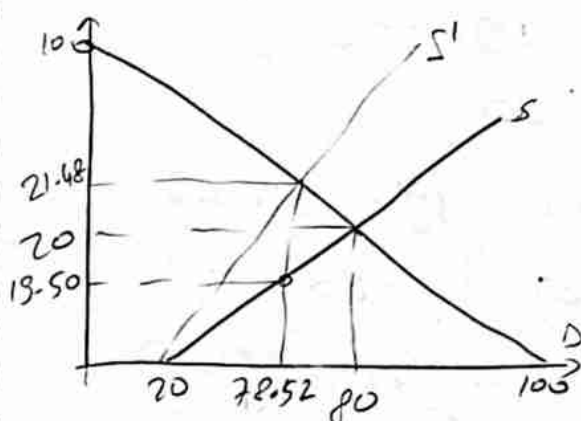
c) 10% tax imposed on supplier

$$Q_s = 20 + 3P \quad P = \frac{Q_s - 20}{3} \quad P = \frac{Q_s - 20}{3} \cdot 1.10$$

$$P^* = \frac{Q_s - 20}{3} \cdot 1.10 \quad Q_d = 100 - P \quad P^* = 0.367Q - 7.733 = 100 - Q$$

$$1.367Q = 107.733$$

$$Q = 78.52$$



$$P_b = 20 \quad P_b^* = 21.48$$

$$P_s = 20 \quad P_s^* = 19.50$$

$$\text{Tax revenue} = 78.52 \times 21.48$$

$$\text{buyer burden} = (21.48 - 20) \cdot 78.52$$

$$\text{seller burden} = (20 - 19.50) \cdot 78.52$$

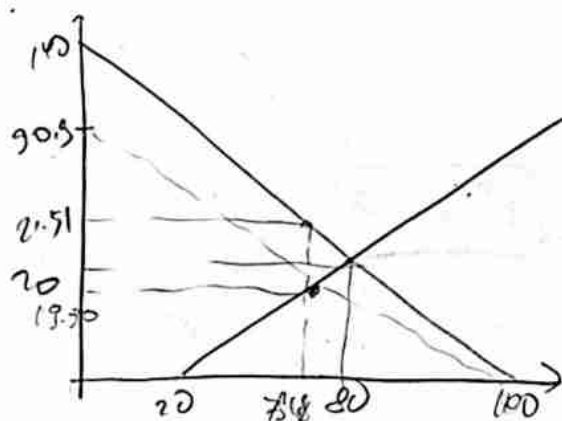
d) 10% tax imposed on buyers

$$P = 100 - Q_d \quad P^* = 90.91 - 0.91Q_d \quad P^s = \frac{Q - 20}{3}$$

$$272.73 - 2.73Q = Q - 20$$

$$3.73Q = 292.73$$

$$Q = 78.48$$



$$P_b = 20 \quad P_b^* = 21.51$$

$$P_s = 20 \quad P_s^* = 19.50$$

Same

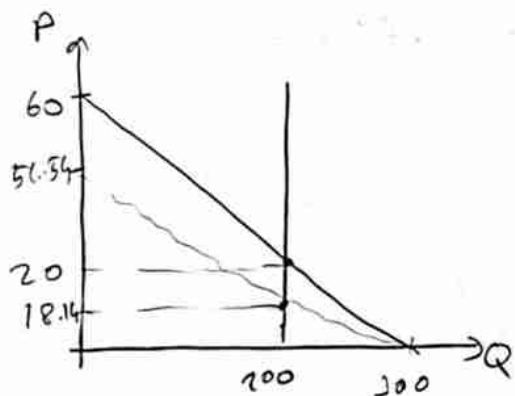
Q6 | $Q_d = 300 - 5p$

a) 10% sales tax supply is perfectly inelastic (buyers pay)

$Q_s = 200$

$P_{eq} = 20 \quad Q_{eq} = 200$

$$P_d = \frac{60 - \frac{Q_d}{5}}{1.1} = 54.54 - 0.182 Q_d$$



$P_{eq}^* = 18.14 \quad Q_{eq}^* = 200$

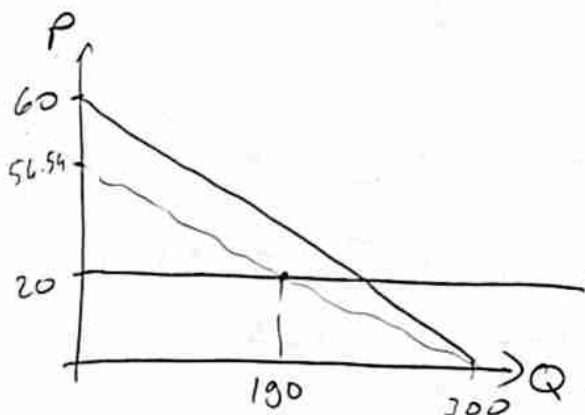
$DWL = (20 - 18.14)(200 - 0) = 364$

$P^b = 20 \quad P^b = 18.14 + 1.86 = 20$
buyers pay same

$P^s = 20 \quad P^s = 18.14$

seller burden = $(20 - 18.14) \cdot 200$

c) perfectly elastic ($P^s = 20$)



$54.54 - 0.182 Q_d = 20$

$Q_d^* = 190$

$P^b = 20 \quad P_b^* = 20 + \frac{\text{tax}}{2}$

$P^s = 20 \quad P_s = 20$

buyer burden = $(22 - 20)(190)$

tax revenue = $(22 - 20)(190)$

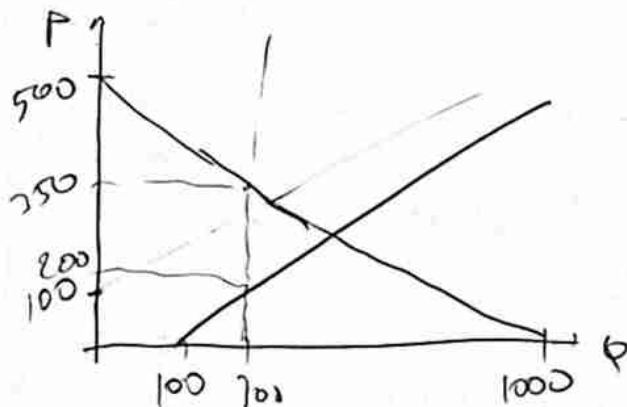
Q7 | $Q_d = 1000 - 2P$

$Q_s = 100 + P$

$P_{eq} = 300$

$Q_{eq} = 400$

b) $Q_{quota} = 300$ units



$P^* = 250$

Q8 | True / False

- 1) True $\alpha \epsilon_d < 1$ increase in price will increase total spending
- 2) False, if unit elastic, total spending would be same
- 3) False, drug interdiction is a supply side shock
- 4) False
- 5) True

18/10/2024 week 3

Ch 3: Preferences

Given any consumption bundles (x_1, x_2) and (y_1, y_2) the consumer can rank these bundles as to their desirability

$(x_1, x_2) \succ (y_1, y_2) \rightarrow$ Bundle (x_1, x_2) is strictly better than (y_1, y_2)

$(x_1, x_2) \sim (y_1, y_2) \rightarrow$ Bundles are indifferent

$(x_1, x_2) \succeq (y_1, y_2) \rightarrow$ Bundle x is at least as good as y

if $x \succeq y$ and $y \succeq x$ then $x \sim y$

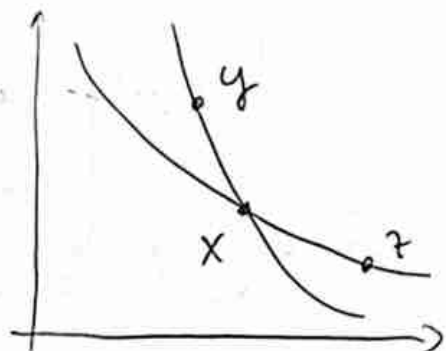
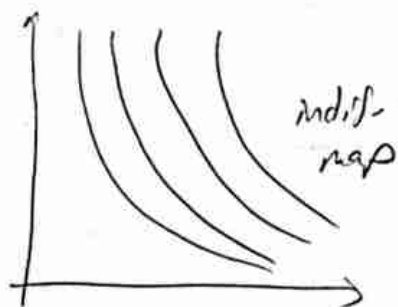
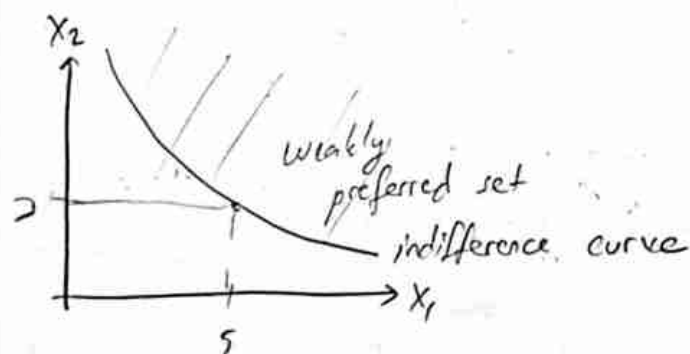
if $x \succeq y$ and not $y \succeq x$ then $x \succ y$

Assumptions about preferences:

* complete: we either have $x \succeq y$ or $y \succeq x$ or both

* transitivity: $[x \succeq y \text{ and } y \succeq z] \Rightarrow x \succeq z$

rock paper scissors situation:

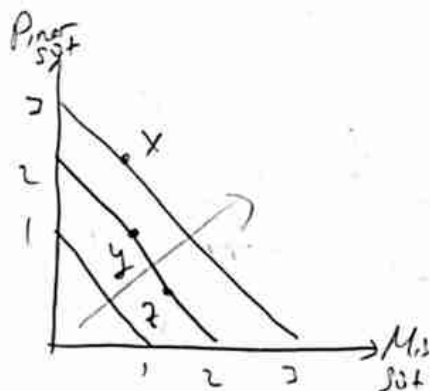


it means $x \sim y$, $x \sim z$

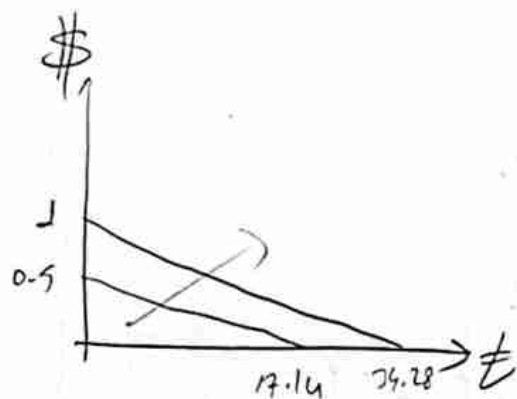
transitivity 'ye gone! $y \sim z$ obviously,
but they are not in the same
indif. curve

* Indif. curves cannot cross

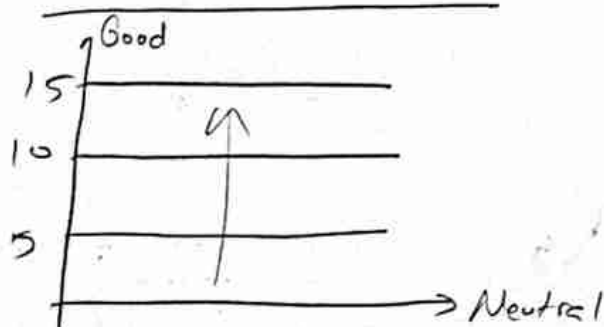
Perfect substitutes: The consumer is willing to substitute one good for another at a constant rate



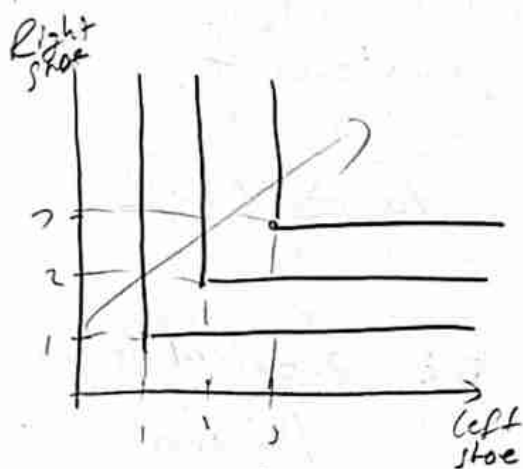
$$U(x, y) = y + 2x$$



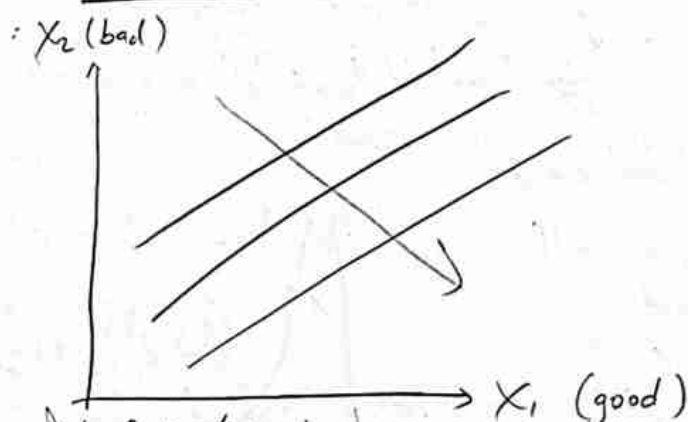
Neutral Goods



Perfect complements:

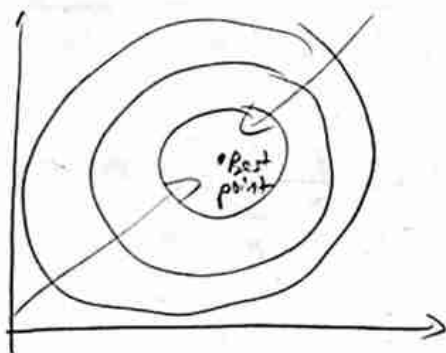


Bad Goods

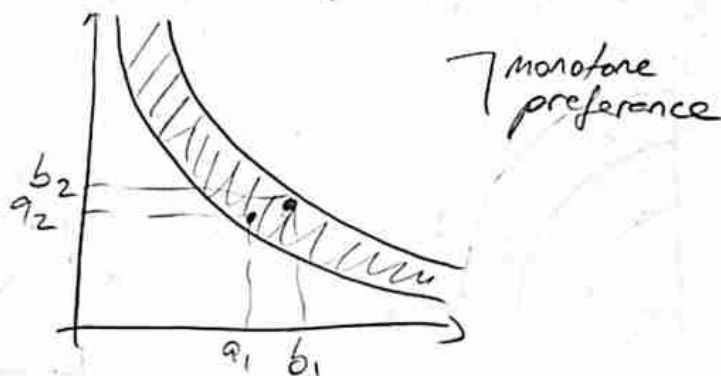


İhtiyaçları karşılamak için malı tüketilmeli
daha çok iyi mal tüketilmeli

Satiation

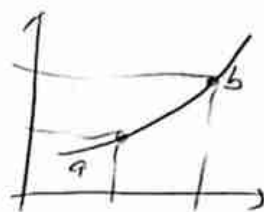


Thick Bundles



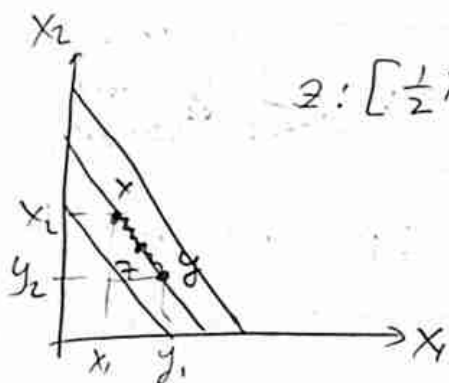
Monotonic preferences: Suppose $x = (x_1, x_2)$ $y = (y_1, y_2)$ are such that $x_1 \geq y_1, x_2 \geq y_2, x \neq y$ then $x \succ y$
 *More is better

$$x = (4, 3) \succ y = (1, 2)$$



b'de daha 902
 mal olmasina
 ragmen U agni
 not/monotone

Convex Preference



$$z: \left[\frac{1}{2}x_1 + \frac{1}{2}y_1, \frac{1}{2}x_2 + \frac{1}{2}y_2 \right]$$

Suppose we have $(x_1, x_2) \sim (y_1, y_2)$
 $t \in (0, 1)$, then

$$[tx_1 + (1-t)y_1, tx_2 + (1-t)y_2]$$

X ile y arasinda bir nokt

X veya y 'den - at least daha iyi' olmal

$\exists$: there exists

$\forall$: for all

$\wedge$: and

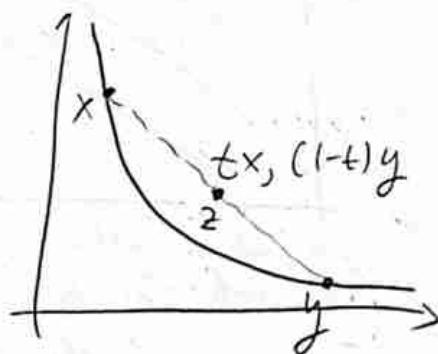
$\vee$: or

$\neg$: not

$$(x_1, x_2) \sim (y_1, y_2)$$

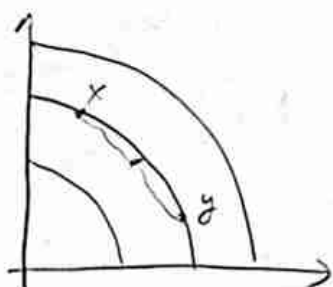
whatever t is
 utility same

$$\forall z \succeq tx \sim ty$$



$\forall t, z = tx + (1-t)y \succ x \sim y$
 convex function

Concave function:

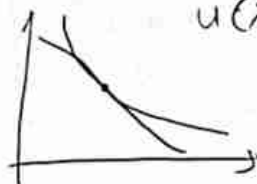


Marginal Rate of Substitution

How can we understand a function
 is concave or convex

Second derivative positive convex

$$u(x, y) = x^\alpha y^\alpha$$

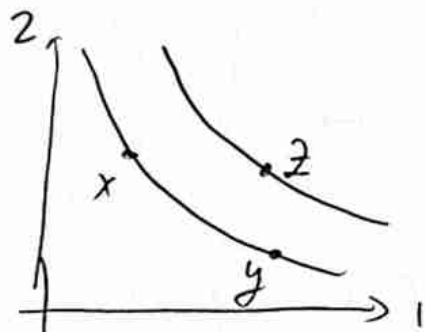


slope = MRS

24/10/2024 week 4

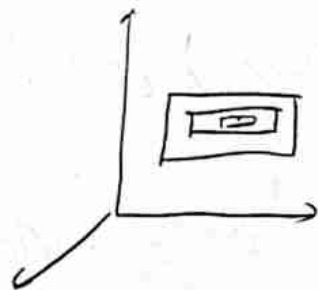
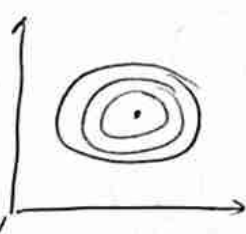
Ch 4. Utility

If utility function $U(x_1, x_2)$ represents the preference relation ($\succ$) then $(x_1, x_2) \succ (y_1, y_2) \Leftrightarrow U(x_1, x_2) > U(y_1, y_2)$



$$U(x) = U(y) = 5 \quad z = U(x, y)$$

Imagine a donut, and cut a slice. It will create a circle. If you increase the height, circle gets smaller. 3 boyutlu grafik çalıs



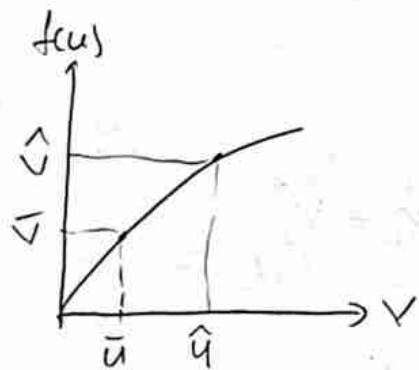
piramit est' mid-term sorusu

★ A monotonic transformation is a way of transforming one set of numbers into another set of numbers in a way that preserves order of the numbers.

$f(U)$ is a monotonic transformation of U :

$$U(x) > U(y) \Leftrightarrow f(U(x)) > f(U(y))$$

ex: $V = 2U$ if multiply with 2, order would be same



$$U = X_1 X_2 \xrightarrow{\text{monotonic transformation}} V = X_1^2 X_2^2$$

or $V' = X_1^2 X_2^3$ order doesn't change

If $f(u)$ is any monotonic transformation of a utility function, that represents some differences, then

$f(u(x_1, x_2))$ is also a utility function representing the same preferences

Proof: $U(x_1, x_2)$ represents some preferences

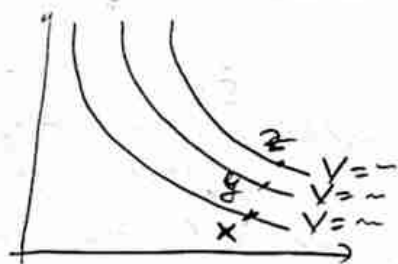
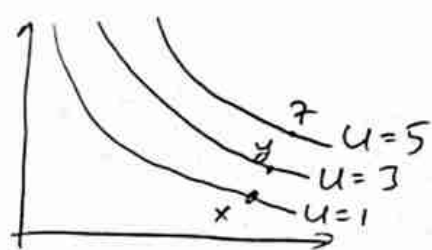
$$1) U(x_1, x_2) > U(y_1, y_2) \Leftrightarrow (x_1, x_2) \succ (y_1, y_2)$$

2) $f(u)$ is a monotonic transformation

$$U(x_1, x_2) > U(y_1, y_2) \Leftrightarrow f(U(x_1, x_2)) > f(U(y_1, y_2))$$

$$3) f(U(x_1, x_2)) > f(U(y_1, y_2)) \Leftrightarrow (x_1, x_2) \succ (y_1, y_2)$$

ex/ $U = x_1 \cdot x_2 \quad V = x_1^2 \cdot x_2^2$



$$U(10, 1) > U(2, 2)$$

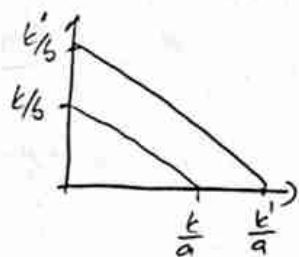
$$V(10, 1) > V(2, 2)$$

Value of utility u changes but the preference doesn't change

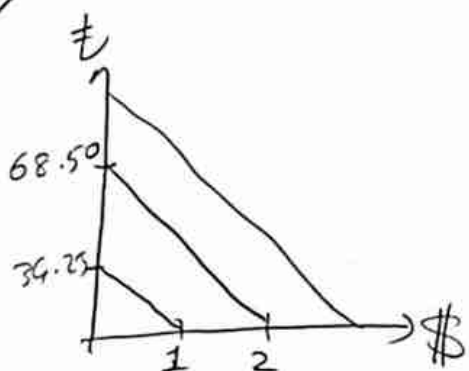
Perfect Substitutes

$U = ax_1 + bx_2$ that is perfect substitutes

$$U = k = ax_1 + bx_2 \quad (0, k/b) \quad (k/a, 0)$$



Ex/



$U(x_1, x_2) = 34.25 X_1 + X_2$
utility is the £ equivalent of your portfolio

$$V(x_1, x_2) = X_1 + \frac{1}{34.25} X_2$$

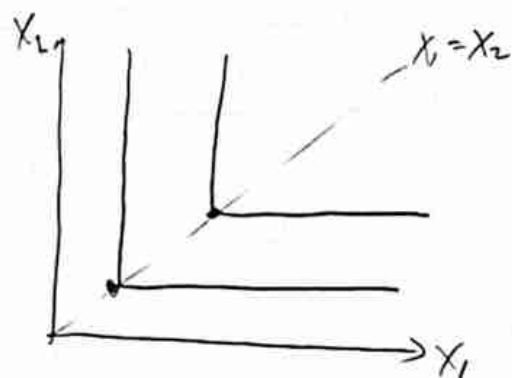
\$ equivalent of the portfolio

25/10/2024 week 4

$$U(X_1, X_2) = aX_1 + bX_2 \quad (\text{perfect substitutes}) \rightarrow$$



$$U(X_1, X_2) = \min\{X_1, X_2\}$$



if $X_2 \geq X_1$, then vertical indifference curve

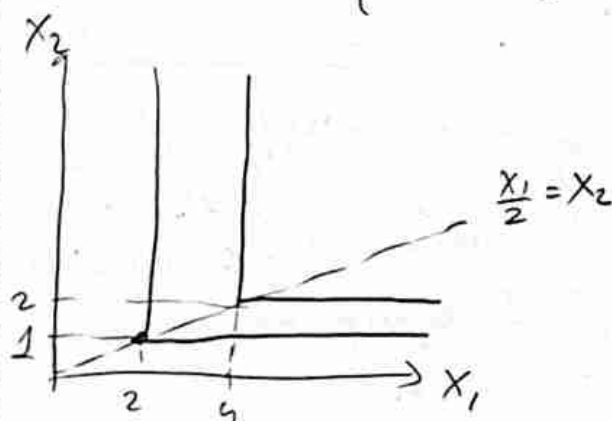
$$U(X_1, X_2) = X_1$$

if $X_1 \geq X_2$, then horizontal indifference curve

$$U(X_1, X_2) = X_2$$

$$U(X_1, X_2) = \min\left\{\frac{X_1}{2}, X_2\right\}$$

2 sugar cubes + 1 glass of tea



$$\text{if } \frac{X_1}{2} < X_2, U = \frac{X_1}{2}$$

$$\text{if } \frac{X_1}{2} > X_2, U = X_2$$

since monotonic transformation

$$V(X_1, X_2) = \min(X_1, 2X_2)$$

$$(5, 1) \sim (2, 1)$$

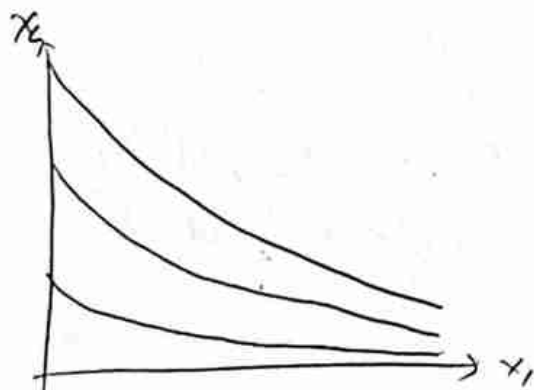
$$(5, 2) \sim (10, 2)$$

Quasi-Linear Preferences

$$U = V(X_1) + X_2$$

$$\text{Ex: } U = X_1^2 + X_2$$

$$U = \sqrt{X_1} + X_2$$



Other preferences with kinks

$$U(X_1, X_2) = \min \{X_1 + 2X_2, 2X_1 + X_2\}$$

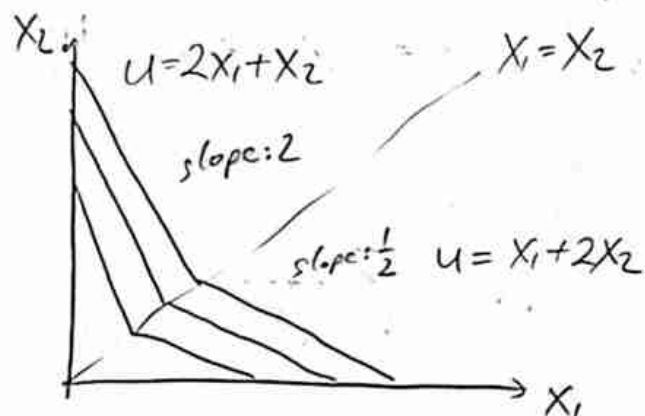
$$U = X_1 + 2X_2 \quad \text{if} \quad X_1 + 2X_2 \leq 2X_1 + X_2$$

$$U = 2X_1 + X_2 \quad \text{if} \quad X_1 + 2X_2 \geq 2X_1 + X_2$$

$$X_1 + 2X_2 = 2X_1 + X_2$$

$$X_1 = X_2$$

critical line

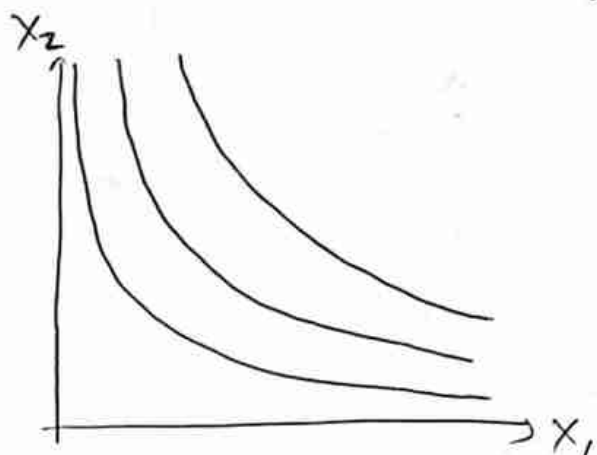


Cobb-Douglas Preferences

$$U(X_1, X_2) = X_1^c X_2^d$$

monotonic
transformation

$$c \ln X_1 + d \ln X_2 = V(X_1, X_2)$$

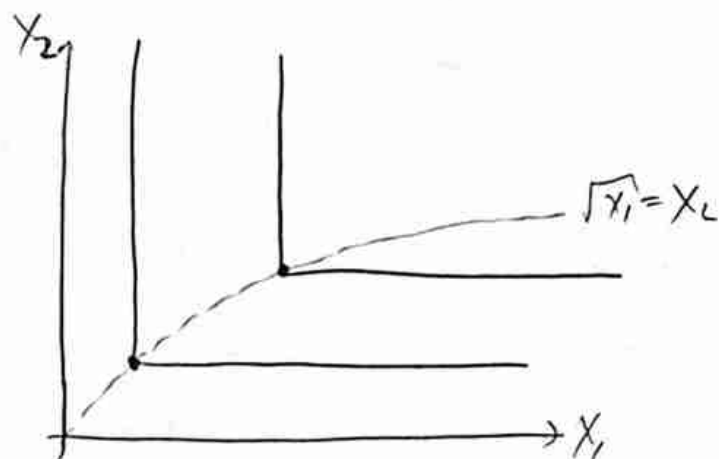


$$(Hw) \quad U(X_1, X_2) = \min \{\sqrt{X_1}, X_2\}$$

$$\sqrt{X_1} = X_2$$

$$\text{if } \sqrt{X_1} > X_2, \quad U = X_2$$

$$\text{if } \sqrt{X_1} < X_2, \quad U = \sqrt{X_1}$$



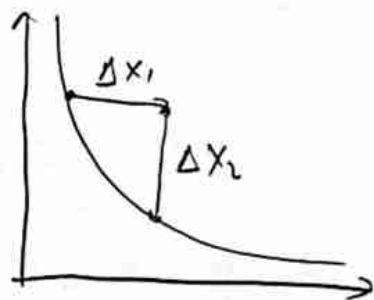
Marginal Utility

$$MU_1 = \frac{\Delta U_1}{\Delta X_1} = \frac{U(X_1 + \Delta X_1, X_2) - U(X_1, X_2)}{\Delta X_1} \quad MU_1 = \frac{\partial U(x)}{\partial X_1}$$

$$\Delta U_1 = MU_1 \Delta X_1$$

Marginal rate of substitution

Consider a change in consumption of each good $(\Delta X_1, \Delta X_2)$ that keeps utility constant



$$MU_1 \Delta X_1 + MU_2 \Delta X_2 = 0$$

$$\frac{MU_1}{MU_2} = \frac{-\Delta X_2}{\Delta X_1} = MRS = \frac{\partial U / \partial X_1}{\partial U / \partial X_2}$$

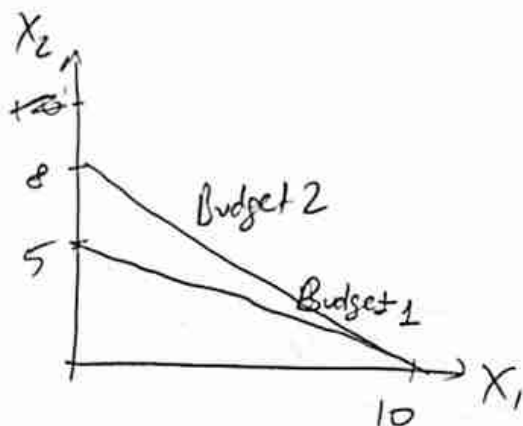
Ex: $\min\{X_1, X_2\}$ there is no MRS. Function is not differentiable

HW: Is this function continuous at all points?

There is no jump. So function is continuous

Problem Set 2

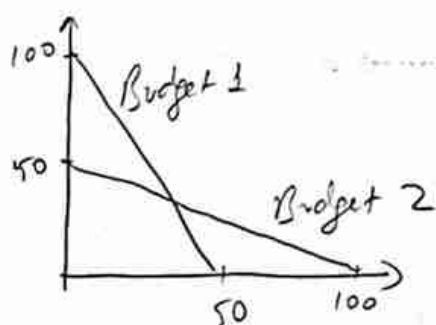
Q1) $P_1 = 10\$$ $P_1' = 20\$$ $I = 100\$$ $I' = 200\$$
 $P_2 = 20\$$ $P_2' = 25\$$



Budget 2 is better

Q2) $P_x = 2$, $P_y = 1$, $m = 100$

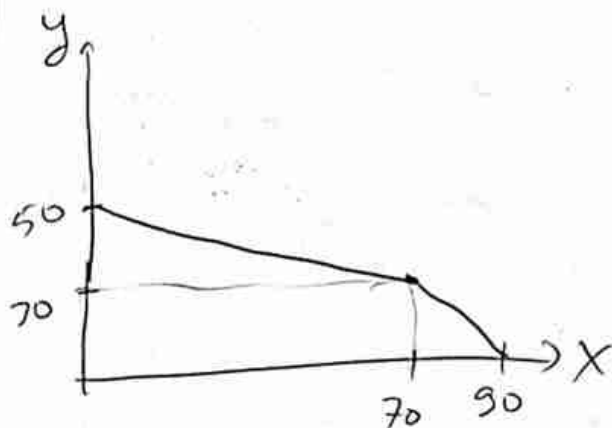
a) $P_x' = 1$, $P_y' = 2$



b) +0.50 tax for each unit of X beyond 70 units

$P_x' = 1$, $X \leq 70$

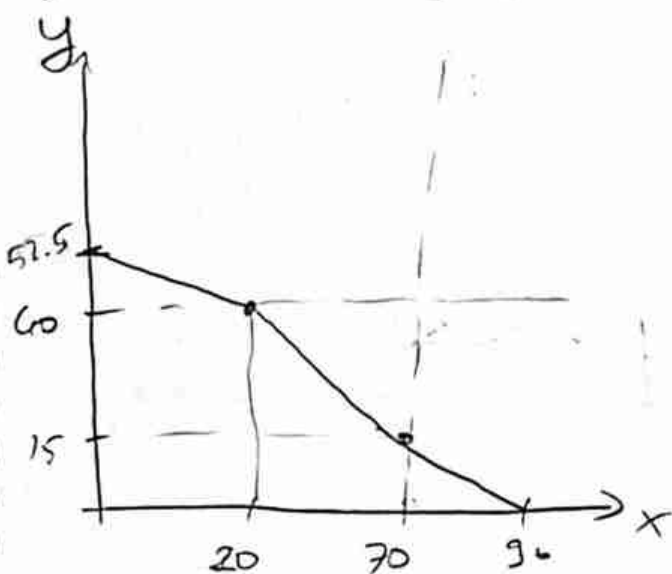
$P_x = 1.5$, $X > 70$



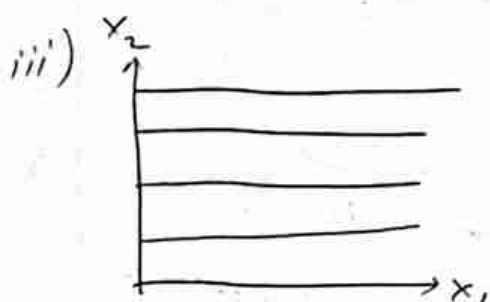
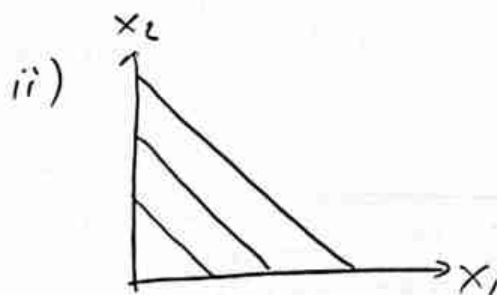
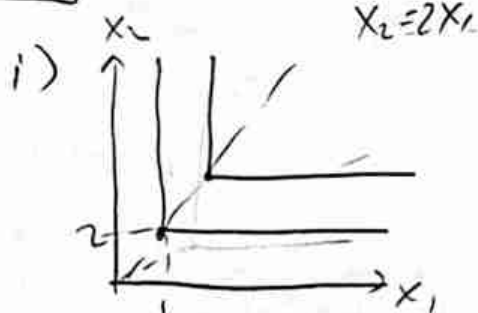
c) -0.40 subsidy for each unit of y beyond 40 units.

$$P_y' = 2 \rightarrow P_y'' = 1.6$$

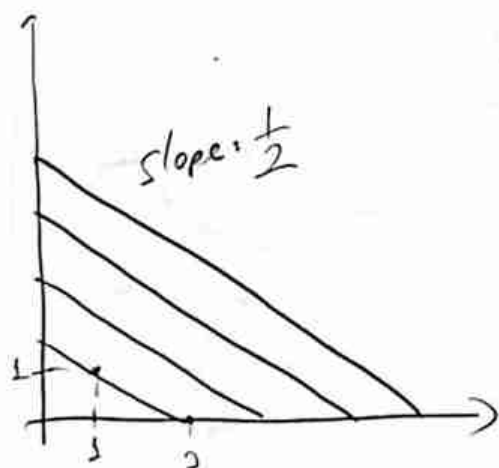
$$P_x' = 1 \rightarrow P_x'' = 1.5$$



Q3 a)



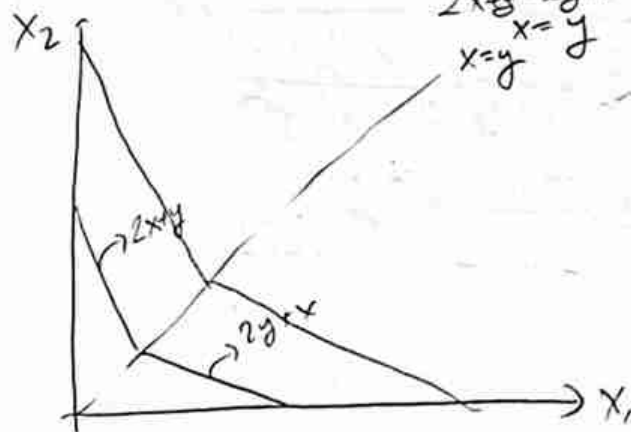
b) i) $U(x,y) = x + 2y$



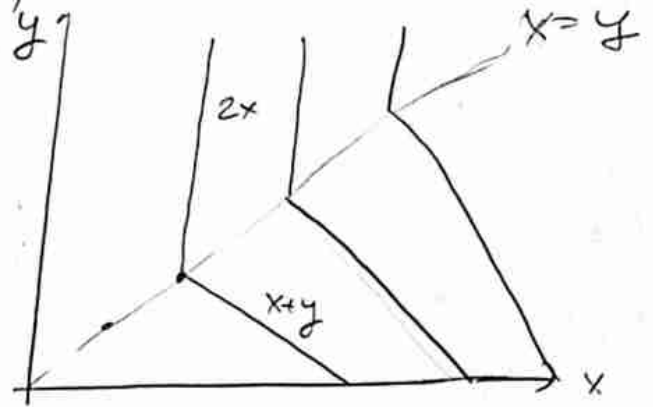
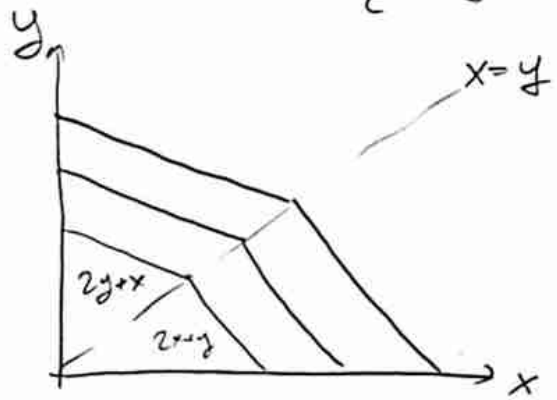
ii) $U = \min \{2x+y, 2y+x\}$

$$2x+y = 2y+x$$

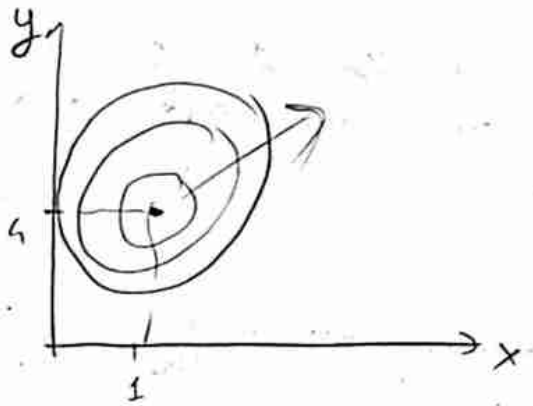
$$x=y$$



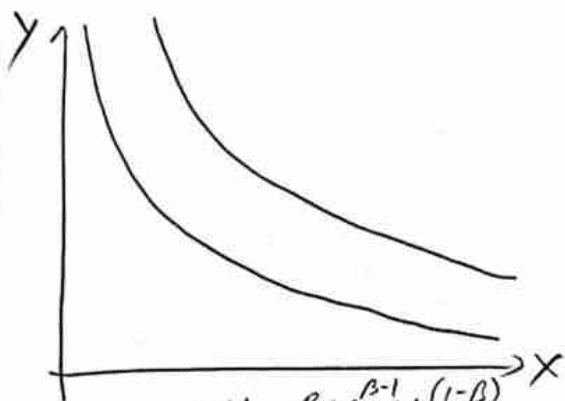
iii) $U(x,y) = \max \{2x+y, 2y+x\}$ iv) $U(x,y) = X + \min\{x,y\}$



v) $U(x,y) = (x-1)^2 + (y-4)^2$



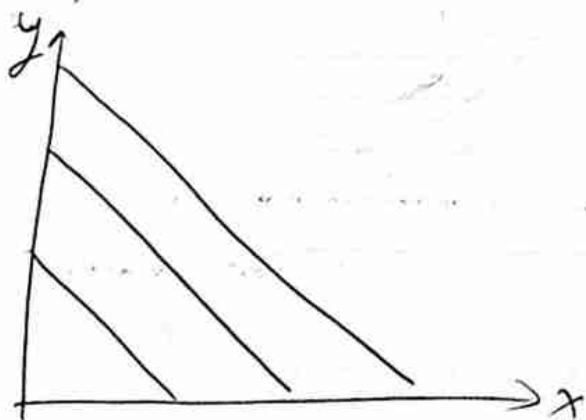
c) i) $U(x,y) = x^\beta y^{(1-\beta)}$



$$MRS = \frac{MU_x}{MU_y} = \frac{\beta x^{\beta-1} y^{(1-\beta)}}{(1-\beta) x^\beta y^{(-\beta)}}$$

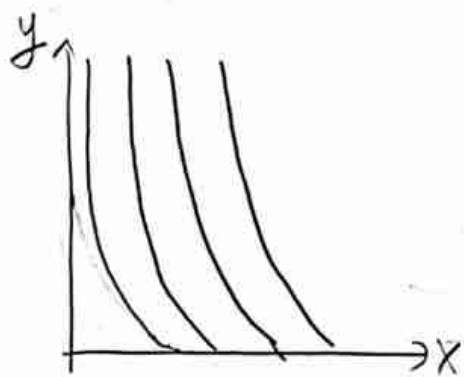
$$MRS = \frac{\beta}{1-\beta} \cdot x^{-1} \cdot y$$

ii) $U(x,y) = ax + by$



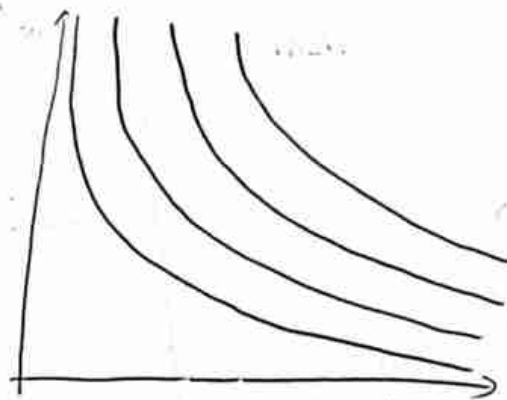
$$MRS = \frac{a}{b}$$

$$\text{iii) } U(x, y) = \ln x + y$$



$$MRS = \frac{1}{x}$$

$$\text{iv) } U(x, y) = a \ln x + b \ln y$$

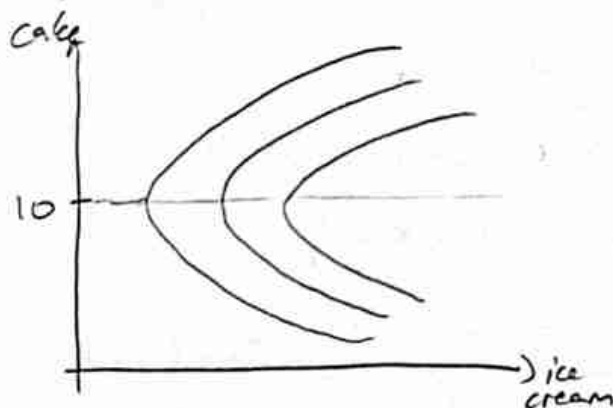


$$MRS = \frac{\frac{a}{x}}{\frac{b}{y}} = \frac{ay}{bx}$$

Q4 |

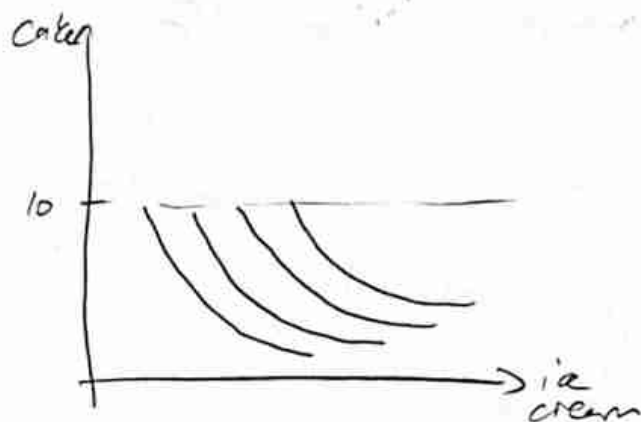
Ghi likes cake and icecream. But after 10 slice of cake, additional slice makes her less happy. Ghi always prefer more ice cream. Ghi must finish what she has

a) Graph



should consume more ice cream to compensate the negative effect of excess cake

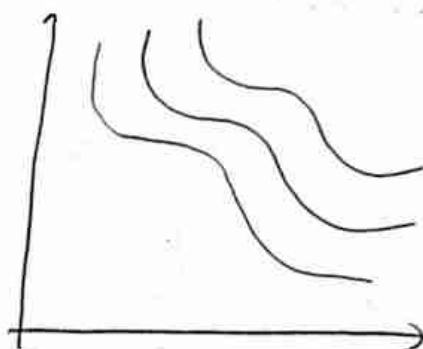
b) She mustn't finish her plate.



Q7

a) Convex but not monotonic

b) monotonic but not convex



Q8

Set of bundles $B = \{B_1, B_2, B_3, B_4, B_5\}$

$$B_1 \succ B_2 \sim B_3 \quad B_4 \succ B_5$$

a) No, we don't know the preferences between any of B_1, B_4, B_5 and any of B_2, B_3 . Once we have the info $B_5 \succ B_1$, we can rank as $B_4 \succ B_5 \succ B_1 \succ B_2 \sim B_3$. Assumption: preferences are transitive

Q9

$$U(x, y) = x + y$$

a) Show that $V(x, y) = 10x + 10y$ represent the same preferences
suppose $x = (x_1, x_2)$, $y = (y_1, y_2)$ and $x \succ y$
2 points

$$U(x_1, x_2) > U(y_1, y_2) \text{ then } V(x_1, x_2) > V(y_1, y_2)$$

$$10x_1 + 10x_2 > 10y_1 + 10y_2 \text{ monotonic transformation}$$

b) show that no monotonic transformation with $Z = x + y^2$

$(1, 1)$ and $(0, 2)$ bundles

$$\begin{array}{llll} U(1, 1) = 2 & U(0, 2) = 2 & \longrightarrow & (1, 1) \sim (0, 2) \\ Z(1, 1) = 2 & Z(0, 2) = 4 & \longrightarrow & (0, 2) \succ (1, 1) \end{array} \quad \text{no monotonic transformation}$$

Q10) $u(x,y) = xy$
 $v(x,y) = x^2y^2$
 $z(x,y) = \ln x + \ln y$

show that they all have same MRS. Why?

$$\frac{MU_1}{MU_2} = \frac{y}{x} \quad \frac{MV_1}{MV_2} = \frac{2xy^2}{x^22y} = \frac{y}{x} \quad \frac{Mz_1}{Mz_2} = \frac{\frac{1}{x}}{\frac{1}{y}} = \frac{y}{x}$$

since they are monotonic transformation of each other

Q11) guns and bangos $u(X_g, X_b) = X_g + \sqrt{X_b}$

a) $u(X_g, X_b) = X_g + \sqrt{X_b} = \bar{u} \quad X_g = \bar{u} - \sqrt{X_b}$

b) $\frac{MU_b}{MU_g} = \frac{\frac{1}{2}X_b^{-1/2}}{1} = \frac{1}{2\sqrt{X_b}}$

c) Convex? In other words, determine whether there is diminishing MRS property.

MRS is decreasing since $\frac{1}{2\sqrt{X_b}}$ is decreasing as X_b increases.

or, we can check the second derivative

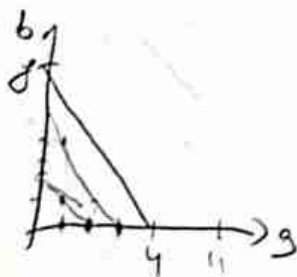
$\frac{dX_g}{dX_b} = -\frac{1}{2}X_b^{-1/2}$, then $\frac{d^2X_g}{d^2X_b} = \frac{1}{4}X_b^{-3/2}$. Second derivative is positive, means, positive = convex slope

d) Show that for a budget constraint $P_b X_b + P_g X_g = I$ at any point of tangency between budget line and indifference curves, it must be the case that $X_b = \frac{1}{4} \left(\frac{P_g}{P_b} \right)^2$

Tangency is equivalent to $MRS = \frac{P_b}{P_g}$ then $\frac{1}{2}X_b^{-1/2} = \frac{P_b}{P_g}$

$X_b^{-1/2} = \frac{2P_b}{P_g} \quad X_b^{1/2} = \frac{P_g}{2P_b} \quad X_b = \frac{P_g^2}{4P_b^2} \quad \checkmark$

e) $P_g = 2\$$, $P_b = 1\$$, $m = 8\$$



Since the function is monotone, convex, continuous and differentiable, tangency condition works (interior solution)

$X_b^* = \frac{1}{4} \cdot \left(\frac{2}{1} \right)^2 = 1$, $X_g = 7/2$

$$f) P_g = 5\$ \quad P_b = 2\$ \quad m = 4\$$$

$X_b^* = \frac{1}{4} \left(\frac{P_g}{P_b} \right)^2 = \frac{1}{4} \cdot 5^2 = \frac{25}{4} \rightarrow$ not in budget line. This implies $\sim$ amount of guns. This can't happen. Spend all money on b.

Corner solution

Q 12 $\left| \right.$ $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$

a) Find demand for x_1 and x_2 in terms of m, P_1, P_2 $\alpha > 0$

$$MRS = \frac{\alpha \cdot x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha) x_1^\alpha x_2^{-\alpha}} = \frac{\alpha}{1-\alpha} \cdot \frac{x_2}{x_1} = \frac{P_1}{P_2}$$

... from utility function from budget line

$$x_2 = \frac{P_1}{P_2} \cdot x_1 \cdot \frac{1-\alpha}{\alpha}$$

$$P_1 x_1 + P_2 x_2 = I$$

$$x_1^* = \alpha \frac{m}{P_1}$$

b) Spending ratio

$$\%x_1 = \frac{P_1 x_1^*}{m} = \alpha \quad \%x_2 = \frac{P_2 x_2^*}{m} = 1-\alpha$$

$$x_2^* = (1-\alpha) \cdot \frac{m}{P_2}$$

31/10/2024 week 5

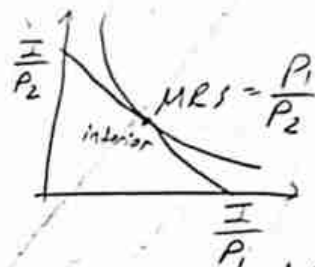
Ch 5. Choice

Consumer choose the best affordable bundle

$$\max U(X_1, X_2) \text{ s.t. } P_1 X_1 + P_2 X_2 \leq I, X_1 \geq 0, X_2 \geq 0$$

interior solution: $X_1^* > 0$ and $X_2^* > 0$

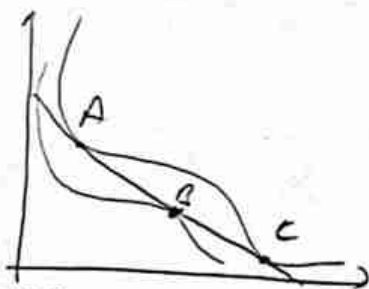
corner solution: either $X_1^* > 0$ and $X_2^* = 0$
or $X_1^* = 0$ and $X_2^* > 0$



If we have interior optimum with smooth indifference curves, then the indif. curve passing through the optimum should be tangent to the budget line (unless preferences are satiated)

- Tangency condition is not sufficient

Ex: (Tangent but not optimal)



Point A and C are optimal.

Point B satisfies tangent condition but it is not optimal

Ex

$$\max U(X_1, X_2, X_3) = X_1 X_2 X_3^2 \text{ s.t. } P_1 X_1 + P_2 X_2 + P_3 X_3 \leq I \quad \begin{matrix} X_1 \geq 0 \\ X_2 \geq 0 \\ X_3 \geq 0 \end{matrix}$$

(1) (1) (2)
tangent power

$$X_1^* = \frac{1}{4} \frac{I}{P_1} \quad X_2^* = \frac{1}{4} \frac{I}{P_2} \quad X_3^* = \frac{2}{4} \frac{I}{P_3}$$

$$P_1 X_1 + P_2 X_2 + P_3 X_3 = I$$

→ Lagrange multiplier

Ex

$$\max U(X_1, X_2) = X_1^2 X_2^3 \rightarrow \text{MRS} = 1$$

$$P_1 X_1 + P_2 X_2 = I \quad X_1^* = \frac{2}{5} \frac{I}{P_1} \quad X_2^* = \frac{3}{5} \frac{I}{P_2}$$

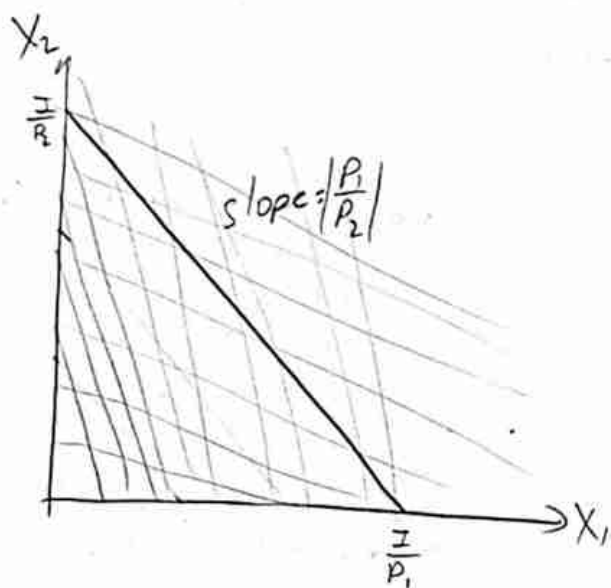
$$\frac{MU_1}{MU_2} = \frac{2 X_1 X_2^3}{3 X_1^2 X_2^2} = \frac{2}{3} \frac{X_2}{X_1} = \frac{P_1}{P_2} \quad X_2 = \frac{P_1}{P_2} \cdot X_1 \cdot \frac{3}{2}$$

$$P_1 X_1 + \frac{P_1}{P_2} \cdot P_2 \cdot X_1 \cdot \frac{3}{2} = I \quad P_1 X_1 \left(1 + \frac{3}{2}\right) = I \quad X_1 = \frac{I}{P_1} \cdot \frac{2}{5}$$

Perfect Substitutes

$$U(X_1, X_2) = aX_1 + bX_2$$

corner solutions



indif slope $\rightarrow \left| \frac{a}{b} \right|$

$$X_1^* = \begin{cases} \frac{I}{P_1} & \text{if } \left| \frac{a}{b} \right| > \frac{P_1}{P_2} \\ 0 & \text{if } \left| \frac{a}{b} \right| < \frac{P_1}{P_2} \end{cases}$$

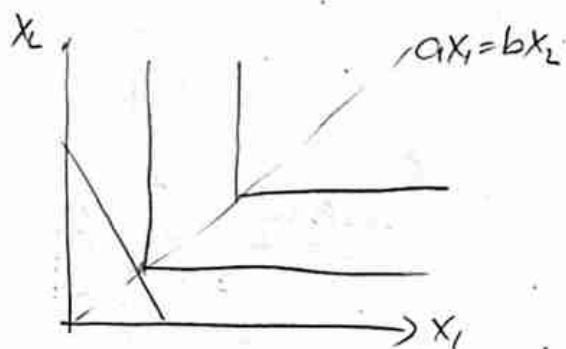
$$X_2^* = \begin{cases} 0 & \text{if } \left| \frac{a}{b} \right| > \frac{P_1}{P_2} \\ \frac{I}{P_2} & \text{if } \left| \frac{a}{b} \right| < \frac{P_1}{P_2} \end{cases}$$

*No tangency condition

if $\left| \frac{a}{b} \right| = \frac{P_1}{P_2}$, any point on the budget line is optimal

Perfect Complements

$$U = \min\{aX_1, bX_2\}$$



$$P_1X_1 + P_2X_2 = I$$

$$aX_1 = bX_2 \Rightarrow X_1 = \frac{b}{a}X_2$$

$$P_1 \cdot \frac{b}{a}X_2 + P_2X_2 = I$$

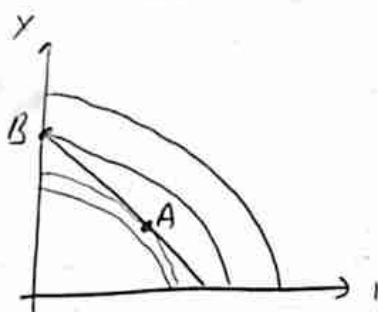
$$X_2 = \frac{I}{\frac{bP_1}{a} + P_2} = \frac{aI}{bP_1 + aP_2}$$

$$X_1 = \frac{bI}{aP_1 + bP_2}$$

Neutrals and Bads

$$U(X_1, X_2) = X_1 - X_2 \rightarrow X_1 = \frac{I}{P_1}, X_2 = 0$$

$$U(X_1, X_2) = X_1 \rightarrow X_1 = \frac{I}{P_1}, X_2 = 0$$

Concave Preferences

A: Tangent condition var fastest optimal degil.

B: Optimal corner solution

$$\text{if } U\left(\frac{I}{P_1}, 0\right) > U\left(0, \frac{I}{P_2}\right) \Rightarrow X = \left(\frac{I}{P_1}, 0\right)$$

Cobb-Douglas Preferences

$$U = X_1^c X_2^d \quad c, d > 0$$

$$1) \max_{X_1, X_2} c \ln X_1 + d \ln X_2 \quad \text{s.t.} \quad \begin{aligned} P_1 X_1 + P_2 X_2 &\leq I \\ P_1 X_1 + P_2 X_2 &= I \end{aligned} \quad \begin{aligned} X_1 &\geq 0 \\ X_2 &\geq 0 \end{aligned}$$

$$2) \max_{X_1, X_2, \lambda} L = c \ln X_1 + d \ln X_2 + \lambda (I - P_1 X_1 - P_2 X_2)$$

$$\frac{\partial L}{\partial X_1} = 0 = \frac{c}{X_1} - \lambda P_1 = 0$$

$$\frac{\partial L}{\partial X_2} = 0 = \frac{d}{X_2} - \lambda P_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = 0 = I - P_1 X_1 - P_2 X_2 = 0$$

$$\begin{aligned} \text{MRS} \\ \frac{c/X_1}{d/X_2} &= \frac{cX_2}{dX_1} = \frac{P_1}{P_2} \end{aligned}$$

$$\boxed{X_2 = \frac{P_1}{P_2} \frac{d}{c} X_1}$$

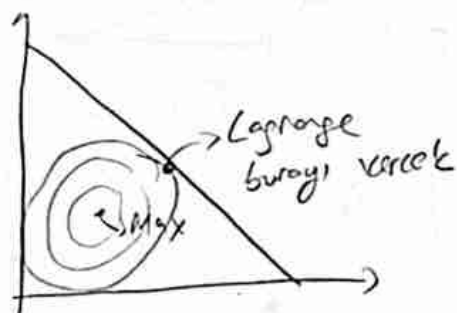
$$I = P_1 X_1 + P_2 X_2$$

$$\boxed{X_1^* = \frac{c}{c+d} \cdot \frac{I}{P_1} \quad X_2^* = \frac{d}{c+d} \cdot \frac{I}{P_2}}$$

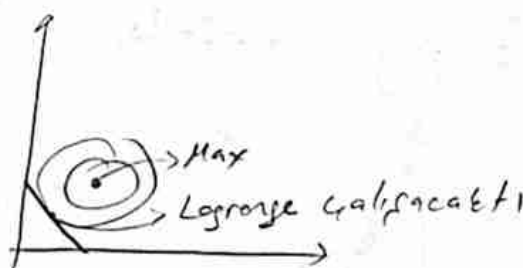
Ex/

$$U(X_1, X_2) = 8X_1 - X_1^2 + 8X_2 - X_2^2 \quad P_1 = P_2 = 1 \quad I = 100$$

$$U = -(X_1 - 4)^2 - (X_2 - 4)^2 + 32 \quad \rightarrow X_1^* = X_2^* = 4$$



Ex: I=2



$$\text{Ex: } U(X_1, X_2) = 20X_1 - 4X_1^2 + X_2 \quad I = 200$$

$$P_1 = 4, P_2 = 1$$

$$X_1^* = ? \quad X_2^* = ?$$

$$\text{convex: } (2, 182)$$

$$MRS = \frac{20 - 8X_1}{1} = \frac{P_1}{P_2} = 4$$

$$20 - 8X_1 = 4 \quad 8X_1 = 16 \quad \boxed{X_1 = 2}$$

$$P_1 X_1 + P_2 X_2 = I \rightarrow \boxed{X_2 = 182}$$

$$\text{Ex: } U(X_1, X_2) = X_2 - (X_1 - 5)^2 = X_2 - X_1^2 + 10X_1 - 25$$

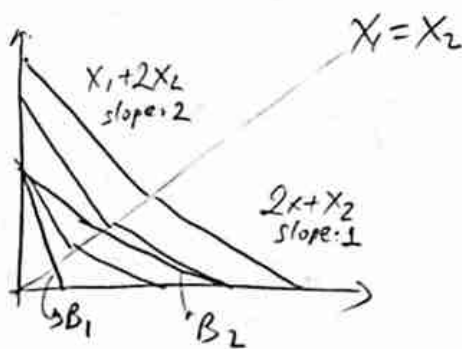
$$X_1^* = 5 - \frac{P_1}{2P_2} \text{ as long as affordable} \quad MRS = \frac{-2X_1 + 10}{1} = \frac{P_1}{P_2} \quad \boxed{X_1 = 5 - \frac{P_1}{2P_2}}$$

$$X_1^* = \begin{cases} 5 - \frac{P_1}{2P_2} & \text{if } I > P_1(5 - \frac{P_1}{2P_2}) \\ I/P_1 & \text{if otherwise} \end{cases}$$

$$\frac{1}{P_2} < \frac{-2(X_1 - 5)}{P_1} \text{ as long as } X_1 < 5 - \frac{P_1}{2P_2}$$

$$\frac{P_1}{P_2} < -2(X_1 - 5)$$

$$\text{Ex: } U = \min(X_1 + 2X_2, 2X_1 + X_2)$$



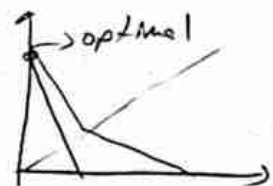
$$1) \text{ if } \frac{P_1}{P_2} < \frac{1}{2}$$

$$X_1 = \frac{I}{P_1} \quad X_2 = 0$$



$$2) \text{ if } \frac{P_1}{P_2} > 2$$

$$X_1 = 0 \quad X_2 = \frac{I}{P_2}$$



$$4) \text{ if } \frac{P_1}{P_2} = \frac{1}{2}$$

$$X_2 \in \left[\frac{I}{P_1 + P_2}, \frac{I}{P_1} \right], \quad X_2 = I - P_1 X_1$$

$$3) \text{ if } \frac{1}{2} < \frac{P_1}{P_2} < 2$$

$$X_1 = X_2 = \frac{I}{P_1 + P_2}$$



$$5) \text{ if } \frac{P_1}{P_2} = 2$$

$$X_1 \in \left[0, \frac{I}{P_1 + P_2} \right], \quad X_2 = I - P_1 X_1$$



$$HW \rightarrow u(x_1, x_2) = \min \{x_1, x_1 + x_2, x_2\}$$

$$HW \rightarrow u(x_1, x_2) = \min \{x_1, x_2^2\}$$

$$x_1 = x_2^2 \quad x_2 = \sqrt{x_1}$$



$$\text{if } x_2^2 > x_1 \rightarrow u = x_1$$

$$\text{if } x_2^2 < x_1 \rightarrow u = x_2^2 = x_2 \quad \text{monotonic transformation}$$

Expenditure Minimization Problem

$$\min_{q_1, q_2} E = p_1 q_1 + p_2 q_2 \quad \text{s.t.} \quad U = U(q_1, q_2)$$

$$E = E(p_1, p_2, u) \quad q_1^* = h(p_1, p_2, u) \quad q_2^* = h(p_1, p_2, u)$$

Hicksian Demand Function
(Compensated demand function)

$$\max_{x_1, x_2} U(x_1, x_2) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = I$$

$$x_1(p_1, p_2, I) \quad x_2(p_1, p_2, I) \quad v(p_1, p_2, I) = U^* \quad \text{indirect utility function}$$

Walrasian Demand Function
(Uncompensated demand function)

$$E_x = U = q_1^a q_2^{1-a} \quad L = p_1 q_1 + p_2 q_2 + \lambda (U - q_1^a q_2^{1-a})$$

$$\frac{\partial L}{\partial q_1} = p_1 - \lambda a q_1^{a-1} q_2^{1-a} = p_1 - \lambda a \frac{U}{q_1} = 0$$

$$\frac{\partial L}{\partial q_2} = p_2 - \lambda (1-a) q_1^a q_2^{-a} = p_2 - \lambda (1-a) \frac{U}{q_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = U - q_1^a q_2^{1-a} = 0$$

$$\frac{P_1}{P_2} = \frac{a}{1-a} \cdot \frac{q_2}{q_1}$$

$$\rightarrow E = q_1 P_1 + q_2 P_2 = P_1 q_1 + P_2 \left(\frac{q_1 (1-a)}{a P_2} \right) = \frac{P_1 q_1}{a}$$

$$q_2 = (1-a) \frac{E}{P_2} \quad U = \left(\frac{a E}{P_1} \right)^a \left(\frac{(1-a) E}{P_2} \right)^{1-a}$$

$$E = U \left(\frac{P_1}{a} \right)^a \left(\frac{P_2}{1-a} \right)^{1-a} \quad q_2 = (1-a) \frac{E}{P_2} \rightarrow \text{Hickian demand function}$$

$$h_1(P_1, P_2, U) = q_1 = U \left(\frac{a}{1-a} \cdot \frac{P_2}{P_1} \right)^{1-a}$$

$$h_2(P_1, P_2, U) = q_2 = U \left(\frac{1-a}{a} \cdot \frac{P_1}{P_2} \right)^a$$

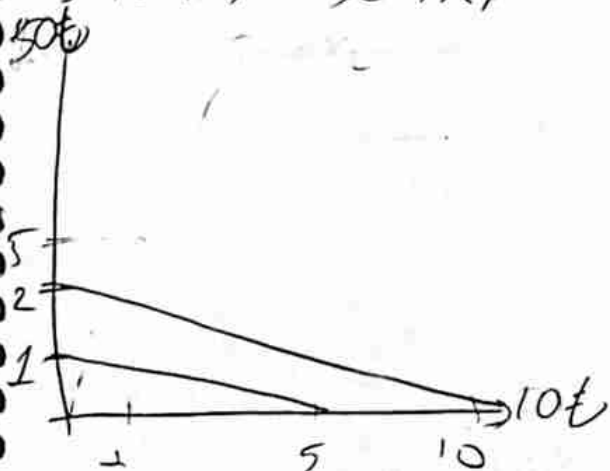
Problem Set 3

Q1 | Draw indif curves

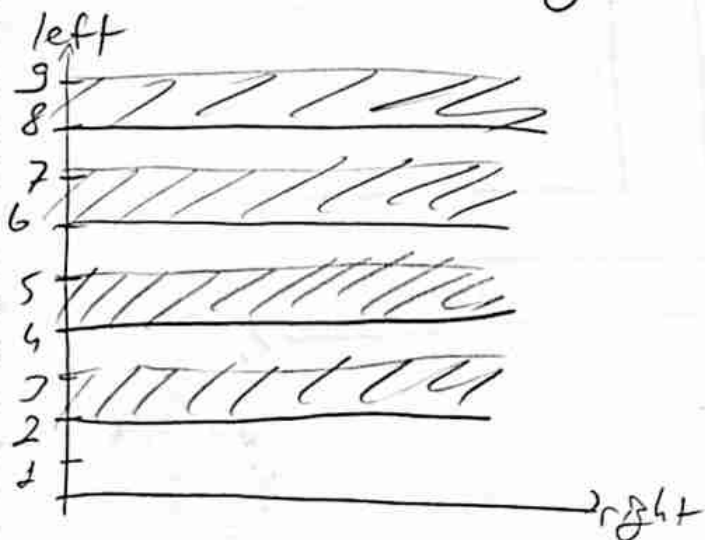
a) 1 left glove + 1 right glove



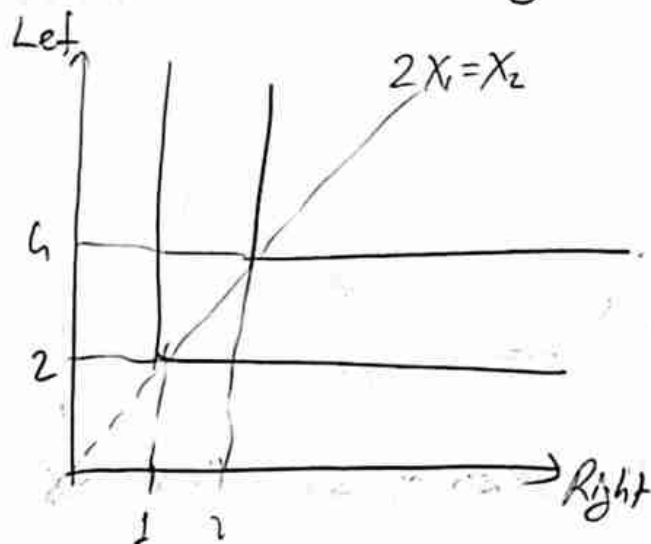
b) 10 TRY + 50 TRY



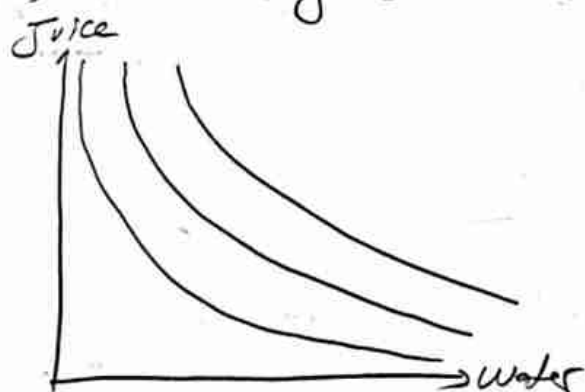
c) 2 left shoes + 1 right shoe



d) 2 left shoes + 1 right shoe

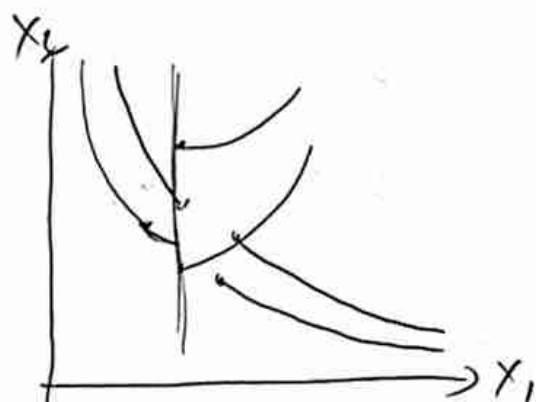


c) Water + Orange Juice



Q2 | $C \subset \mathbb{R}_+^2$ draw indif curves:

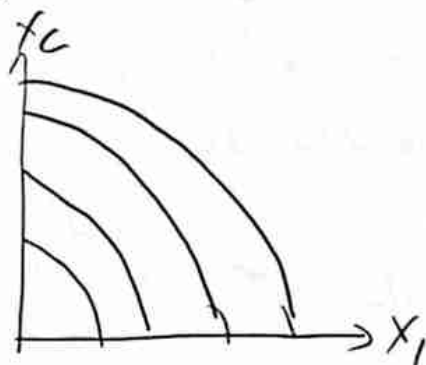
a) Not continuous



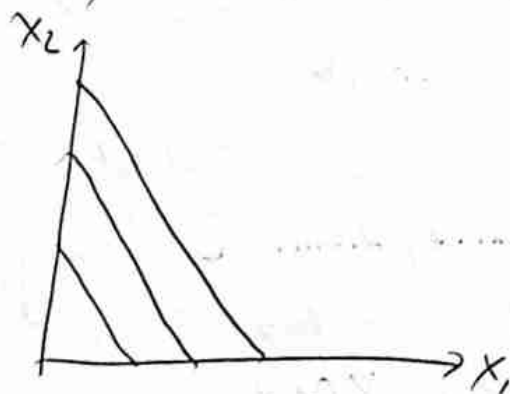
b) Not strictly monotone



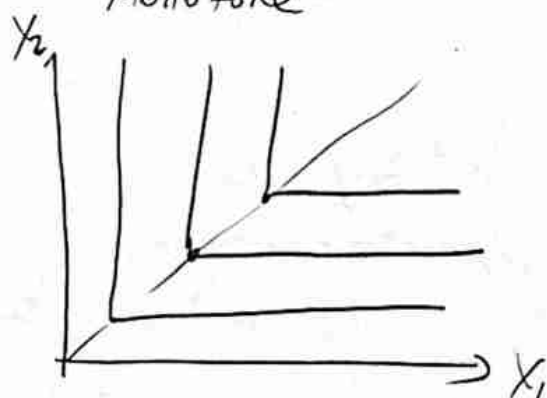
c) Not convex



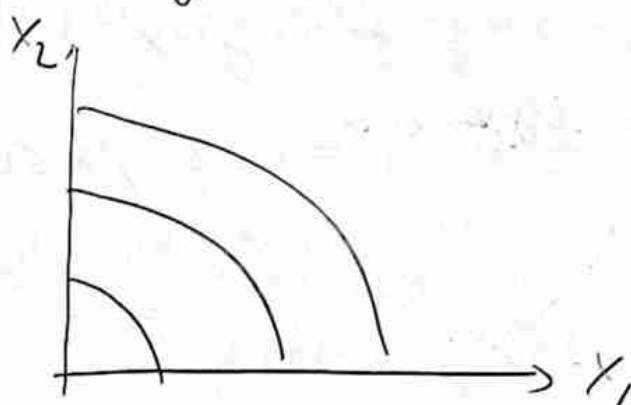
d) Convex but not strictly convex



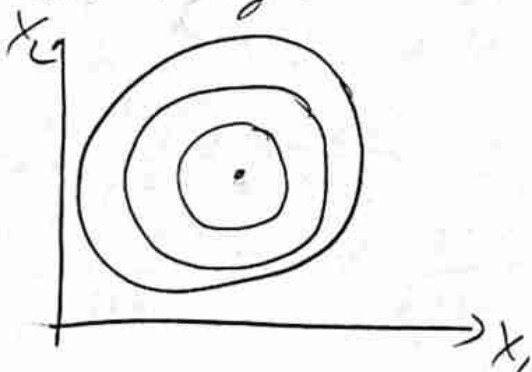
e) Convex but not strictly monotone



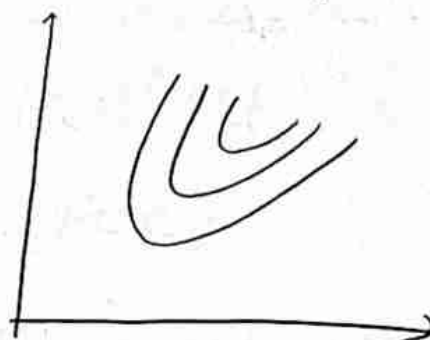
f) Strictly monotone but not convex



g) Not locally satiated



h) Convex, locally non satiated, not monotone



Q3] Prove that every strictly convex preferences (prove convexity)

$$\begin{aligned}
 & x \neq y \quad x \succ y \\
 & \quad \downarrow \\
 & \alpha x + (1-\alpha)y \succ y \quad \Rightarrow \quad \exists z \succ y \quad \forall y \succ z \\
 & \quad \downarrow \\
 & \alpha x + (1-\alpha)y \succ y \quad \text{convexity} \quad \checkmark
 \end{aligned}$$

Q4 | $C \subset \mathbb{R}_+^2$ $B = \{x \in C : p \cdot x \leq w\}$ $w > 0$

Show that if C is convex, then B is also convex. Gec.

Q5 | Show that if $\succsim$ is a rational preference relation, then

a) $\sim$ is reflexive, transitive and symmetric

Reflexivity: x is at least as good as itself

$x \succsim x$ then $x \sim x$

Transitivity: if $x \succsim y$ and $y \succsim z$, then $x \succsim z$

Symmetry: $x \sim y$ then $y \sim x$

Q6 | $C \subset \mathbb{R}_+^2$ $B = \{x \in C : p \cdot x \leq w\}$ $x^* \succsim y$ then $p \cdot x^* = w$ Gec.

07/11/2024 week 6

Unconstrained Optimization (one variable)

$$Z = f(x)$$

FOC: $dz = f'(x)dx = 0$ for an arbitrary non zero dx
(first order condition)

$$\Rightarrow f'(x) = 0$$

$$\text{SOC: } d^2(z) = d(dz) = d[f'(x)dx] = [df'(x)]dx = [f''(x)dx]dx$$

$$f''(x)(dx)^2 < 0$$

should always
be
negative positive



max point: negative slope along
every direction

$$f''(x) < 0 \quad f'(x) = 0$$

Min point: positive slope along
every direction

$$f'(x) = 0 \quad f''(x) > 0$$

FOC:

At a peak, the surface should be flat so that very small move
in any direction should leave us at the same altitude

$$df = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 = 0 \quad \text{for arbitrary } dx_i$$

$$\Rightarrow \frac{\partial f}{\partial x_1} = 0 \quad \text{and} \quad \frac{\partial f}{\partial x_2} = 0 \quad (\text{FOC})$$

SOC: If we move away from the peak, the surface should
slope downward in every direction

* We also need to take the cross effects into account. Even if
 $f_{11} < 0$, an increasing x_1 may raise the marginal effects
of other variables on f , e.g. f_2

$$d(df) = \frac{\partial(df)}{\partial x_1} dx_1 + \frac{\partial(df)}{\partial x_2} dx_2 < 0$$

$$\frac{\partial}{\partial x_1} (f_1 dx_1 + f_2 dx_2) dx_1 + \frac{\partial}{\partial x_2} (f_1 dx_1 + f_2 dx_2) dx_2$$
$$(f_{11} dx_1 + f_{12} dx_2) dx_1 + (f_{21} dx_1 + f_{22} dx_2) dx_2$$

$$f_{11}(dx_1)^2 + 2f_{12}dx_1dx_2 + f_{22}(dx_2)^2$$

$$= \begin{bmatrix} dx_1 & dx_2 \end{bmatrix}_{1 \times 2} \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}_{2 \times 2} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix}_{2 \times 1}$$

let $u = dx_1$
 $v = dx_2$
 $a = f_{11}$
 $b = f_{22}$
 $h = f_{12}$

$$q = d^2f = au^2 + 2huv + bv^2 = \frac{h^2}{a^2}v^2$$

$$q = a\left(u^2 + \frac{2huv}{a} + \frac{h^2}{a^2}v^2\right) + \left(b - \frac{h^2}{a}\right)v^2$$

$$= a \underbrace{\left(u + \frac{h}{a}v\right)^2}_{+} + \underbrace{\frac{ab - h^2}{a}v^2}_{+} < 0$$

iff $a > 0$ and $ab - h^2 > 0$, q is positive definite
 (always positive)
 iff $a < 0$ and $ab - h^2 > 0$, q is negative definite
 (always negative)
 $f_{11} < 0$ det of matrix

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} > 0 = f_{11}f_{22} - (f_{12})^2$$

Determinant of 3×3 matrix:

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

E_x

$$Z = 2x_1^2 + x_1x_2 + 4x_2^2 + x_1x_3 + x_3^2 + 2 = u(x_1, x_2, x_3)$$

$$\frac{\partial u}{\partial x_1} = 4x_1 + x_2 + x_3 = 0$$

$$\frac{\partial u}{\partial x_2} = x_1 + 8x_2 = 0$$

$$\frac{\partial u}{\partial x_3} = x_1 + 2x_3 = 0$$

$$x_1^* = x_2^* = x_3^* = 0$$

$$|H| = \begin{vmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{vmatrix} = \begin{vmatrix} 4 & 1 & 1 \\ 1 & 8 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 54$$

$q = f_{11} > 0$ (condition)
function is positive definite
It is a minimum point

N variable case

$$Z = f(x_1, x_2, x_3, \dots, x_n) \quad dz = f_1 dx_1 + f_2 dx_2 + \dots + f_n dx_n = 0$$

$$\Delta u = \mu u_1 \Delta x_1 + \mu u_2 \Delta x_2$$

$$du = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0$$

Maximum

$$f_1 = f_2 = \dots = f_n = 0$$

$$|H_1| < 0 \quad (1 \times 1 \text{ matrix})$$

$$|H_2| > 0 \quad (2 \times 2 \text{ matrix})$$

$$|H_n| \cdot (-1)^n < 0$$

Minimum

$$f_1 = f_2 = \dots = f_n = 0$$

$$|H_1|, |H_2|, \dots, |H_n| > 0$$

Local minimum: $\det(H) > 0$, $f_{11} > 0$ (positive definite)

Local maximum: $\det(H) > 0$, $f_{11} < 0$ (negative definite)

Saddle point: $\det(H) < 0$, (indefinite)

Optimization with equality constraint

Lagrangian method

$$1) \max_{q_1, q_2} U(q_1, q_2) \text{ s.t. } p_1 q_1 + p_2 q_2 = I$$

$$2) \max_{q_1, q_2, \lambda} L = U(q_1, q_2) + \lambda (I - p_1 q_1 - p_2 q_2)$$

$$\frac{\partial L}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = MU_1 - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda p_2 = MU_2 - \lambda p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = I - p_1 q_1 - p_2 q_2 = 0$$

$U_{12} \stackrel{?}{=} U_{21}$ HW
Must be continuous and 1st
second order derivatives

$$\begin{vmatrix} 0 & -p_1 & -p_2 \\ -p_1 & -U_{11} & U_{12} \\ -p_2 & U_{21} & -U_{22} \end{vmatrix} < 0$$

Max point

Optimization with equality constraints

$$\max U(q_1, q_2) \text{ s.t. } p_1 q_1 + p_2 q_2 \leq I$$

08/11/2024 week 6

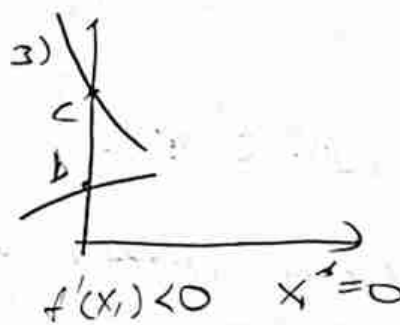
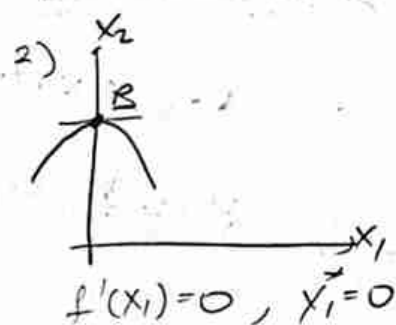
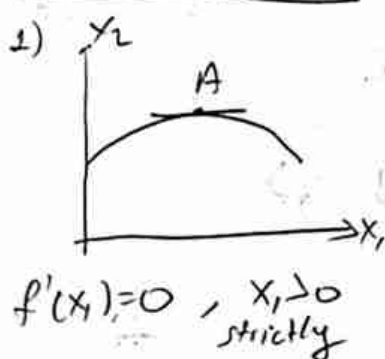
$q_1 \geq 0$
 $q_2 \geq 0$ 3 constraints

Hint: $x^2 + y^2 = [x \ y] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Kuhn-Tucker Conditions

$$\max \pi = f(x_1) \text{ s.t. } x_1 \geq 0$$

interior solution: Corner solution:



for all (1,2,3) situations

$$\boxed{\begin{matrix} f'(x_1) \leq 0 & x_1 \geq 0 \\ x_1 f'(x_1) = 0 \end{matrix}}$$

$$\max \pi = f(x_1, x_2, x_3) \quad \text{s.t.} \quad g_i(x_1, x_2, x_3) \leq \Gamma_i \\ \text{and} \quad x_1, x_2, x_3 \geq 0$$

$$S_i = \Gamma_i - g_i(x_1, x_2, x_3)$$

$$\max f(x_1, x_2, x_3) \quad \text{s.t.} \quad g_i(x) + S_i = \Gamma_i \quad \text{and} \quad x_i \geq 0 \quad \forall i = 1, 2, 3$$

$$L = f(x) + \lambda_i (\Gamma_i - g_i(x) - S_i) \quad x_i \geq 0$$

$$\frac{\partial L}{\partial x_i} \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial L}{\partial x_i} = 0$$

$$\frac{\partial L}{\partial S_i} \leq 0, \quad S_i \geq 0, \quad S_i \frac{\partial L}{\partial S_i} = 0$$

$$\frac{\partial L}{\partial \lambda} = 0$$

$$\frac{\partial L}{\partial S_i} = -\lambda \leq 0$$

$$\lambda_i \geq 0, \quad S_i \geq 0, \quad S_i \lambda_i = 0 \\ \lambda_i \geq 0, \quad \Gamma_i - g_i(x) \geq 0 \\ (\Gamma_i - g_i(x)) \lambda_i = 0$$

$$\frac{\partial L}{\partial x_i}, \quad x_i \geq 0, \quad x_i \frac{\partial L}{\partial x_i} = 0$$

$$\Gamma_i - g_i(x) \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i [\Gamma_i - g_i(x)] = 0$$

$$\text{Ex: } \max f(x_1, x_2) = 4x_1 + 3x_2 \quad \text{s.t.} \quad g(x_1, x_2) = 2x_1 + x_2 \leq 10 \\ x_1, x_2 \geq 0$$

$$L = 4x_1 + 3x_2 + \lambda (10 - 2x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 4 - 2\lambda \leq 0, \quad x_1 \geq 0, \quad x_1(4 - 2\lambda) = 0$$

$$\frac{\partial L}{\partial x_2} = 3 - \lambda \leq 0, \quad x_2 \geq 0, \quad x_2(3 - \lambda) = 0$$

$$\frac{\partial L}{\partial \lambda} = 10 - 2x_1 - x_2 \geq 0, \quad \lambda \geq 0, \quad \lambda(10 - 2x_1 - x_2) = 0$$

① Examine $x_1(4 - 2\lambda) = 0$, Assume $(4 - 2\lambda)$ term is zero $\rightarrow \lambda = 2$
it doesn't fit the constraint $(3 - \lambda \leq 0)$ which is $1 \leq 0$ ✗
Hence $x_1^* = 0$

② Examine $x_2(3 - \lambda) = 0$, Assume $x_2^* = 0$, It doesn't fit the constraint $\lambda(10 - 2x_1 - x_2) = 0$ $10\lambda \neq 0$. Suppose $\lambda = 3$

③ $\lambda(10 - 2x_1 - x_2) = 0 \rightarrow 3(10 - x_2) = 0$ $x_2^* = 10$

Ex / $\max f(x_1, x_2) = 10x_1 + 3x_2$ s.t. $g(x_1, x_2) = 2x_1 + x_2 \leq 10$
 $x_1 \geq 0 \quad x_2 \geq 0$

$$L = 10x_1 + 3x_2 + \lambda(10 - 2x_1 - x_2)$$

$$\frac{\partial L}{\partial x_1} = 10 - 2\lambda \leq 0, \quad x_1 \geq 0, \quad x_1(10 - 2\lambda) = 0$$

$$\frac{\partial L}{\partial x_2} = 3 - \lambda \leq 0, \quad x_2 \geq 0, \quad x_2(3 - \lambda) = 0$$

$$\frac{\partial L}{\partial \lambda} = 10 - 2x_1 - x_2 \geq 0, \quad \lambda \geq 0, \quad \lambda(10 - 2x_1 - x_2) = 0$$

① $x_1(10 - 2\lambda) \rightarrow x_1^* = 0$ or $\lambda = 5$

Suppose $\lambda = 5$, it fits all the equations
 $x_1^* = 0$? sounds good for now

② $x_2(3 - \lambda) \rightarrow x_2^* = 0$ or $\lambda = 3$

suppose $\lambda = 3$, it doesn't fit the equation $(10 - 2\lambda \leq 0)$
then $x_2^* = 0$ ✓

③ $\lambda(10 - 2x_1 - x_2) = 0 \rightarrow \lambda(10 - 2x_1) = 0$ $x_1 = 5$
or $\lambda = 0$

$\lambda = 0$ doesn't fit equations $(10 - 2\lambda \leq 0, 3 - \lambda \leq 0)$
then $x_1^* = 5$

The critical point found is $(5, 0)$ check max or min

$$|H| = \begin{vmatrix} L_{xx} & L_{xy} & L_{x\lambda} \\ L_{xy} & L_{yy} & L_{y\lambda} \\ L_{x\lambda} & L_{y\lambda} & L_{\lambda\lambda} \end{vmatrix} = \begin{vmatrix} 0 & -2 & -1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 0 \cdot \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} - (-2) \begin{vmatrix} -2 & 0 \\ -1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} -2 & 0 \\ -1 & 0 \end{vmatrix}$$

$= 0 // \quad f''_{11} = 0$ and $|H| < 0$ Max

Ex/ $f(x,y) = 2x + 3y$ and $g(x,y) = x^2 + y^2 \leq 2$, $x, y \geq 0$

$$L = 2x + 3y + \lambda(2 - x^2 - y^2)$$

$$L_x = 2 - 2\lambda x \leq 0, x \geq 0, x(2 - 2\lambda x) = 0$$

$$L_y = 3 - 2\lambda y \leq 0, y \geq 0, y(3 - 2\lambda y) = 0$$

$$L_\lambda = 2 - x^2 - y^2 \geq 0, \lambda \geq 0, \lambda(2 - x^2 - y^2) = 0$$

$$(1) \rightarrow x(2 - 2\lambda x) = 0 \quad x^* = 0 \text{ or } \lambda x = 1$$

suppose $x^* = 0 \rightarrow 2 - 2\lambda x \leq 0$ or $\lambda x = 1$ $\lambda x = 1$

$$(2) \rightarrow y(3 - 2\lambda y) = 0, y^* = 0 \text{ or } \lambda y = \frac{3}{2}$$

suppose $y^* = 0 \rightarrow 3 - 2\lambda y \leq 0$ or $\lambda y = \frac{3}{2}$ $\lambda y = \frac{3}{2}$

$$(3) \rightarrow 2 - \left(\frac{1}{2}\right)^2 - \left(\frac{3}{2\lambda}\right)^2 = 0 \rightarrow \lambda = \sqrt{\frac{13}{18}}, x = \sqrt{\frac{18}{13}}, y = \sqrt{\frac{8}{13}}$$

Max min check

check convexity? $g = x^2 + y^2 \leq 2$ d^2g is positive definite

$$H = \begin{bmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad ? \text{ bumpy, sor}$$

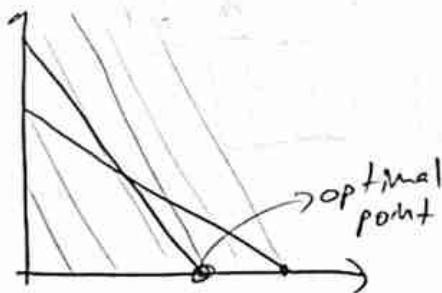
Ex/ Max $U = 4S + D$ s.t.

$$S + D \leq 10$$

$$S + 2D \leq 12$$

$$S, D \geq 0$$

Lagrange
Bumpy, sor



$$\text{Minimize } f(x) = x_1^2 + x_2^2 \quad \text{s.t.} \quad x_1 + 4x_2 \leq 2 \quad x_1 \geq 0, x_2 \geq 0$$

$$L = x_1^2 + x_2^2 + \lambda(2 - x_1 - 4x_2)$$

$$L_{x_1} = 2x_1 - \lambda \leq 0, \quad x_1 \geq 0, \quad x_1(2x_1 - \lambda) = 0$$

$$L_{x_2} = 2x_2 - 4\lambda \leq 0, \quad x_2 \geq 0, \quad x_2(2x_2 - 4\lambda) = 0$$

$$L_{\lambda} = 2 - x_1 - 4x_2 \geq 0, \quad \lambda \geq 0, \quad \lambda(2 - x_1 - 4x_2) = 0$$

$$(1) \rightarrow x_1(2x_1 - \lambda) = 0 \quad x_1 = 0 \text{ or } 2x_1 = \lambda$$

$x_1 = 0$ sounds good
 $\lambda = 2x_1 \rightarrow$ sounds good

$$(2) \rightarrow x_2(2x_2 - 4\lambda) = 0 \quad x_2 = 0 \text{ or } 2x_2 = 4\lambda$$

$x_2 = 0$ sound good
 $2x_2 = 4\lambda \rightarrow$ sounds good

Case 1

$$x_1 > 0, x_2 > 0 \rightarrow \lambda = 2x_1, \quad 4\lambda = 2x_2$$

$$(3) \rightarrow \lambda(2 - \frac{\lambda}{2} - 8\lambda) = 0 \quad 2\lambda - \frac{\lambda^2}{2} - 8\lambda^2 = 0$$

$$-17\lambda^2 + 4\lambda = 0 \quad -17\lambda + 4 = 0 \quad \lambda = 0 \text{ or } \lambda = \frac{4}{17}$$

Case 2

$$x_1 > 0, x_2 = 0 \rightarrow \lambda(2 - x_1) = 0 \quad 2\lambda - 2\lambda x_1 = 0$$

$$2 - x_1 = 0 \quad \boxed{x_1 = 2}$$

$$\boxed{\lambda = 4}$$

Case 3

$$x_1 = 0, x_2 > 0 \rightarrow \lambda(2 - 4x_2) = 0 \quad \boxed{x_2 = \frac{1}{2}} \rightarrow \boxed{\lambda = \frac{1}{4}}$$

Case 4

$$x_1 = 0, x_2 = 0, \lambda = 0$$

Case 4

checking a utility function is convex or not:

$H = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$ For 2 variable case,
if $f_{11} > 0$, $\det|H| \geq 0$, function is convex

ex/ $\text{Max } U = 4x + y$ s.t. $x + y \leq 10$ $x + 2y \leq 12$ $x, y \geq 0$ (preference convex olamak zorunda değil)

$$L = 4x + y + \lambda_1(10 - x - y) + \lambda_2(12 - x - 2y)$$

$$\frac{\partial L}{\partial x} = 4 - \lambda_1 - \lambda_2 = 0 \quad \left. \begin{array}{l} 3 + \lambda_2 = 0 \\ \lambda_1 = 7 \end{array} \right\} \lambda_2 = -3$$

$$\frac{\partial L}{\partial y} = 1 - \lambda_1 - 2\lambda_2 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = 10 - x - y = 0 \quad \left. \begin{array}{l} -2 + y = 0 \\ x = 8 \end{array} \right\} y = 2$$

$$\frac{\partial L}{\partial \lambda_2} = 12 - x - 2y = 0$$

$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ means that all eigenvalues are zero. Hessian is positive semi-definite but not positive definite. The function is convex but not strictly convex

Problem Set 4

Q1) $U(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ $m=36$ $P_1=2$ $P_2=4$

a) Max $x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$ subject to $2x_1 + 4x_2 = 36$
 $x_1 \geq 0$ $x_2 \geq 0$

b) $L = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} + \lambda(36 - 2x_1 - 4x_2)$

FOC:

$$\frac{\partial L}{\partial x_1} = \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} - 2\lambda = 0 \rightarrow \lambda = \frac{1}{6} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}$$

$$\frac{\partial L}{\partial x_2} = \frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}} - 4\lambda = 0 \rightarrow \lambda = \frac{1}{6} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}$$

$$\frac{\partial L}{\partial \lambda} = 36 - 2x_1 - 4x_2 = 0 \rightarrow 36 - 6x_1 = 0$$

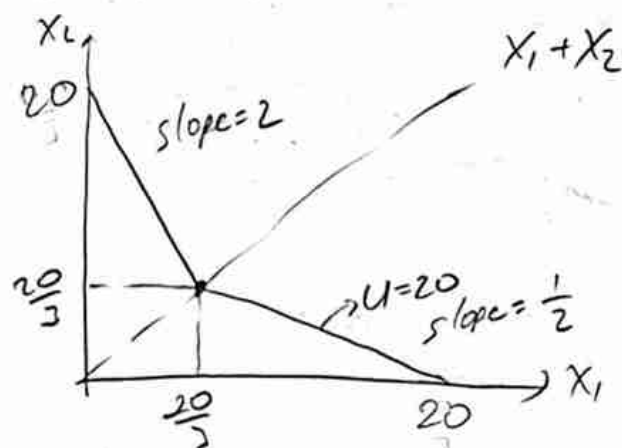
$$\left. \begin{aligned} \frac{1}{6} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} &= \frac{1}{6} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}} \\ x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} &= x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}} \\ x_1^{-1} &= x_2^{-1} \end{aligned} \right\}$$

$$\boxed{x_1 = x_2}$$

c) $x_1 = x_2 = 6$

Q2) $U(x_1, x_2) = \min\{x_2 + 2x_1, x_1 + 2x_2\}$

a) Draw $U=20$

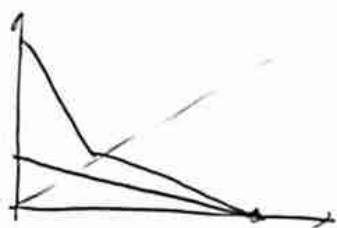


$$U = x_2 + 2x_1 = 2x_1 = 20$$

b) $P_1 = P_2$ what is the condition $x_1 = 0$

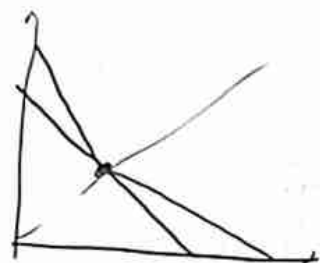


c) again $X_2 = 0$



slope $< \frac{1}{2}$

d) if solution is unique and $X_1 > 0, X_2 > 0$



$\frac{1}{2} < \text{slope} < 2$

Q3 | $U = G^{\frac{1}{2}} + M^{\frac{1}{2}}$ $P_G = 20, P_M = 10, I = 60$

$$L = G^{\frac{1}{2}} + M^{\frac{1}{2}} + \lambda(60 - 20G - 10M)$$

$$\frac{\partial L}{\partial G} = \frac{1}{2} G^{-\frac{1}{2}} - 20\lambda = 0 \quad \left. \begin{array}{l} 20\lambda = \frac{1}{2} G^{-\frac{1}{2}} \\ \lambda = \frac{1}{40} G^{-\frac{1}{2}} \end{array} \right\}$$

$$\frac{\partial L}{\partial M} = \frac{1}{2} M^{-\frac{1}{2}} - 10\lambda = 0 \quad \left. \begin{array}{l} 10\lambda = \frac{1}{2} M^{-\frac{1}{2}} \\ \lambda = \frac{1}{20} M^{-\frac{1}{2}} \end{array} \right\}$$

$$\frac{\partial L}{\partial \lambda} = 60 - 20G - 10M = 0$$

$$\frac{1}{2 \cdot 40 \sqrt{G}} = \frac{1}{20 \sqrt{M}} \quad \boxed{4G = M}$$

$$60 - 20G - 40G = 0$$

$$60G = 60$$

$$\boxed{G = 1}$$

$$\boxed{M = 4}$$

b) $L = \ln(G^{\frac{1}{2}} + M^{\frac{1}{2}}) + \lambda(60 - 20G - 10M)$

$$\frac{\partial L}{\partial G} = \frac{1}{2} \frac{G^{\frac{1}{2}}}{G^{\frac{1}{2}} + M^{\frac{1}{2}}} - 20\lambda = 0$$

$$\frac{\partial L}{\partial M} = \frac{1}{2} \frac{M^{\frac{1}{2}}}{G^{\frac{1}{2}} + M^{\frac{1}{2}}} - 10\lambda = 0$$

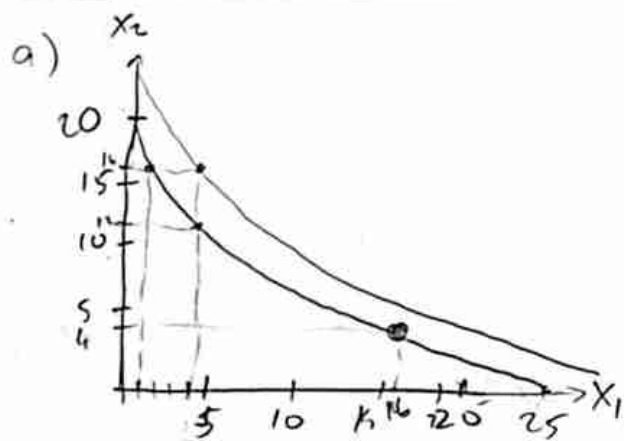
$$\frac{\partial L}{\partial \lambda} = 60 - 20G - 10M$$

$$G = 1 \quad M = 4$$

again

Since it is a monotonic transformation, result does not change

Q4) $(X_1, X_2) \sim (16, 4)$ $X_1 \geq 0$ $X_2 \geq 0$ $X_2 = 20 - 4\sqrt{X_1}$



b) $U = 4\sqrt{X_1} + X_2$ $P_{X_1} = 1$ $P_{X_2} = 2$ $m = 24$

$$X_1 + 2X_2 \leq 24$$

$$L = 4X_1^{\frac{1}{2}} + X_2 + \lambda(24 - X_1 - 2X_2)$$

$$\frac{\partial L}{\partial X_1} = 2X_1^{-\frac{1}{2}} - \lambda = 0$$

$$\lambda = \frac{1}{2}$$

$$X_1^{-\frac{1}{2}} = \frac{1}{4}$$

$$\sqrt{X} = 4$$

$$\boxed{X = 16}$$

$$\boxed{X_2 = 4}$$

$$\frac{\partial L}{\partial X_2} = 1 - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 24 - X_1 - 2X_2 = 0$$

c) $m^* = 9$

$$L = 4X_1^{\frac{1}{2}} + X_2 + \lambda(9 - X_1 - 2X_2)$$

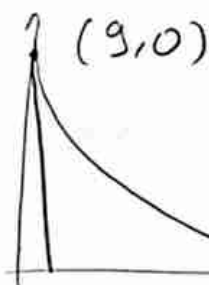
$$\frac{\partial L}{\partial X_1} = 2X_1^{-\frac{1}{2}} - \lambda = 0$$

$$\lambda = \frac{1}{2} \rightarrow \boxed{X_1 = 16} \quad X_2 = \frac{-7}{2} \quad \times$$

There is a corner solution

$$\frac{\partial L}{\partial X_2} = 1 - 2\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 9 - X_1 - 2X_2 = 0$$



Q5 | $U = \sqrt{X_1} + X_2$

a) $P_1 = 1$ Find demand function

$$L = X_1^{\frac{1}{2}} + X_2 + \lambda (m - X_1 - P_2 X_2)$$

$$\frac{\partial L}{\partial X_1} = \frac{1}{2} X_1^{-\frac{1}{2}} - \lambda = 0$$

$$\lambda = \frac{1}{P_2}$$

$$\frac{1}{\sqrt{X_1}} = \frac{2}{P_2}$$

$$X_1 = \frac{P_2^2}{4}$$

$$\frac{\partial L}{\partial X_2} = 1 - P_2 \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = m - X_1 - P_2 X_2 = 0 \rightarrow m - \frac{P_2^2}{4} - P_2 X_2 = 0 \quad X_2 = \frac{m}{P_2} - \frac{P_2}{4}$$

$\frac{\partial L}{\partial \lambda}$

$X \leq 0$ ok

corner solution $\rightarrow (X_1, X_2) = (m, 0)$

$$X_1(1, P_2, m) = \begin{cases} \frac{P_2^2}{4} & \text{if } \frac{m}{P_2} \geq \frac{P_2}{4} \\ m & \text{otherwise} \end{cases}$$

$$X_2(1, P_2, m) = \begin{cases} \frac{m}{P_2} - \frac{P_2}{4} & \text{if } \frac{m}{P_2} \geq \frac{P_2}{4} \\ 0 & \text{otherwise} \end{cases}$$

Same back

Problem Set 5

Q1 $U = X_1^{\alpha_1} X_2^{\alpha_2} X_3^{\alpha_3}$ $0 < \alpha_i < 1$

a) $\mathcal{L} = X_1^{\alpha_1} X_2^{\alpha_2} X_3^{\alpha_3} + \lambda (M - P_1 X_1 - P_2 X_2 - P_3 X_3)$

$$\left. \begin{aligned} \frac{\partial \mathcal{L}}{\partial X_1} &= \alpha_1 X_1^{\alpha_1-1} X_2^{\alpha_2} X_3^{\alpha_3} - P_1 \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial X_2} &= \alpha_2 X_1^{\alpha_1} X_2^{\alpha_2-1} X_3^{\alpha_3} - P_2 \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial X_3} &= \alpha_3 X_1^{\alpha_1} X_2^{\alpha_2} X_3^{\alpha_3-1} - P_3 \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= M - P_1 X_1 - P_2 X_2 - P_3 X_3 = 0 \end{aligned} \right\} \begin{aligned} \frac{P_1 \lambda}{P_2 \lambda} &= \frac{\alpha_1 X_1^{\alpha_1-1} X_2^{\alpha_2} X_3^{\alpha_3}}{\alpha_2 X_1^{\alpha_1} X_2^{\alpha_2-1} X_3^{\alpha_3}} \\ \frac{P_1}{P_2} &= \frac{\alpha_1 X_2}{\alpha_2 X_1} \\ X_2 &= \frac{P_1 \cdot \alpha_2}{P_2 \alpha_1} X_1 \\ X_3 &= \frac{P_1}{P_3} \frac{\alpha_3}{\alpha_1} X_1 \end{aligned}$$

$$M = P_1 X_1 + P_2 \cdot \frac{P_1 \cdot \alpha_2}{P_2 \alpha_1} X_1 + P_3 \cdot \frac{P_1}{P_3} \frac{\alpha_3}{\alpha_1} X_1$$

$$M = X_1 \left(P_1 + P_1 \frac{\alpha_2}{\alpha_1} + P_1 \frac{\alpha_3}{\alpha_1} \right) \quad X_1^* = \frac{M}{P_1 (1 + \frac{\alpha_2}{\alpha_1} + \frac{\alpha_3}{\alpha_1})} = \frac{M}{P_1} \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3}$$

$$X_2^* = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \alpha_3} \frac{M}{P_2} \quad X_3^* = \frac{\alpha_3}{\alpha_1 + \alpha_2 + \alpha_3} \frac{M}{P_3}$$

b) $X_1 \geq 0 \quad X_2 \geq 0 \quad X_3 \geq 0$

c) $\frac{\partial X_1^*}{\partial M} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_3} \cdot \frac{1}{P_1}$ which is always positive

Q2) $u(x,y) = \sqrt{x} + \sqrt{y}$

$M > 0$

$x > 0$
 $y > 0$

a) Is the u increasing in x ?

$\frac{\partial u}{\partial x} = \frac{1}{2\sqrt{x}} > 0$ yes, increasing

$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{4} x^{-\frac{3}{2}} < 0$, u is concave in x

b) Max $\sqrt{x} + \sqrt{y}$ subject to $P_x x + P_y y = M$

since u is increasing in both x and y , budget constraint is satisfied

c) $\mathcal{L} = \sqrt{x} + \sqrt{y} + \lambda(M - P_x x - P_y y)$

d) FOCs:

$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} - P_x \lambda = 0$

$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2} y^{-\frac{1}{2}} - P_y \lambda = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda} = M - P_x x - P_y y = 0$

$\frac{P_x \lambda}{P_y \lambda} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{\frac{1}{2} y^{-\frac{1}{2}}} \quad \frac{P_x}{P_y} = \frac{\sqrt{y}}{\sqrt{x}}$

$y = \frac{P_x^2}{P_y^2} x$

$M - P_x x - P_y \frac{P_x^2}{P_y^2} x = 0$

$M = P_x x + \frac{P_x^2}{P_y} x = x \left(P_x + \frac{P_x^2}{P_y} \right)$

$x^* = \frac{M}{P_x + \left(\frac{P_x^2}{P_y} \right)}$

$y^* = \frac{P_x^2}{P_y^2} \cdot \frac{M}{P_x + \left(\frac{P_x^2}{P_y} \right)}$

f) $f_{11} = -\frac{1}{4} x^{-\frac{3}{2}}$

$f_{12} = 0$

$f_{21} = 0$

$f_{22} = -\frac{1}{4} y^{-\frac{3}{2}}$

$f_{1\lambda} = -P_x$

$f_{2\lambda} = -P_y$

$f_{\lambda 1} = -P_x$

$f_{\lambda 2} = -P_y$

$f_{\lambda \lambda} = 0$

$\det(H) = \begin{vmatrix} f_{\lambda \lambda} & f_{\lambda 1} & f_{\lambda 2} \\ f_{1\lambda} & f_{11} & f_{12} \\ f_{2\lambda} & f_{21} & f_{22} \end{vmatrix} = \begin{vmatrix} 0 & -P_x & -P_y \\ -P_x & -\frac{1}{4} x^{-\frac{3}{2}} & 0 \\ -P_y & 0 & -\frac{1}{4} y^{-\frac{3}{2}} \end{vmatrix}$

$= +0 - (-P_x) \left(-P_x \cdot \frac{1}{4} y^{-\frac{3}{2}} - 0 \right) + \left(0 - \left(-\frac{1}{4} x^{-\frac{3}{2}} \cdot (-P_y) \right) \right)$
 $= 0 + P_x^2 \frac{1}{4} y^{-\frac{3}{2}} - P_y \frac{1}{4} x^{-\frac{3}{2}} > 0$

$f_{\lambda \lambda} = 0$ and $|H| > 0$ negative definite
there exists a max

Q4) Check if Kuhn-Tucker satisfies

$$a) U = \ln(x_1+1) + \ln(x_2+1) \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq M \quad \begin{matrix} x_1 \geq 0 \\ x_2 \geq 0 \\ p_1, p_2, M > 0 \end{matrix}$$

$$L = \ln(x_1+1) + \ln(x_2+1) + \lambda(M - p_1 x_1 - p_2 x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{1}{x_1+1} - \lambda p_1 \leq 0, \quad x_1 \geq 0, \quad x_1 \left(\frac{1}{x_1+1} - \lambda p_1 \right) = 0$$

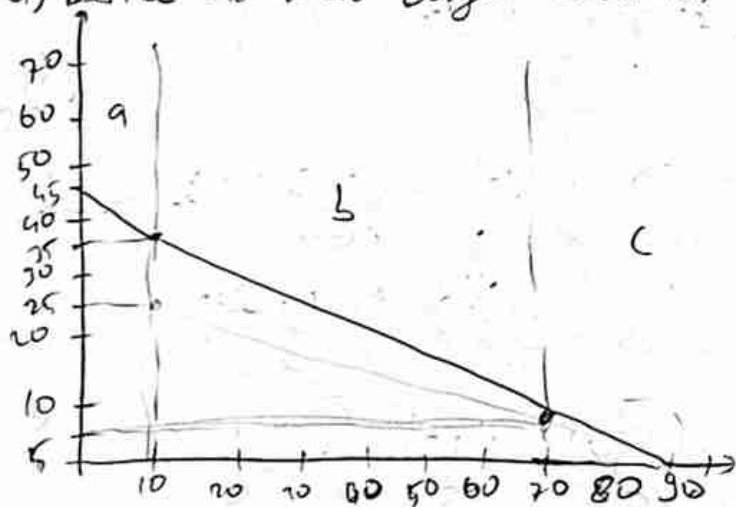
$$\frac{\partial L}{\partial x_2} = \frac{1}{x_2+1} - \lambda p_2 \leq 0, \quad x_2 \geq 0, \quad x_2 \left(\frac{1}{x_2+1} - \lambda p_2 \right) = 0$$

$$\frac{\partial L}{\partial \lambda} = M - p_1 x_1 - p_2 x_2 \geq 0, \quad \lambda \geq 0, \quad \lambda(M - p_1 x_1 - p_2 x_2) = 0$$

Sample Midterm.

Q1) $U = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \quad M = 100$

a) Derive and draw budget constraint



$$\begin{matrix} p_1 = 2 & x_1 \leq 10 \\ p_1 = 1 & x_1 > 10 \end{matrix}$$

$$\begin{matrix} p_2 = 4 & x_2 \leq 5 \\ p_2 = 2 & x_2 > 5 \end{matrix}$$

$$a \rightarrow 2x_1 + 2x_2 = 45$$

$$b \rightarrow x_1 + 2x_2 = 80$$

$$c \rightarrow x_1 + 4x_2 = 90$$

b) Compute optimal solution

Tangency method works $MRS_a = \frac{x_2}{x_1} = \frac{p_1}{p_2} \quad \text{s.t.} \quad x_1 + x_2 = 45$
 $x_1^* = x_2^* = 22.5$

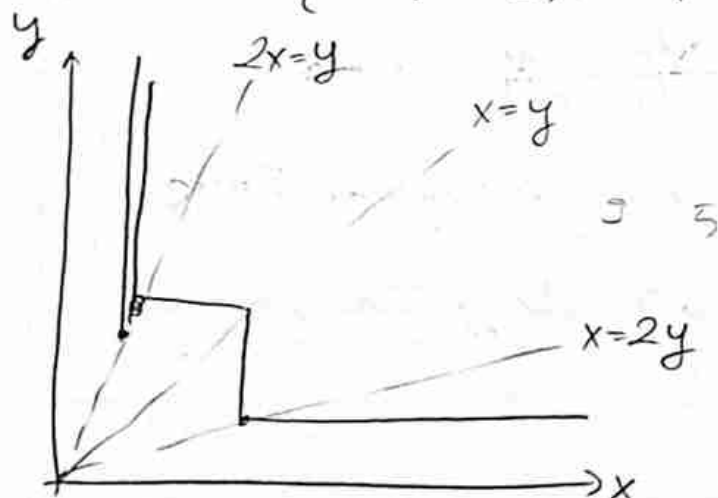
$MRS_b = \frac{x_2}{x_1} = \frac{p_1}{p_2} = \frac{1}{2} \quad \text{s.t.} \quad x_1 + 2x_2 = 80$
 $4x_2 = 80$
 $x_2 = 20$
 $x_1 = 40$
 $(40, 20) \checkmark$

$MRS_c = \frac{x_2}{x_1} = \frac{1}{4} \quad \text{s.t.} \quad x_1 + 4x_2 = 90$
 $4x_2 = 90$
 $x_2 = \frac{90}{4}$
 $x_1 = 45$
 $(45, 22.5)$

Q2] $U = 2 \min\{x_1, x_2\} + 3 \max\{x_3, x_4\}$

find demand functions for each good

Q3] $U = \max\{\min\{x, \frac{1}{2}y\}, \min\{\frac{1}{2}x, y\}\}$



Q4] $U(E, J) = (E+1)^2(J+2)$ $P_E = 240$ $I = 1760$
 $P_J = 80$

Max $(E+1)^2(J+2)$ s.t. $240E + 80J = 1760$

$L = (E+1)^2(J+2) + \lambda(1760 - 240E - 80J)$

$\frac{\partial L}{\partial E} = 2(E+1)(J+2) - 240\lambda = 0$ $\frac{240\lambda}{80\lambda} = \frac{2 \cdot (E+1)(J+2)}{(E+1)^2} = \frac{2J+4}{E+1}$

$\frac{\partial L}{\partial J} = (E+1)^2 - 80\lambda = 0$

$240E + 240 = 160J + 320$

$3E + 3 = 2J + 4$

$\frac{\partial L}{\partial \lambda} = 1760 - 240E - 80J = 0$

$J = \frac{3E-1}{2}$

$1760 - 240E - 120E + 40 = 0$

$1800 = 360E$

$E = 5$ $J = 7$

days spend: $5(240) = 1200$
 $7(80) = 560$

Q5] a) $U(x,y)$ and $V(x,y)$ if $V(x,y) = 2U(x,y)$ then the marginal rate of subst. V is twice of U

False. V is monotonic transformation of U . Slopes of the indif. curves are same. Slope would be same every point

$$MRS^U = \frac{MU_x}{MU_y} \quad MRS^V = \frac{2MU_x}{2MU_y} = \frac{MU_x}{MU_y} \text{ same}$$

b) Price elasticity of demand is always negative. False

$$\frac{\% \Delta Q_D}{\% \Delta P} = \frac{dQ}{dP} \cdot \left(\frac{P}{Q} \right) \quad \frac{dQ}{dP} > 0 \text{ for giffen goods}$$

always positive

Q6] $\max_{x_1 \geq 0, x_2 \geq 0} U(x_1, x_2) = \sqrt{x_1} + x_2 \quad \text{s.t.} \quad x_1 + 2x_2 \leq I$

a) $L = \sqrt{x_1} + x_2 + \lambda(I - x_1 - 2x_2)$

FOCs:

$$\frac{\partial L}{\partial x_1} = \frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \leq 0, \quad x_1 \geq 0, \quad x_1 \left(\frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \right) = 0$$

$$\frac{\partial L}{\partial x_2} = 1 - 2\lambda \leq 0, \quad x_2 \geq 0, \quad x_2(1 - 2\lambda) = 0$$

$$\frac{\partial L}{\partial \lambda} = I - x_1 - 2x_2 \geq 0, \quad \lambda \geq 0, \quad \lambda(I - x_1 - 2x_2) = 0$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad \lambda \geq 0$$

$$\lambda \left(\frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \right) = 0 \quad \lambda(1 - 2\lambda) = 0 \quad \lambda(I - x_1 - 2x_2) = 0$$

$$\lambda \left(\frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \right) = 0 \quad \lambda(1 - 2\lambda) = 0 \quad \lambda(I - x_1 - 2x_2) = 0$$

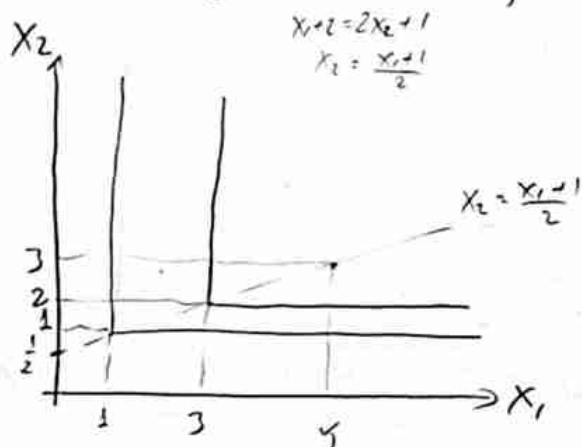
$$\lambda \left(\frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \right) = 0 \quad \lambda(1 - 2\lambda) = 0 \quad \lambda(I - x_1 - 2x_2) = 0$$

$$\lambda \left(\frac{1}{2}x_1^{-\frac{1}{2}} - \lambda \right) = 0 \quad \lambda(1 - 2\lambda) = 0 \quad \lambda(I - x_1 - 2x_2) = 0$$

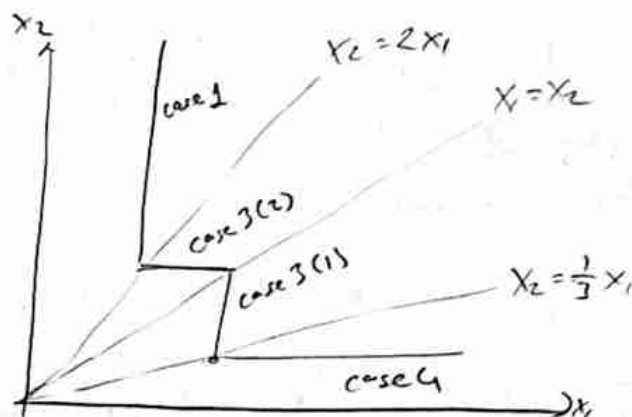
MT1 Exercises

1) Drawing indifference curves

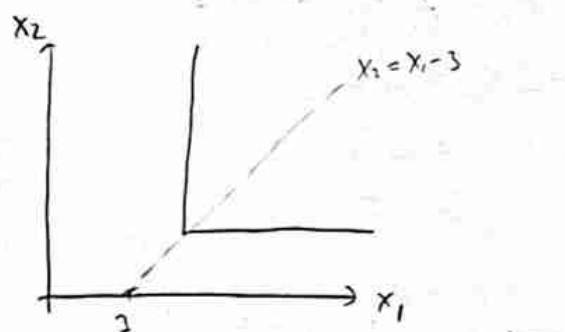
a) $U = \min\{X_1 + 2, 2X_2 + 1\}$



b) $U = \max\{\min(2X_1, X_2), \min(X_1, 3X_2)\}$



c) $U = \min\{X_1, X_2 + 3\}$



d) $U = \max\{\min(X_1, 2X_2), X_2\}$

Case 1

$X_1 \geq 2X_2 \rightarrow U = \max(2X_2, X_2) = 2X_2$

Case 2

$2X_2 \geq X_1 \rightarrow U = \max(X_1, X_2)$

Case 2(1)

$2X_2 \geq X_1$
 $X_2 \geq X_1 \rightarrow U = X_2$

Case 2(2)

$2X_2 \geq X_1$
 $X_1 \geq X_2 \rightarrow U = X_1$

Case 1

$2X_1 \leq X_2, X_1 \leq 3X_2 \rightarrow U = 2X_1$

Case 2

$2X_1 \leq X_2, X_1 \geq 3X_2$ not applicable

Case 3

$2X_1 \geq X_2, X_1 \leq 3X_2 \rightarrow U = \max\{X_1, X_2\}$

Case 3(1)

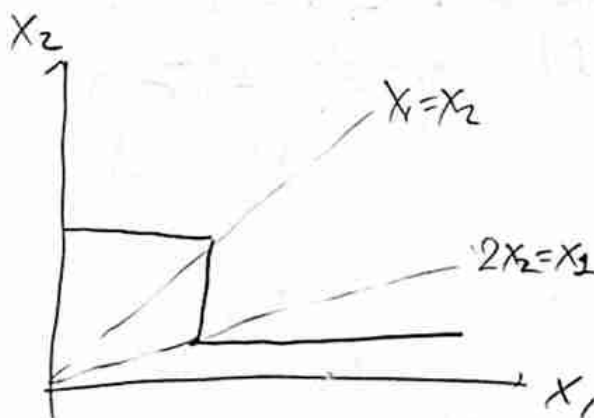
$X_1 \geq X_2 \rightarrow U = X_1$

Case 3(2)

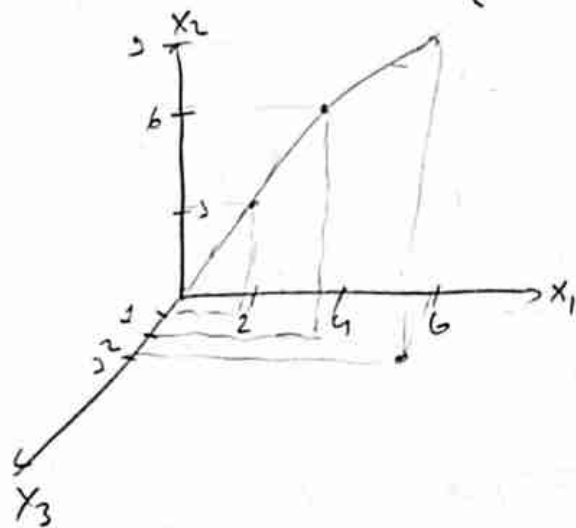
$X_2 \geq X_1 \rightarrow U = X_2$

Case 4

$2X_1 \geq X_2, X_1 \geq 3X_2 \rightarrow U = 3X_2$



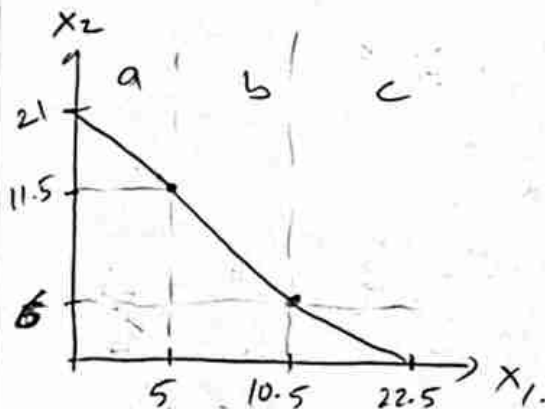
e) $U(x_1, x_2, x_3) = \min \left\{ \frac{x_1}{2}, \frac{x_2}{3}, x_3 \right\}$. $(x_1, x_2, x_3) = (24, 34, 4)$



for $U=1$, indif surface includes $(2, 3, 1)$
 for $U=2$, indif " " $(4, 6, 2)$
 for $U=3$, indif " " $(6, 9, 3)$

2) Budget Constraint and Choice (simple) $I = 50$

a) $U = x_1^{0.6} x_2^{0.4}$ $P_1 = 3$ if $x_1 \leq 5$ $P_2 = 4$ if $x_2 \leq 6$
 $P_1 = 2$ if $x_1 > 5$ $P_2 = 2$ if $x_2 > 6$



a $\rightarrow 3x_1 + 2x_2 = 42$

b $\rightarrow 2x_1 + 2x_2 = 33$

c $\rightarrow 2x_1 + 4x_2 = 45$

Cobb-Douglas tangency

$MRS_a = \frac{0.6 x_1^{-0.4} x_2^{0.4}}{0.4 x_1^{0.6} x_2^{-0.6}} = \frac{3}{2} = \frac{x_1}{x_2}$ $x_1 = x_2$
 no such point

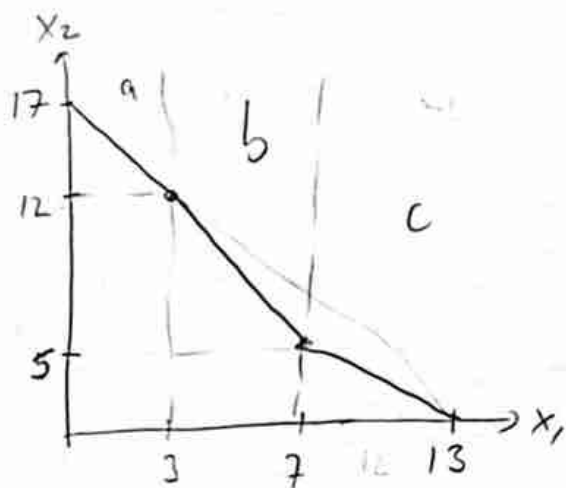
$MRS_b = \frac{0.6 x_1^{-0.4} x_2^{0.4}}{0.4 x_1^{0.6} x_2^{-0.6}} = 1$ $\frac{2}{3} = \frac{x_2}{x_1}$ $2x_2 = 3x_1$
 ✓

$MRS_c = \frac{0.6 x_1^{-0.4} x_2^{0.4}}{0.4 x_1^{0.6} x_2^{-0.6}} = \frac{1}{2}$ $\frac{1}{3} = \frac{x_2}{x_1}$ $x_1 = 3x_2$
 no such point

Then we continue with $2x_1 + 2x_2 = 33$ and $2x_2 = 3x_1$

$2x_1 + 3x_1 = 33$ $x_1 = \frac{33}{5}$ $x_2 = \frac{33}{15}$

6) $U = \sqrt{X_1 + 1} + X_2$ $I = 80$ $P_1 = 10$ $X_1 \leq 3$ $P_2 = 6$ $X_2 \leq 5$
 $P_1 = 5$ $X_1 \geq 3$ $P_2 = 3$ $X_2 \geq 5$



$a \rightarrow 10X_1 + 3X_2 =$
 $b \rightarrow 5X_1 + 3X_2 =$
 $c \rightarrow 5X_1 + 6X_2 =$
 corner solution: $(0, 17)$

3) Constrained Expenditure Minimization

a) Minimize $E = P_x X + P_y Y$ s. to $\bar{U}_1 = X^{0.5} Y^{0.5}$ $P_x = 4$
 $P_y = 1$
 $\bar{U}_1 = 200$

$L = P_x X + P_y Y + \lambda (U - X^{0.5} Y^{0.5})$

$\frac{\partial L}{\partial X} = P_x - 0.5 \lambda X^{-0.5} Y^{0.5} = 0$ $P_x = 0.5 \lambda X^{-0.5} Y^{0.5}$

$\frac{\partial L}{\partial Y} = P_y - 0.5 \lambda X^{0.5} Y^{-0.5} = 0$ $P_y = 0.5 \lambda X^{0.5} Y^{-0.5}$

$\frac{P_x}{P_y} = \frac{0.5 \lambda X^{-0.5} Y^{0.5}}{0.5 \lambda X^{0.5} Y^{-0.5}} = \frac{Y}{X} = \frac{4}{1}$

$\frac{\partial L}{\partial \lambda} = U - X^{0.5} Y^{0.5} = 0$ $y = 4x$

$200 - \sqrt{4x} \sqrt{x} = 0$ $200 - \sqrt{4x^2} = 0 = 200 - 2x$ $x = 100$
 $y = 400$

$E^* = 4 \cdot 100 + 1 \cdot 400 = 800$

4) Hicksian Demand

$U = X_1 X_2^2$ P_1, P_2

a) Set up expenditure minimization problem

Minimize $E = P_1 X_1 + P_2 X_2$ s. to $\bar{U} = X_1 X_2^2$
 X_1, X_2

$L = P_1 X_1 + P_2 X_2 + \lambda (U - X_1 X_2^2)$

$L_{X_1} = P_1 - \lambda X_2^2 = 0$ $P_1 = \lambda X_2^2$
 $L_{X_2} = P_2 - 2\lambda X_1 X_2 = 0$ $P_2 = 2\lambda X_1 X_2$

$\frac{P_1}{P_2} = \frac{\lambda X_2^2}{2\lambda X_1 X_2} = \frac{X_2}{2X_1}$ $X_2 = \frac{P_1}{P_2} \cdot 2X_1$

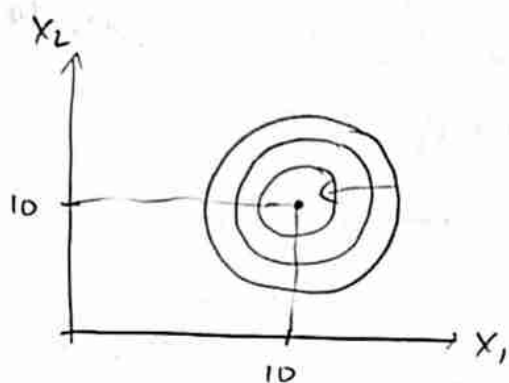
$L_Y = P_2 - 2\lambda X_1 X_2 = 0$ $U = \frac{P_1}{P_2} \cdot 2X_1^2, X_1 = \sqrt{\frac{U P_1}{2 P_2}}$
 $L_\lambda = U - X_1 X_2^2 = 0$ $U - X_1 \left(\frac{P_1}{P_2} 2X_1 \right) = 0$

$X_1 = \sqrt{\frac{U P_1}{2 P_2}} \cdot \frac{2 P_1}{P_2}$

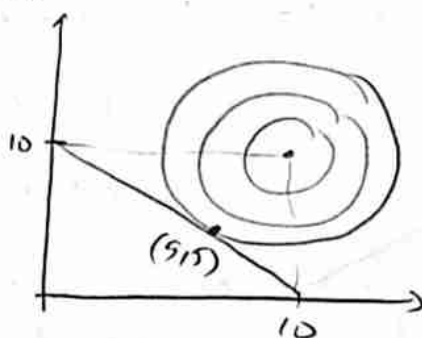
Choice with satiation

$$U = -\frac{1}{2}(x_1 - 10)^2 - \frac{1}{2}(x_2 - 10)^2$$

$$P_1 = 1 \quad P_2 = 1 \quad I = M$$

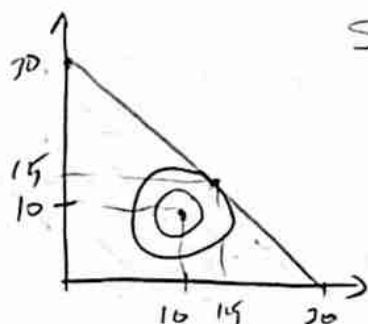


1) Suppose $M = 10$



Tangency
condition
holds
(5, 5)

b) Suppose $M = 30$



Solution is within the budget line (10, 10)

Preferences Assumptions:

* Complete: we either have $x \succeq y$, $y \succeq x$ or both

Any two bundles can be compared

* Reflexive: Any bundle is at least as good as itself $(x, x_2) \succeq (x, x_2)$

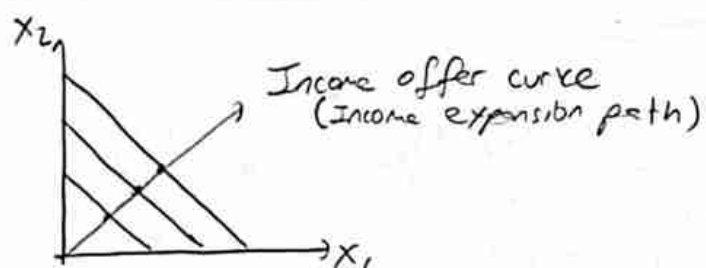
* Transitivity: if $x \succeq y$, $y \succeq z$ then $x \succeq z$

Ch 6: Demand

Demand functions give the optimal amount of the goods as a function of prices and income

$$X_i = X_i(p_1, p_2, I)$$

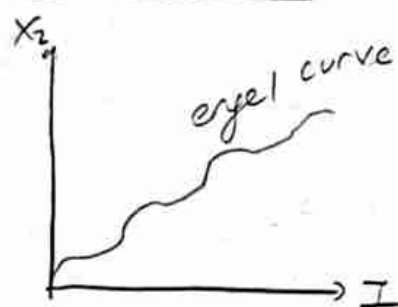
* Income Changes



if $\frac{\Delta X_i}{\Delta I} > 0$ X_i is a normal good

if $\frac{\Delta X_i}{\Delta I} < 0$ X_i is an inferior good

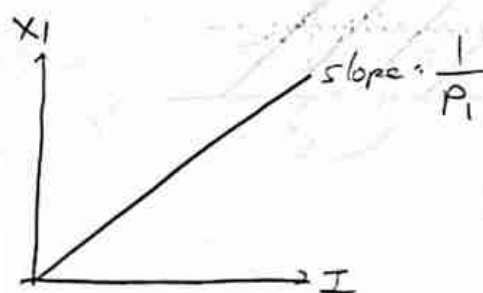
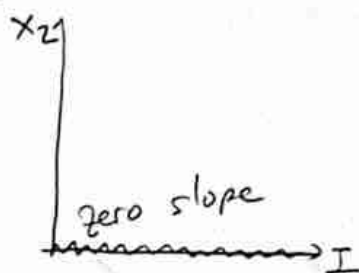
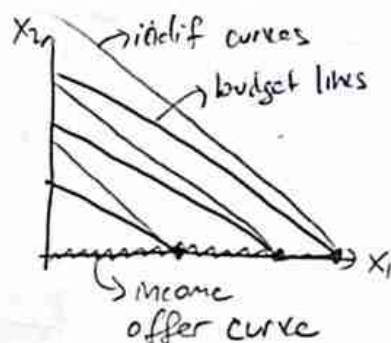
Engel Curves



The graph between income and good

Perfect Substitutes:

$$U(x) = aX_1 + bX_2 \quad \frac{p_1}{p_2} < \frac{a}{b}$$

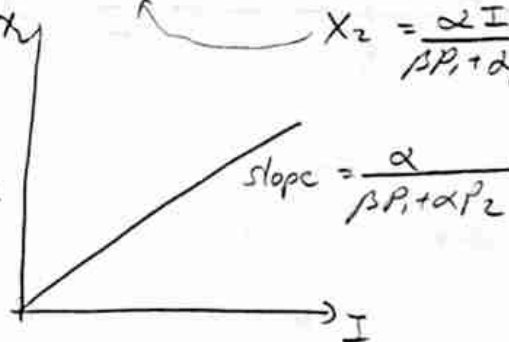
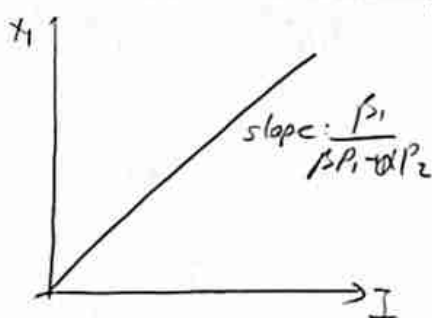
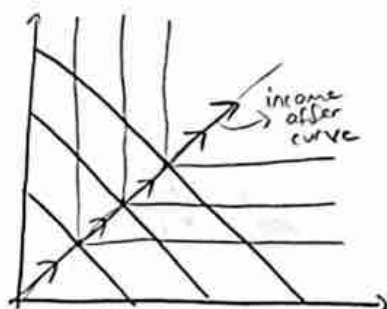


Perfect Complements: $U(x) = \{\alpha X_1, \beta X_2\}$

$$p_1 X_1 + p_2 X_2 = I$$

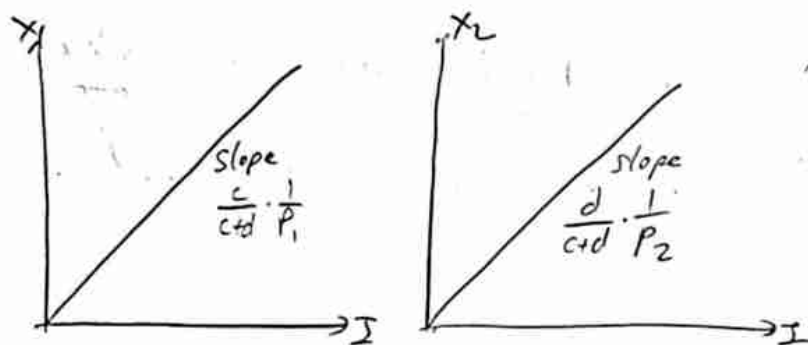
$$X_1 = \frac{\beta I}{\beta p_1 + \alpha p_2}$$

$$X_2 = \frac{\alpha I}{\beta p_1 + \alpha p_2}$$



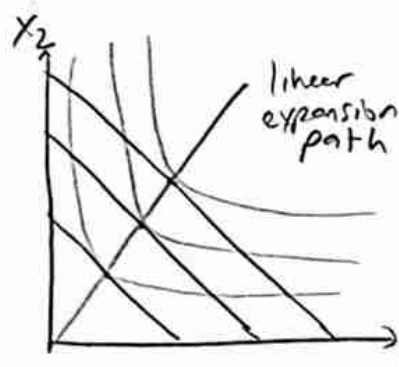
* 3 goods calcs

Cobb Douglas Preferences: $U(x) = x_1^c x_2^d$ $x_1(p, I) = \frac{c}{c+d} \cdot \frac{I}{p_1}$



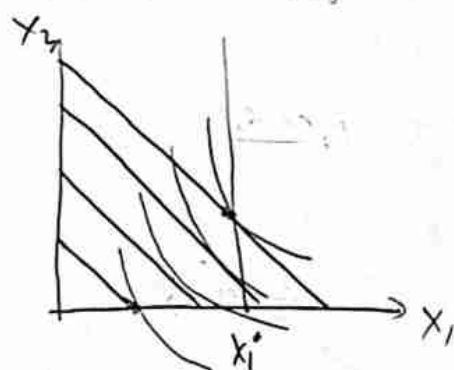
$$x_2 = \frac{d}{c+d} \cdot \frac{I}{p_2}$$

Homothetic Preferences: $(x_1, x_2) \succ (y_1, y_2) \Leftrightarrow (tx_1, tx_2) \succ (ty_1, ty_2) \forall t > 0$



HW: What is the preference type that does not satisfy this condition?

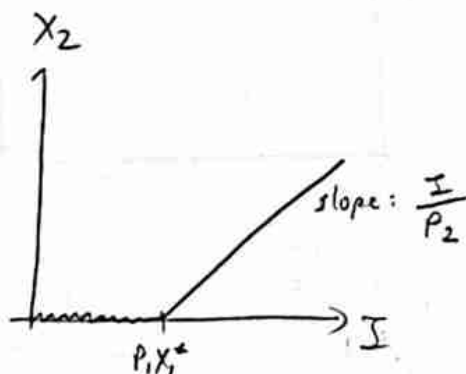
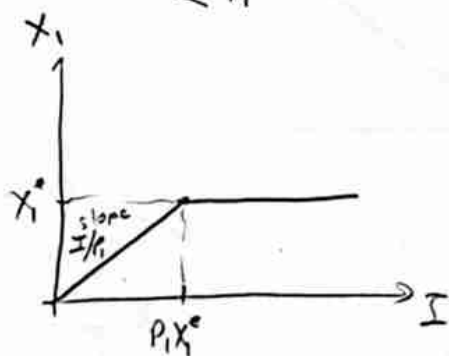
Quasi-linear Preferences: $U = V(x_1) + x_2$ Ex: $\ln x_1 + x_2$



$$\frac{1}{x_1} = \frac{p_1}{p_2} \Rightarrow x_1 = \frac{p_2}{p_1} \text{ if interior}$$

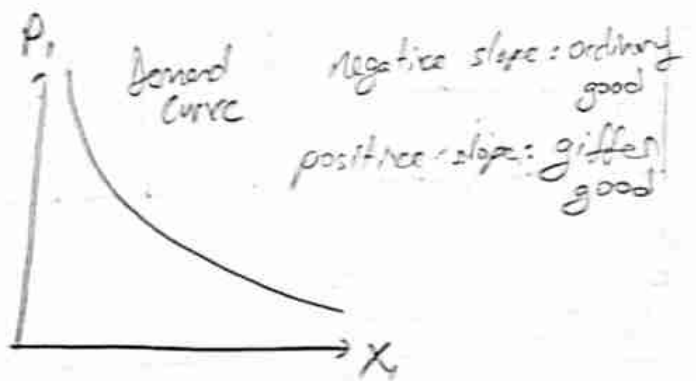
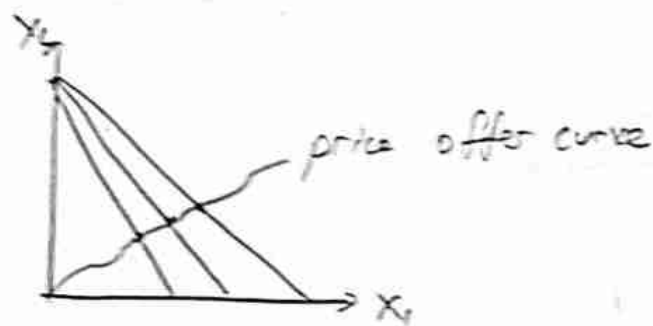
$$p_1 \cdot \frac{p_2}{p_1} + p_2 x_2 = I \quad x_2 = \frac{I}{p_2} - 1$$

$I > p_2$ to have positive x_2 otherwise corner solution

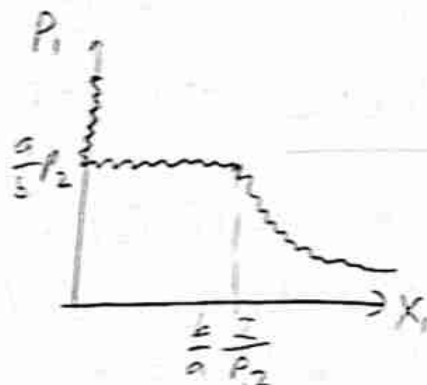
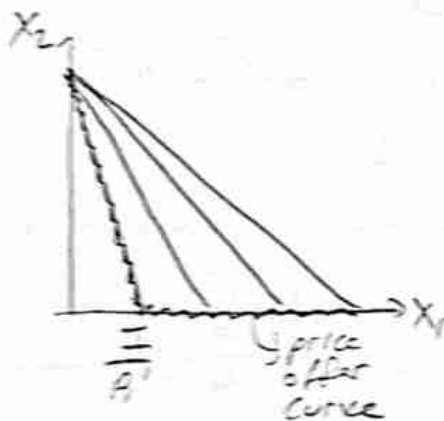


Price Charges

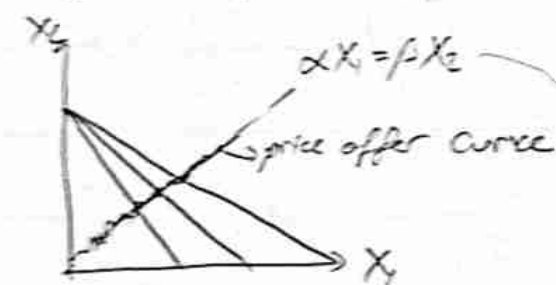
Fix $I \wedge P_2$



Perfect Substitutes: $U = \alpha X_1 + \beta X_2$

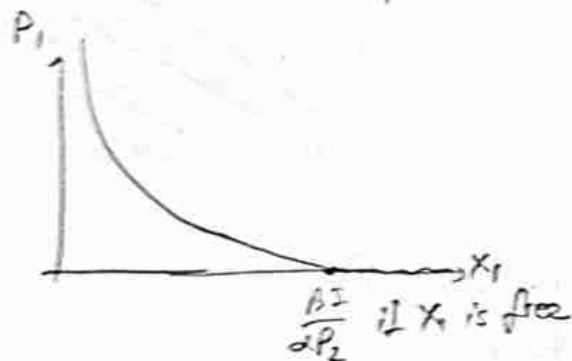


Perfect Complements: $U = \min \{ \alpha X_1, \beta X_2 \}$

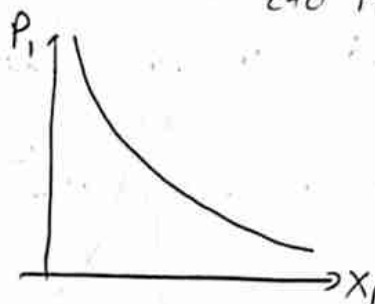
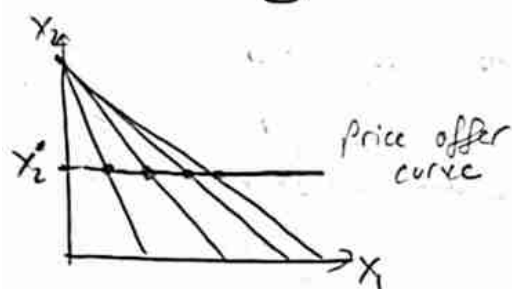


$$I = p_1 X_1 + p_2 X_2 = p_1 X_1 + p_2 \frac{\alpha X_1}{\beta}$$

$$X_1 \left(p_1 + p_2 \frac{\alpha}{\beta} \right) = I \quad X_1 = \frac{\beta I}{\beta p_1 + \alpha p_2}$$



Cobb-Douglas: $U = X_1^c X_2^d$ $X_1 = \frac{c}{c+d} \frac{I}{P_1}$ $X_2 = \frac{d}{c+d} \frac{I}{P_2}$



$\frac{dX_2}{dP_1} = 0$ X_2, P_1 don't change

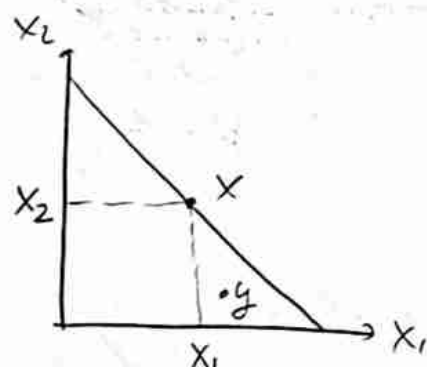
Cross-Price Effects

$\frac{\Delta X_1}{\Delta P_2} > 0$ substitutes $\frac{\Delta X_1}{\Delta P_2} < 0$ complements

Ex: $U = \min\{X_1, X_2\}$ $X_1 = \frac{I}{P_1 + P_2}$ $\frac{\partial X_1}{\partial P_2} = \frac{-I}{(P_1 + P_2)^2} < 0$

Ch 7: Revealed Preference

Suppose that when prices are (P_1, P_2) the consumer choose (X_1, X_2)



X is directly revealed preferred to y

if (X_1, X_2) is an observed choice at prices (P_1, P_2)

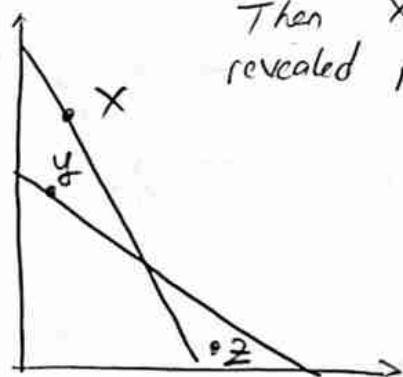
and $P_1 X_1 + P_2 X_2 \geq P_1 y_1 + P_2 y_2$ then (X_1, X_2) is directly revealed preferred to (y_1, y_2)

* The principle of revealed preference:

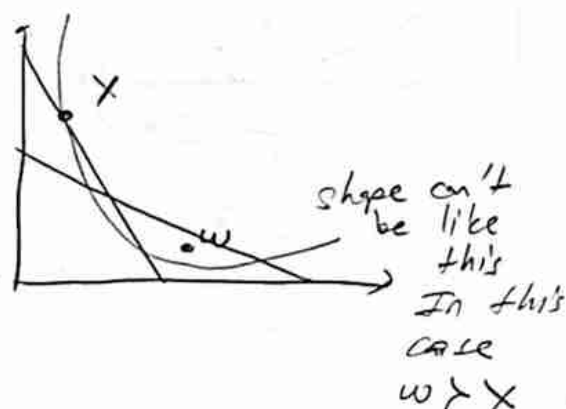
If X is directly revealed preferred to y , then $X \succ y$

Suppose X is chosen at prices (P_1, P_2) and y is chosen at prices (q_1, q_2) . Moreover, suppose X is d.r.p. to y and y is d.r.p. to z

Then X is indirectly revealed preferred to z



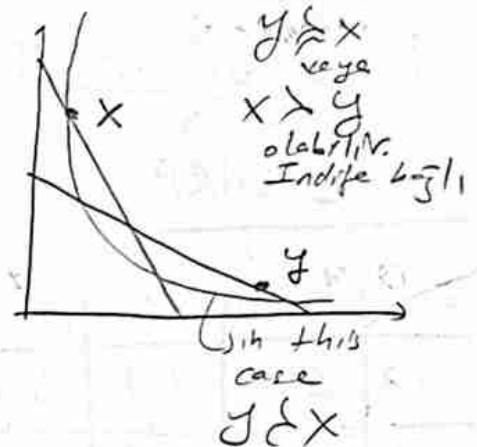
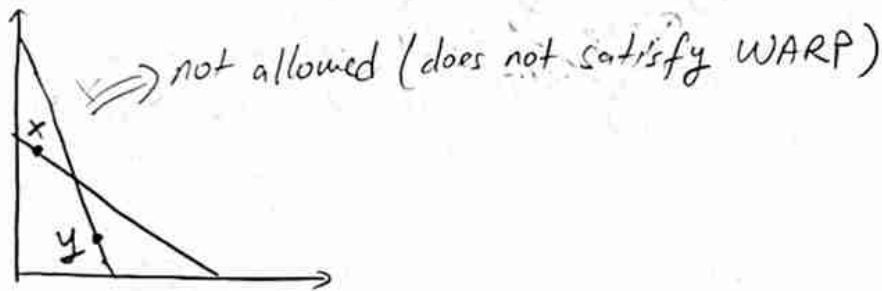
Recovering Preferences



Weak Axiom of Revealed Preferences (WARP)

If X is d.r.p. to y , then y should not be d.r.p. to X .
That is, if X is purchased at prices (p_1, p_2) and y is purchased at (q_1, q_2) , then $p_1 X_1 + p_2 X_2 \geq p_1 y_1 + p_2 y_2$, then we should have $q_1 y_1 + q_2 y_2 < q_1 X_1 + q_2 X_2$.

$$\left. \begin{array}{l} p_1 X_1 + p_2 X_2 \geq p_1 y_1 + p_2 y_2 \\ q_1 X_1 + q_2 X_2 \geq q_1 y_1 + q_2 y_2 \end{array} \right\} \text{ cannot hold at the same time}$$



Checking WARP

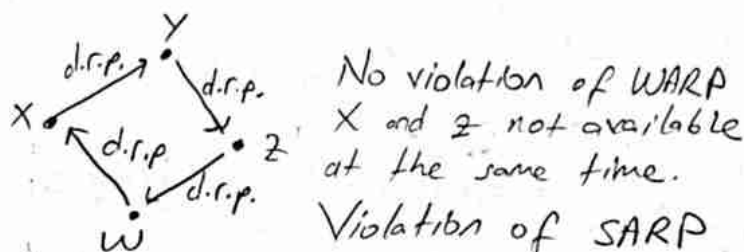
	X			Z	
P \ Q	(10, 1)	(5, 5)	(5, 4)		
(2, 2)	22	20	18	X d.r.p. y X d.r.p. Z	
(2, 1)	21	15	14	y d.r.p. Z	
(1, 2)	12	15	13	Z d.r.p. X	→ violation of WARP

22/11/2024 week 8

Strong Axiom of Revealed Preference (SARP)

The WARP requires that if X is directly revealed preferred to Y , then Y should not be directly revealed preferred to X .

The SARP requires that X is directly or indirectly revealed preferred to Y , then Y should not be directly or indirectly revealed preferred to X .



Checking SARP

$P \backslash Q$	X (3, 1, 4)	Y (2, 5, 3)	Z (6, 4, 3)
1, 3, 10	46	47	46
4, 3, 6	39	41	46
1, 1, 5	24	22	23

$X \succ Z$
 $Y \succ X$
 $Z \succ Y$

No violation of WARP

Violation of SARP

Index Numbers

b : base period $(P_1^b, P_2^b) : (X_1^b, X_2^b)$
 t : another time $(P_1^t, P_2^t) : (X_1^t, X_2^t)$

How the average consumption changed?

$$I_q = \frac{w_1 X_1^t + w_2 X_2^t}{w_1 X_1^b + w_2 X_2^b} \quad \text{if } I_q > 1 \text{ average consumption gone up}$$

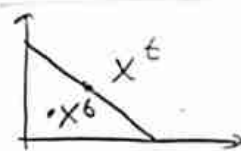
Pasche quantity index: use time t prices (current)

$$P_q = \frac{P_1^t X_1^t + P_2^t X_2^t}{P_1^t X_1^b + P_2^t X_2^b}$$

Laspeyres quantity index: use time b , prices (base)

$$L_q = \frac{P_1^b X_1^t + P_2^b X_2^t}{P_1^b X_1^b + P_2^b X_2^b}$$

suppose $P_q > 1$

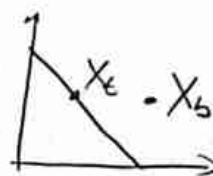


Suppose $P_q > 1$

$$P_1^t X_1^t + P_2^t X_2^t > P_1^t X_1^b + P_2^t X_2^b \Rightarrow X^t \text{ is d.r.p. to } X^b$$

Suppose $P_q < 1$

$$P_1^t X_1^t + P_2^t X_2^t < P_1^t X_1^b + P_2^t X_2^b \text{ we cannot say anything}$$



X^t may or may not be preferred to X^b

if $L_q < 1$

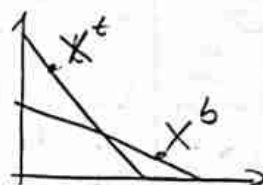
$$P_1^b X_1^t + P_2^b X_2^t < P_1^b X_1^b + P_2^b X_2^b \Rightarrow X^b \text{ is d.r.p. } X^t$$

if $L_q > 1$

$$P_1^b X_1^t + P_2^b X_2^t > P_1^b X_1^b + P_2^b X_2^b \text{ cannot say, inconclusive}$$

Suppose

$$L_q > 1 \text{ and } P_q < 1$$



inconclusive

Suppose $P_q > 1 \rightarrow X^t \text{ drp } X^b$
 $L_q > 1 \rightarrow \text{inconclusive}$

Suppose $P_q > 1$ $X^t \text{ drp } X^b$
 $L_q < 1$ $X^b \text{ drp } X^t$ WARP is violated

Problem Set 6

Q1) $U(x_1, x_2) = \ln(x_1) + \ln(x_2)$

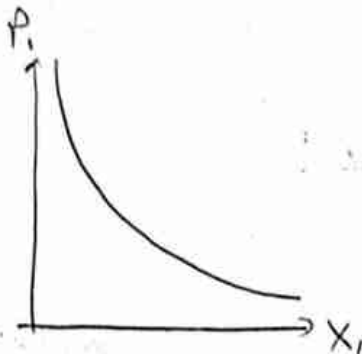
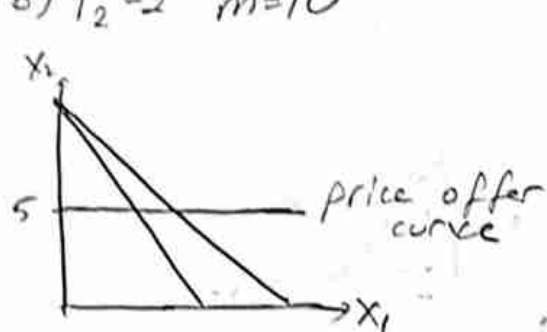
a) $MRS_{12} = \frac{P_1}{P_2} = \frac{1/x_1}{1/x_2} = \frac{x_2}{x_1} = \frac{P_1}{P_2}$

$x_1 P_1 = x_2 P_2$

BC: $P_1 x_1 + P_2 x_2 = m$

$x_1 = \frac{m}{2P_1}$
 $x_2 = \frac{m}{2P_2}$

b) $P_2 = 1$ $m = 10$

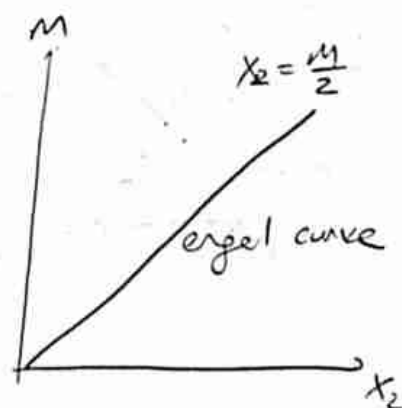
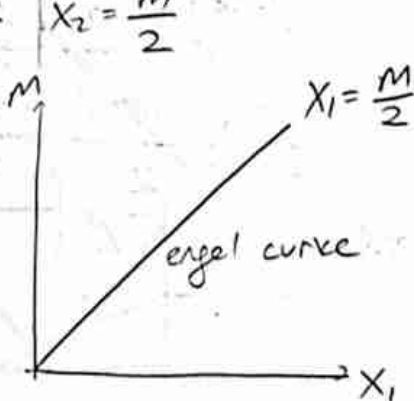
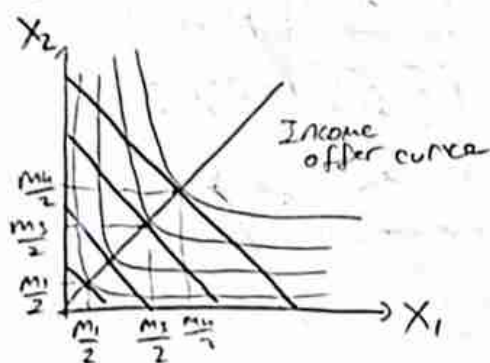


$P_1 = \frac{m}{2x_1} = \frac{5}{x_1}$

c) $\frac{dx_1}{dP_1} < 0$ ordinary good

$\frac{dx_1}{dP_1} > 0$ giffen good

d) $P_1 = 1, P_2 = 1$ $x_1 = \frac{m}{2}$ $x_2 = \frac{m}{2}$



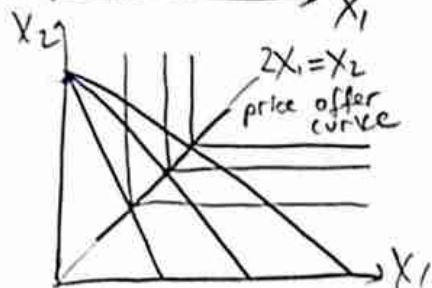
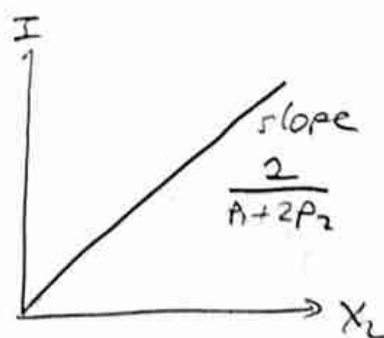
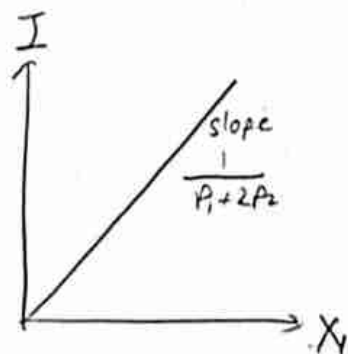
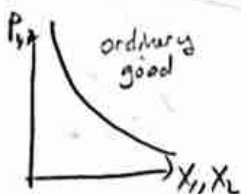
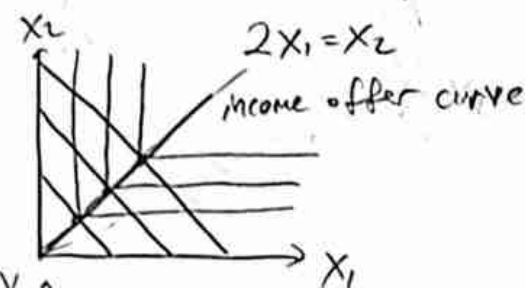
Q2) $U = \min\{2x_1, x_2\}$ $2x_1 = x_2$

$2x_1 = x_2$

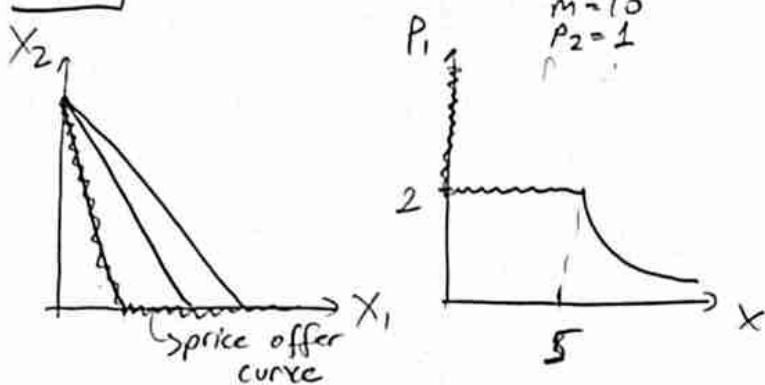
$I = P_1 x_1 + P_2 x_2$

$I = P_1 x_1 + 2P_2 x_1$ $x_1(P_1 + 2P_2) = I$

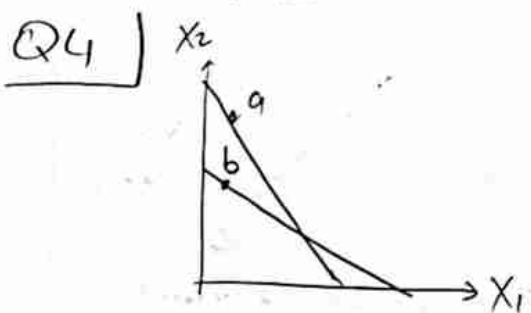
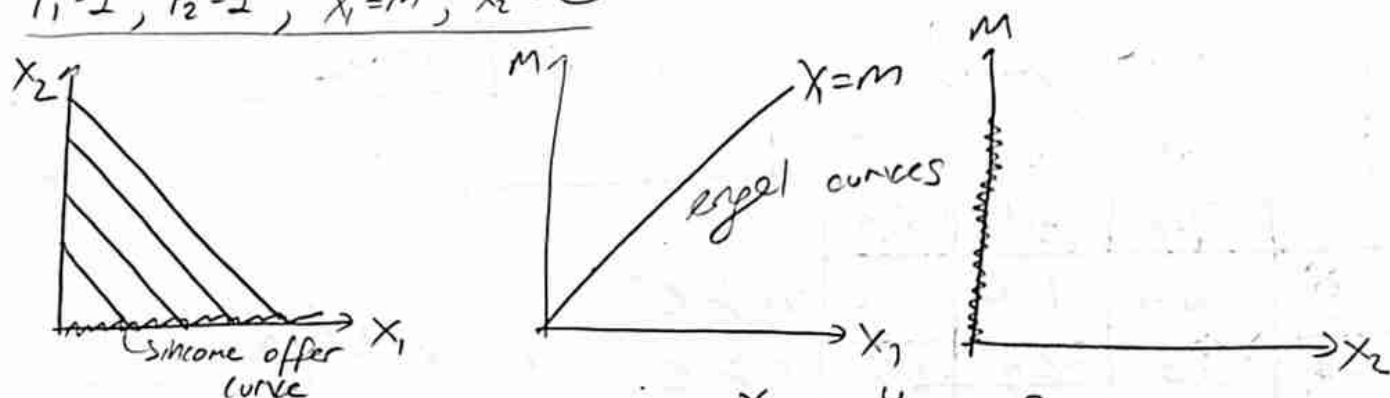
$x_1 = \frac{I}{P_1 + 2P_2}$ $x_2 = \frac{2I}{P_1 + 2P_2}$



Q3) $U(X_1, X_2) = 2X_1 + X_2$ $MRS = 2$ if $2 < \frac{P_1}{P_2}$, $X_1 = 0$, $X_2 = \frac{M}{P_2}$
 if $2 = \frac{P_1}{P_2}$, $X_1 = \alpha$, $X_2 = \frac{M - P_1 \alpha}{P_2}$
 if $2 > \frac{P_1}{P_2}$, $X_1 = \frac{M}{P_1}$, $X_2 = 0$



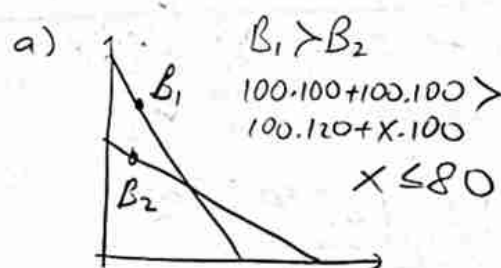
$P_1 = 1, P_2 = 1, X_1 = M, X_2 = 0$



	X	Y	Z	
P/Q	1, 90	0, 1, 0	0, 0, 1	
1, 1/2	1	1	2	$X > Y$
2, 1, 1	2	1	1	$Y > Z$
1, 2, 1	1	2	1	$Z > X$

- a) Violation of SARP
 b) Not consistent

	Year 1	Year 2
Good 1	$\frac{Q}{100} \quad \frac{P}{100}$	$\frac{Q}{120} \quad \frac{P}{100}$
Good 2	$100 \quad 100$	$X \quad 80$



b) $120 \cdot 100 + 80X \geq 100 \cdot 100 + 100 \cdot 80$
 $X \geq 75$

c) $75 \leq X \leq 80$
 WARP violated

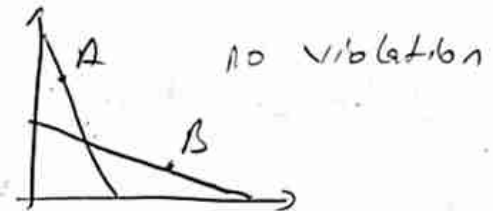
d) $\frac{\partial M}{\partial Q} < 0$ Quota on income α at $M=11$
 $X \leq 80$
 $X \leq 75$

e) $I_{Year 2} > I_{Year 1}$
 $X \geq 80$

Q6

a) $P_1(6,3)$ $P_2(1,2)$ $I_1 = 108 > 8 \cdot 6 + 14 \cdot 3 = 90 \quad 1 \succ 2$
 $B_1(3,18)$ $B_2(2,14)$ $I_2 = 36 < 3 \cdot 1 + 18 \cdot 2 = 45$ affordable değil
 WARP affordable değil

b) Bundle A = (6,6) } $I_A = 48$
 Prices A = (6,2) } $10 \cdot 6 + 0 \cdot 2 = 60$
 affordable değil no comment
 Bundle B = (10,0) } $I_B = 20$
 Prices B = (2,5) } $6 \cdot 2 + 6 \cdot 5 = 42$
 affordable değil no comment



c)

	A	B	C	
	7,8	3,4	7,5	
16,2	144	160	132	$A \succ C$
5,6	83	69	65	$B \succ C$
3,3	45	39	36	no comment

no violation

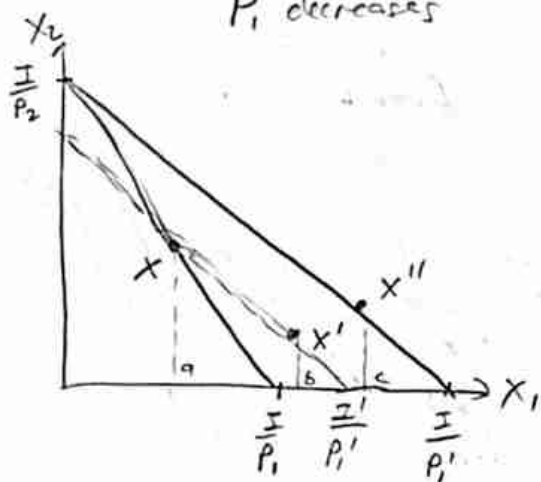
d)

	A	B	C	
	2,20	10,12	14,11	
4,2	72	66	78	$A \succ B$
2,4	36	72	72	$B \succ C$
P_1, P_2	$8P_1 + 20P_2$	$10P_1 + 12P_2$	$14P_1 + 11P_2$	

SARP violation için
 $C \succ A$ için
 ya
 $8P_1 + 20P_2 < 14P_1 + 11P_2$
 $3P_2 < 6P_1$
 $3P_2 < 2P_1$
 $P_2 < \frac{2}{3}P_1$

28/11/2024 week 9

P_1 decreases



Create an imaginary budget line that passes through point X and parallel to new budget line

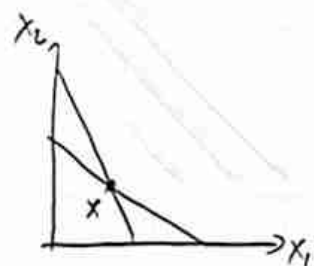
$|ab|$: substitution effect

$|bc|$: income effect

$|ac|$: total effect

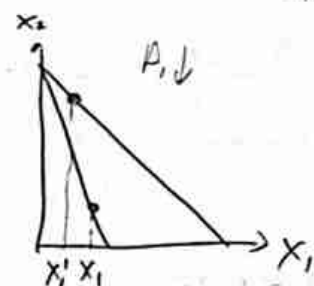
$$I - I' = (P_1 - P_1')X_1 \quad I' < I$$

If WARP satisfies, S.E. always positive/negative?



Compensated price change

$P_1 \downarrow$ so $X_1 \uparrow$ compensated law of demand



if $X'_1 < X_1$ Giffen good

if $X'_1 > X_1$ ordinary good

uncompensated demand

ex/ Demand for milk

$$X_1 = 10 + \frac{I}{10P_1} \quad I_{\text{initial}} = 120 \quad P_1 = 3 \quad X_1 = 14$$

$$\text{suppose } P_1' = 2 \rightarrow X_1' = 10 + \frac{120}{20} = 16$$

initial milk spending: $14 \times 3 = 42$
Milk consumption with new prices: $14 \times 2 = 28$

initially $120 - 42 = 78$ liras left for other goods

$$I' = 2 \times 14 + 78 = 106 \text{ liras to buy initial bundle}$$

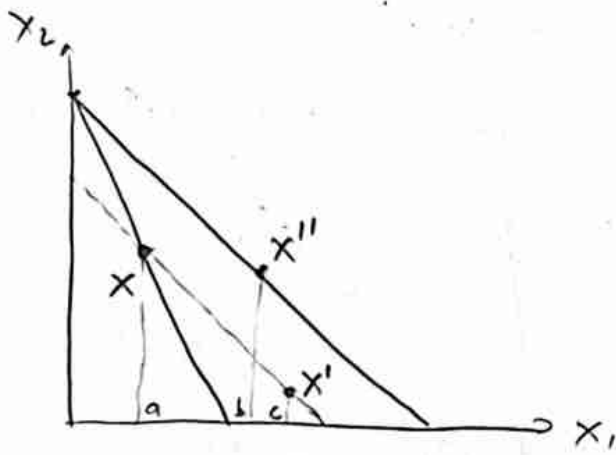
$$X_1' = 10 + \frac{106}{10 \times 2} = 15.3$$

$14 \rightarrow 15.3$ substitution effect

$15.3 \rightarrow 16$ income effect

both positive, primary good

Inferior goods:



$|ac|$ = substitution effect

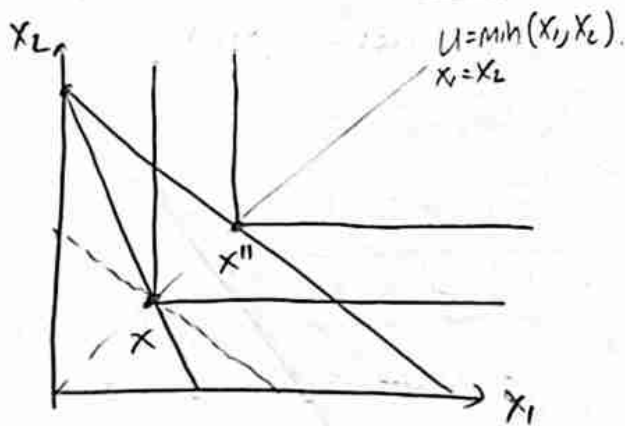
$|cb|$ = income effect

$|ab|$ = total effect

positive substitution effect
negative income effect

Ortadaki fiyat düşüşü değil
artışı citabilir.

Perfect Complements



No substitution effect
Only income effect

EX

$$U = X_1^{0.3} X_2^{0.7} \text{ (Cobb-Douglas)} \quad P_1 = 2 \quad P_2 = 1 \quad I = 100$$

$$P_1' = 1 \quad P_2' = 1$$

$$\left. \begin{aligned} X_1 &= 0.3 \cdot \frac{100}{2} = 15 \\ X_1' &= 0.3 \cdot \frac{100}{1} = 30 \end{aligned} \right\} \text{Total change} = 15$$

$I_1' = 15 \cdot 1 + 70 = 85$ with this income, I can buy the same bundle
indirildiden sonra harcadığım para (ilk bundle için)

$$X_1' = 0.3 \cdot \frac{85}{1} = 25.5 \quad 15 \rightarrow 25.5 \rightarrow 30$$

s.e. i.e.

Hicks Decomposition (keep utility same)

$$U = X_1^{0.4} X_2^{0.6}$$

$$P_1 = 0.5$$

$$P_2 = 1$$

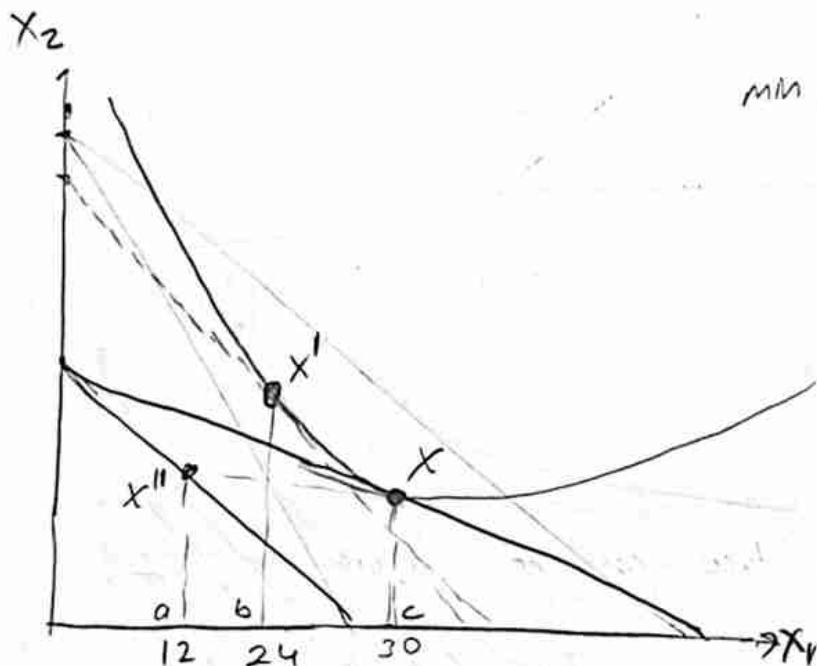
$$I = 30$$

$$P_1' = 1$$

$$P_2' = 1$$

$$h(P, U) = U \left(\frac{0.4}{0.6} \cdot \frac{P_2}{P_1} \right)^{0.6}$$

$$\min P_1 X_1 + P_2 X_2 \text{ s.t. } U(X_1, X_2)$$



$$U^0 = 24^{0.4} 18^{0.6}$$

|cb| = substitution effect

|ba| = income effect

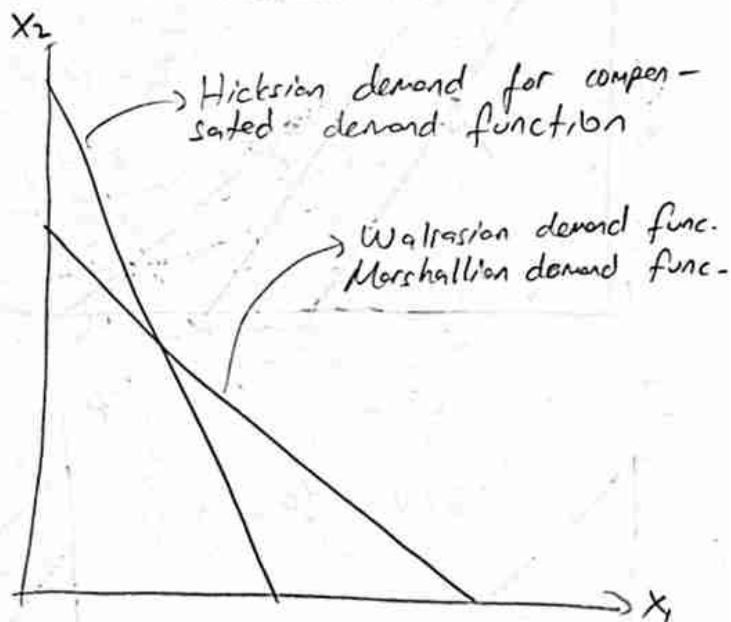
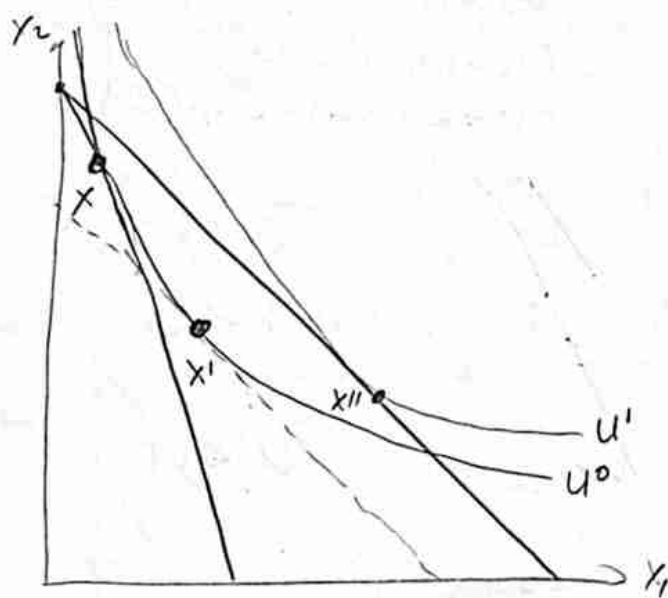
$$HW: U = 2\sqrt{x} + y$$

$$P_x = 0.5$$

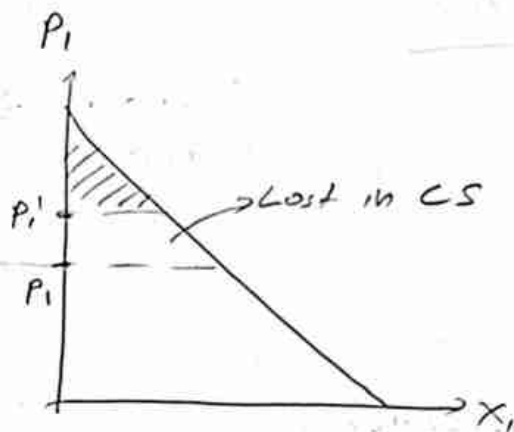
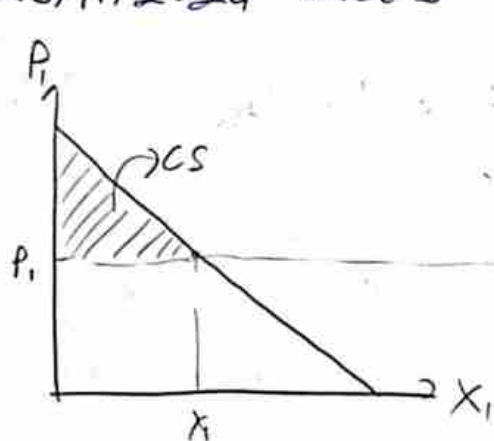
$$P_x' = 0.2$$

$$I = 10$$

$$P_y = 1$$



23/11/2024 week 9



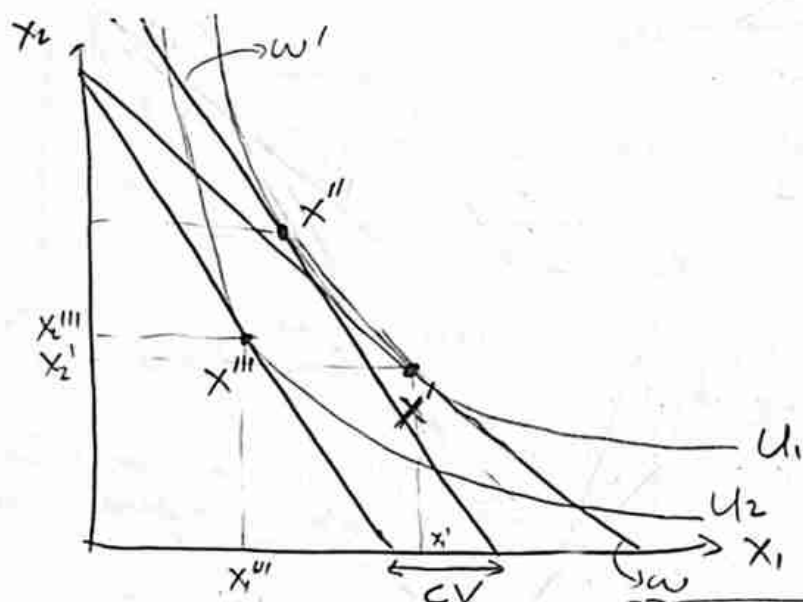
$$0 > \Delta CS$$

$$= \int_{P_1}^{P_2} X(P, I) dP$$

We want to measure how much a consumer is hurt by a price increase

Compensating Variation

$P_1 \uparrow$ What is the smallest amount of additional income which at new prices would just restore consumer's original utility



$$I_1 = P_1 X_1' + P_2 X_2'$$

$$I_2 = P_1' X_1'' + P_2 X_2'' = e(P', u)$$

$$CV = w - e(P', u) < 0$$

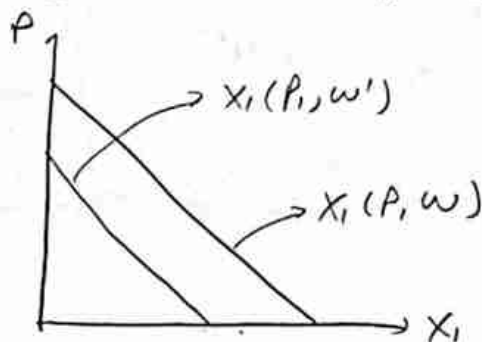
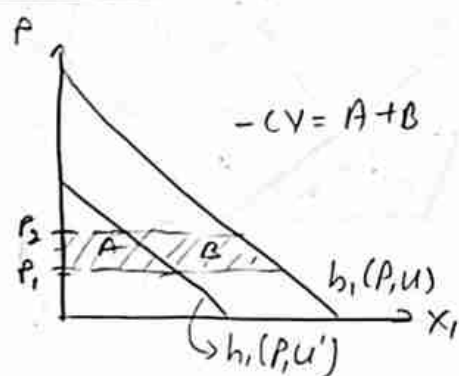
$$U_2 < U_1$$

$$e(P', u) = w' > w$$

$$CV = e(P, u) - e(P', u)$$

$$CV = \int_{P_1}^{P_2} \frac{\partial e(P, u)}{\partial P_1} dP_1$$

$$CV = \int_{P_1}^{P_2} h_1(P, u) dP_1 < 0$$



$$U' < U$$

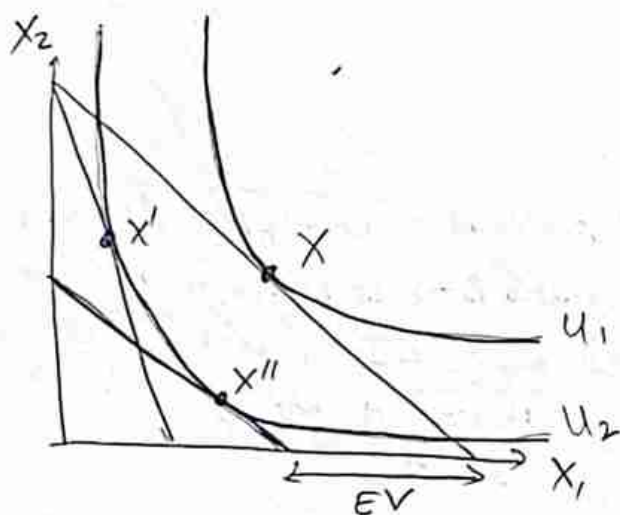
These are for normal goods. in HT, could be giffen goods

Equivalent Variation

What is the smallest amount of income which, at the original prices, would just restore consumer's original utility

How much money would we have to take from consumers to harm this consumer by as much as the price increase?

Fiyatı artırarak bir zarar verdik. Fiyatı değiştirmeyip income'ı düşürerek aynı zararı nasıl verebiliriz?

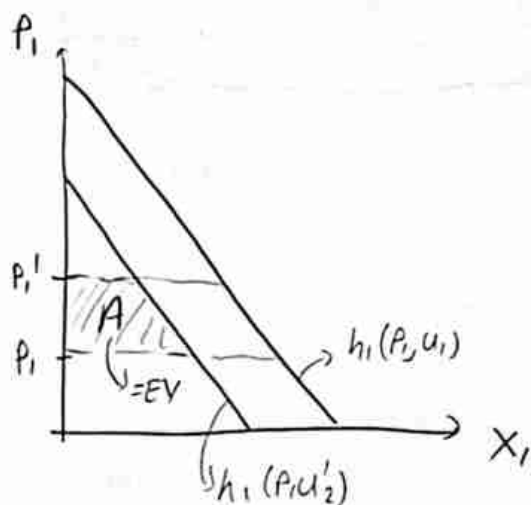


Fiyatı düşürünce X den X' noktasına geldik. Fiyat düşürmeyip geliri azaltırsaydık X'' noktasına gelecektik. X' ve X'' noktalarının utility'si aynı. Aynı zararı verdik.

$$\begin{aligned} X' &= \arg\max U(X_1, X_2) \text{ s.t. } p_1' X_1 + p_2 X_2 = I \\ X &= \arg\max U(X_1, X_2) \text{ s.t. } p_1 X_1 + p_2 X_2 = I \\ X'' &= \arg\max U(X_1, X_2) \text{ s.t. } p_1 X_1 + p_2 X_2 = I' \end{aligned}$$

$$I = p_1 X_1 + p_2 X_2 = p_1' X_1' + p_2 X_2' = e(p_1, u_1) = e(p', u_1)$$

$$I' = p_1 X_1'' + p_2 X_2'' \quad EV = I' - I < 0 \quad I' = e(p, u_2)$$



$$EV = e(p, u_2) - \frac{e(p', u_2)}{I}$$

$$\int_{P_1}^{P_2} h_1(p, u_2) dp < 0$$

for a price increase (normal good)

$$|CV| > |\Delta CS| > |EV|$$

For a price increase:

Normal good $\rightarrow |CV| > |\Delta CS| > |EV|$

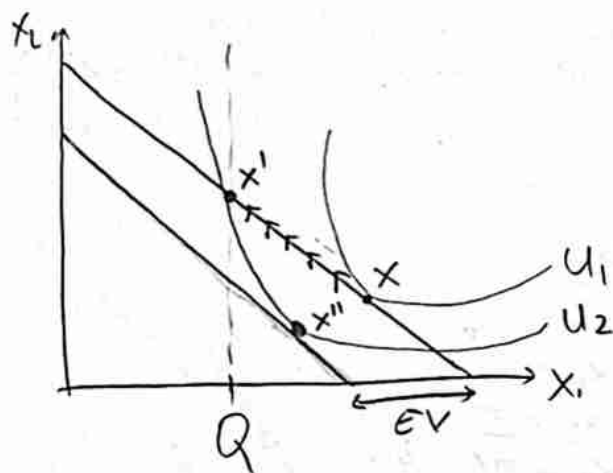
Inferior good $\rightarrow |CV| < |\Delta CS| < |EV|$

HW: Show this case is valid

HW: The case for giffen goods

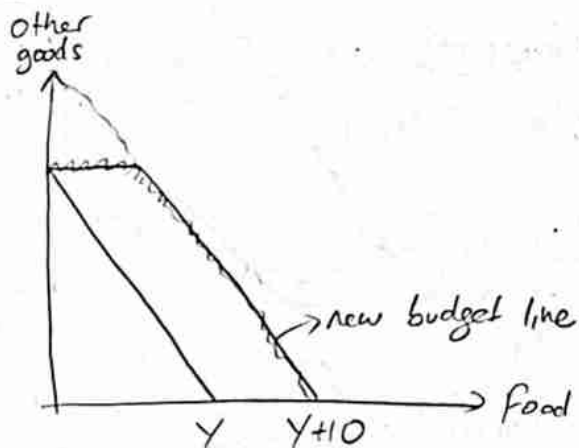
Effect of Government Policies on Consumer Welfare

- Quotas
- Food Aid
- Child care subsidies



Scenario A $\rightarrow$ Quota on X_1 (Q)

Scenario B $\rightarrow$ Decrease in income
if $\Delta EV = \Delta I$, agent is indifferent
to choose A and B



+10 € coupon. Bought other goods again.
Would it make any difference to give cash?

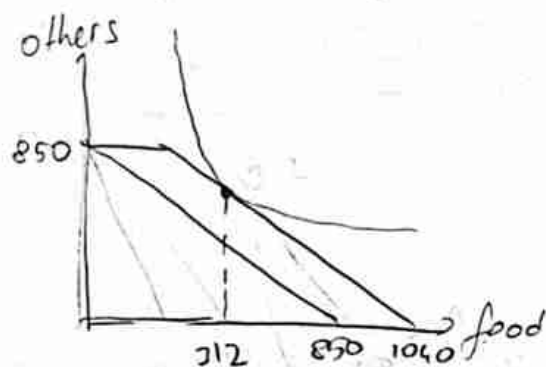
Ex/ Assume that low income families spend 30% of income on food.

$I = 850\$$ and they receive 130 \$ food coupon

If I give them cash, $0.3(850 + 130) = 312\$$ they will spend on food.

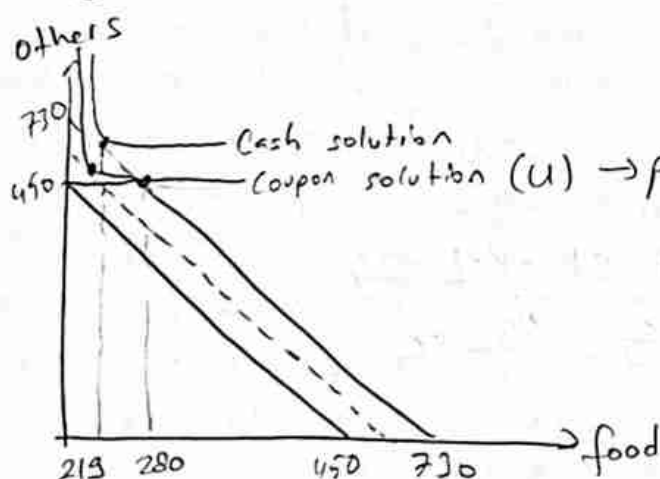
So, in both cases, they will spend 312 \$ *costo similar dağırmedir*

No welfare loss for the family



Ex: Income is 450\$ and they receive 280\$ food stamps. If I give them cash $450 + 280 = 730\$$

They spend 30% $730 \times 0.3 = 219\$$ on food



Cash solution
Coupon solution (U) $\rightarrow$ passes through both cash and coupon solution

Government wastes by giving food stamps rather than cash

A $\rightarrow 280\$$ food stamp

B $\rightarrow 219\$$ cash

Same effect for food expenditure

05/12/2024 week 10

Labor Market

M = money income

C = consumption level

w = wage rate

P = Unit price of consumption

$\bar{L}$ = Labor endowment (24 hours)

L = Labor supplied

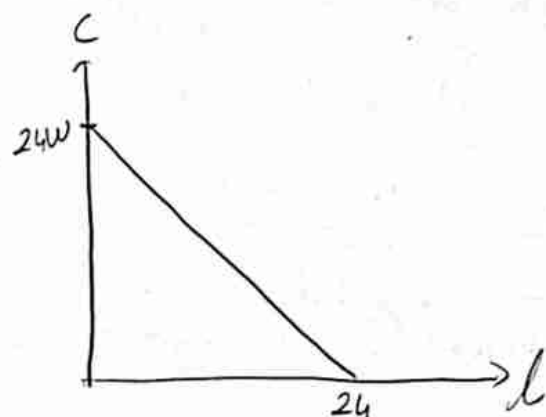
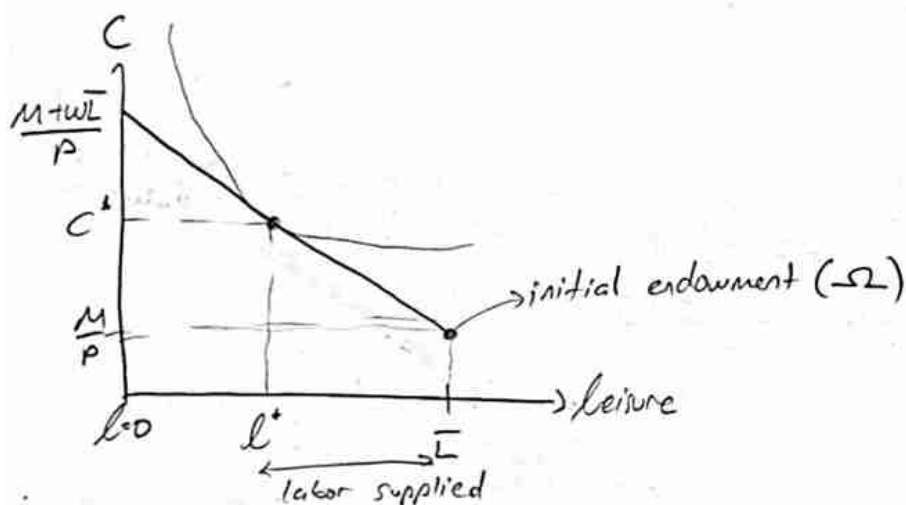
l = Leisure

$$PC = M + w \cdot L$$

$$PC - w(\bar{L} - L) = M + wL + w(\bar{L} - L)$$

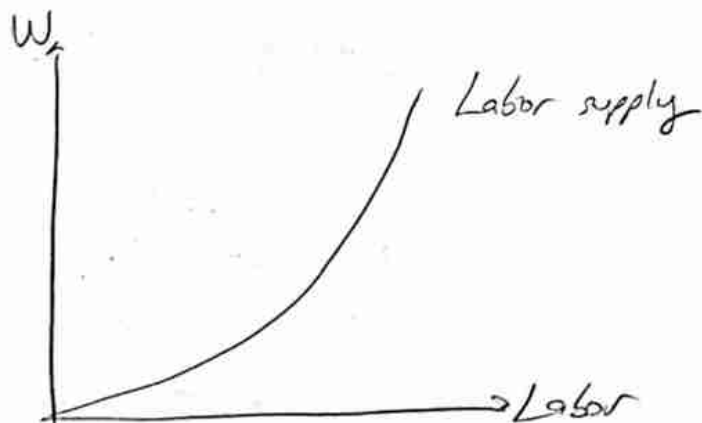
$$PC + wL = M + w\bar{L}$$

Budget
Constraint



$$\text{Max } U(l, C) \text{ s.t. } l + L = 24$$

$$24 = l + \frac{C}{w} \quad wL = C$$

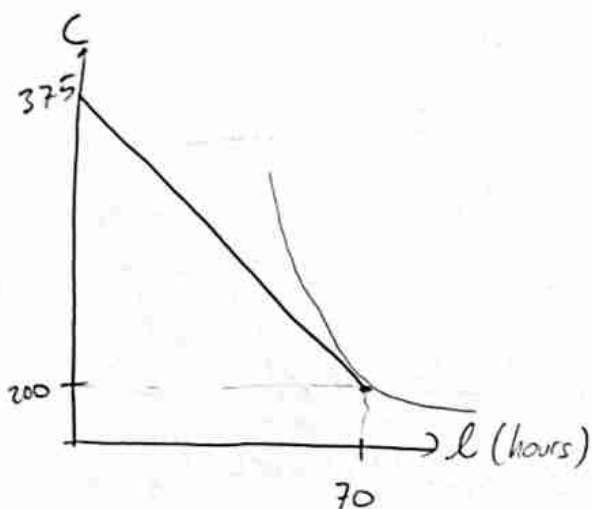


EX

Ece is a waitress and has 70 hours every week to divide btw work and leisure

$$U = C^{0.5} l^{0.5} \quad w = 5$$

Government gives her 200 no matter she works or not but taxes her wage income 50%



$$MRS = \frac{\frac{1}{2} l^{-1/2} C^{1/2}}{\frac{1}{2} l^{1/2} C^{-1/2}} = \frac{175}{70} \text{ slope}$$

$$\frac{175}{70} = \frac{C}{l} = \frac{375 - C}{l}$$

$$\frac{375 - C}{l} = \frac{175}{70}$$

$$l = 75 ?$$

Max time was 70
Corner solution

$$(l^*, C^*) = (70, 200)$$

EX Critique Socu

Ece is a single parent with two dependent children. She is also a waitress and has 24 hours every week to divide between work and leisure. Her utility function is:

$$U(l, C) = l^2 C \quad w = 9 \quad PC = 1$$

$$\text{Max } l^2 C \quad \text{s.t.} \quad C + 9l = 24w$$

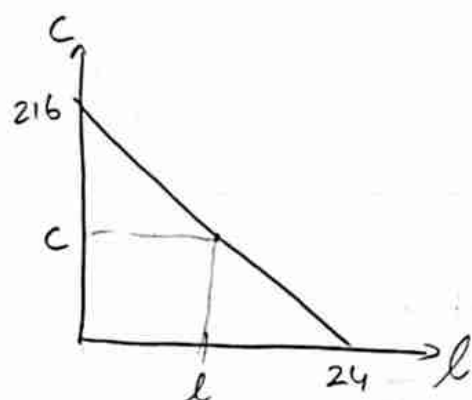
a) What is her optimal leisure-consumption choice?

b) Under AFDC program, gov gives her X no matter whether she works or not but impose %100 tax rate on the wage earnings of the aid recipient. If her wage earnings are higher than X, she is not eligible for the government welfare program. Draw her new budget set.

c) Find the critical level of X such that she will stop working.

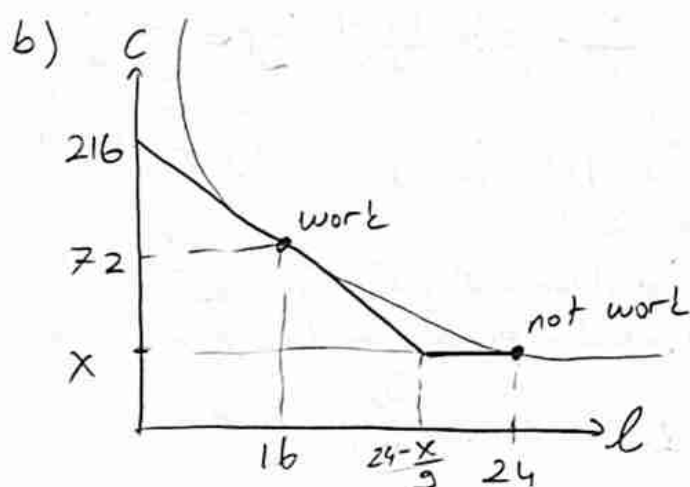
$$U(l, c) = l^2 c \quad w=9 \quad PC=1 \quad \max l^2 c \text{ s.t. } c+9l=24w$$

$$\frac{2lc}{l^2} = 9 \rightarrow 2c = 9l$$



$$\frac{216 - c}{l} = 9 \quad \cdot \quad 216 - c = 9l = 2c$$

$$c = 72 \quad l = 16$$



$$\frac{16^2 \cdot 72}{U_{\text{work}}} = \frac{24^2 X}{U_{\text{not work}}}$$

$$X = 32$$

EX: c : childcare measured in hours
 f : food measured in units
 e : entertainment measured in hours

$$U = c \cdot f \cdot e \quad P_c = 1$$

$$P_f = 1$$

$$P_e = 1$$

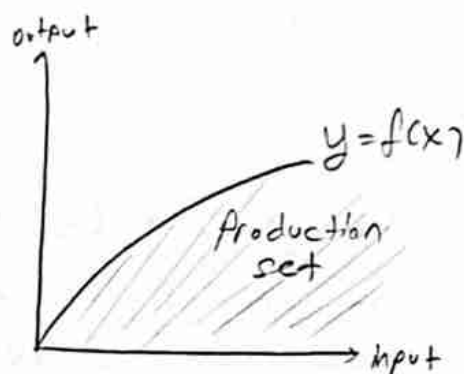
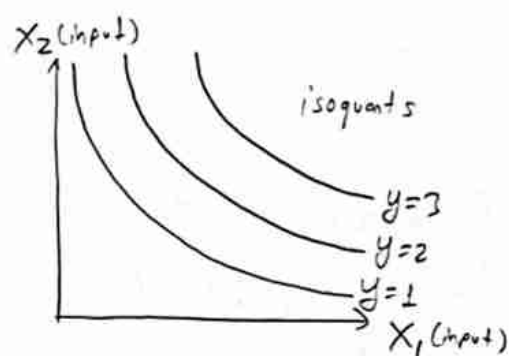
$$I = 5 \text{ £}$$

Suppose the gov decides to give 20£ worth of childcare coupon and 10£ worth of food coupon. Assume coupons cannot be resold.

a) Would the consumer be better off if the gov gave him 30£ of cash, instead of coupons

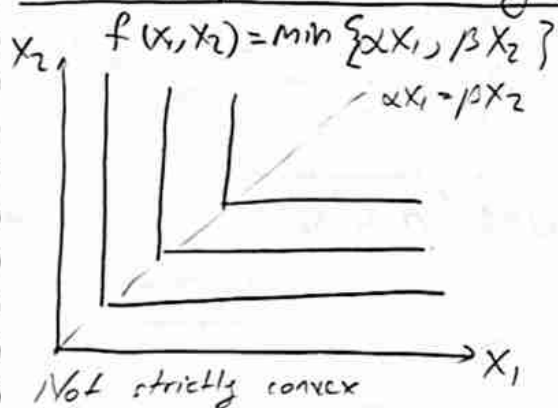
b) What is the amount of the minimum cash transfer to the consumer which would make him as happy as receiving coupons

Production



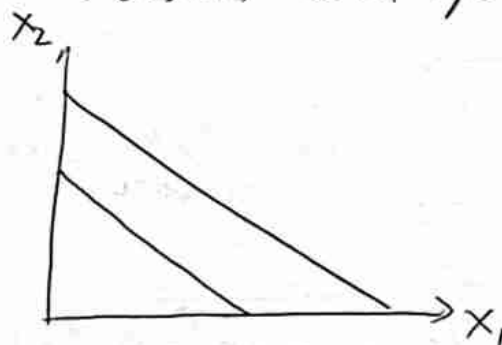
No monotonic transformation!
Magnitude of y matters

Fixed Proportions Technology

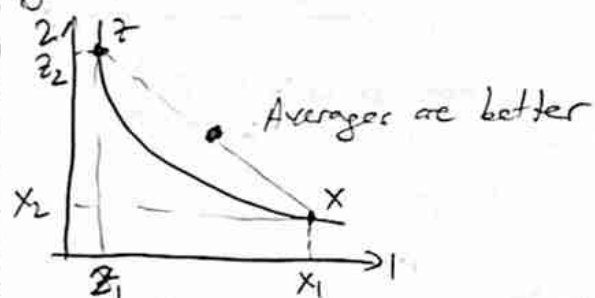


Perfect Substitutes

$$f(x_1, x_2) = \alpha x_1 + \beta x_2$$



Convex technology: Let (x_1, x_2) and (z_1, z_2) be two ways to produce y units of output. Then any linear combination will produce at least y units.



Monotonic technology: If at least one of the technology input increase then the output should not decrease

The marginal product: $MP_i = \frac{\partial f}{\partial x_i}$ $y = f(x_1, x_2, \dots, x_i)$

The technical rate of substitution: $TRS = - \frac{MP_1}{MP_2}$

Diminishing marginal product: $\frac{\partial^2 f}{\partial x_1^2} < 0$

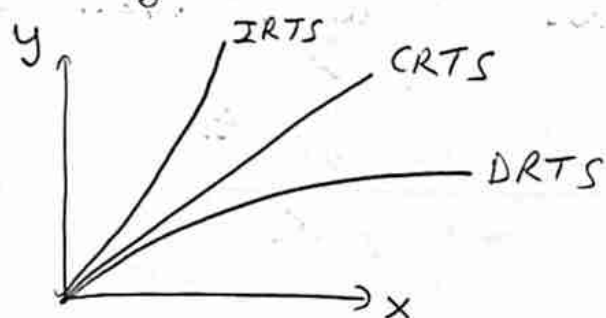
Long run: every input can vary
Short run: every input cannot change

Returns to Scale:

Constant returns to scale (CRTS): $f(tx_1, tx_2) = t f(x_1, x_2)$ ($t > 0$)

Increasing returns to scale (IRTS): $f(tx_1, tx_2) > t f(x_1, x_2)$ ($t > 1$)

Decreasing returns to scale (DRTS): $f(tx_1, tx_2) < t f(x_1, x_2)$ ($t > 1$)



Cobb-Douglas $\rightarrow f(x_1, x_2) = A x_1^a x_2^b$ $f(tx_1, tx_2) = t^{a+b} A x_1^a x_2^b$

if $a+b = 1$ CRTS
 $a+b > 1$ IRTS
 $a+b < 1$ DRTS

Fixed proportions $\rightarrow f(x_1, x_2) = \min \{ \alpha x_1, \beta x_2 \}$

$f(2x_1, 2x_2) = \min \{ 2\alpha x_1, 2\beta x_2 \} = 2 f(x_1, x_2)$ CRTS

* Markets are competitive

Suppose n outputs $(y_1, y_2, \dots, y_n)$

Using m inputs $(x_1, \dots, x_m)$

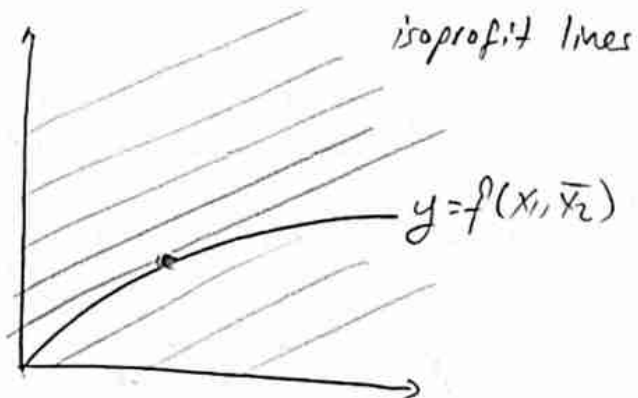
$(P_1, P_2, \dots, P_n) \rightarrow$ output prices

$(w_1, \dots, w_m) \rightarrow$ input prices

$$\pi = \sum_y P_i y_i - \sum_x w_j x_j \quad \text{py s.t. } y \in Y$$

$\max p \cdot f(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$ SR profit maximization

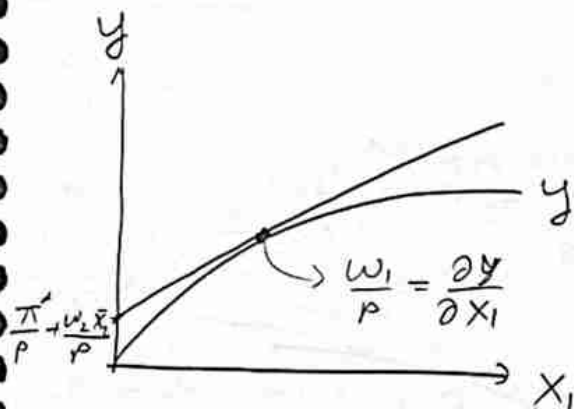
$$\frac{\partial \pi}{\partial x_1} = p \frac{\partial f}{\partial x_1} = w_1 \quad (\text{interior solution}) \quad \frac{pMP_1}{MR_1} = MC_1$$



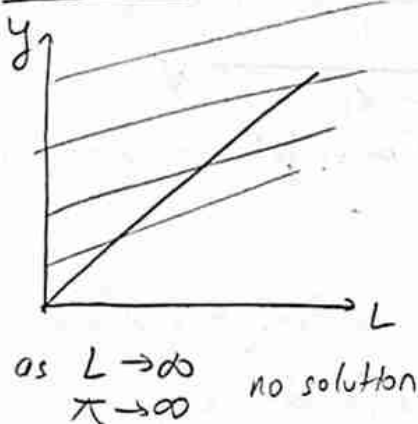
$$\pi = py - w_1 x_1 - w_2 \bar{x}_2$$

$$y = \frac{\pi + w_2 \bar{x}_2 + w_1 x_1}{p} = \underbrace{\frac{\pi + w_2 \bar{x}_2}{p}}_{\text{constant}} + \frac{w_1}{p} x_1$$

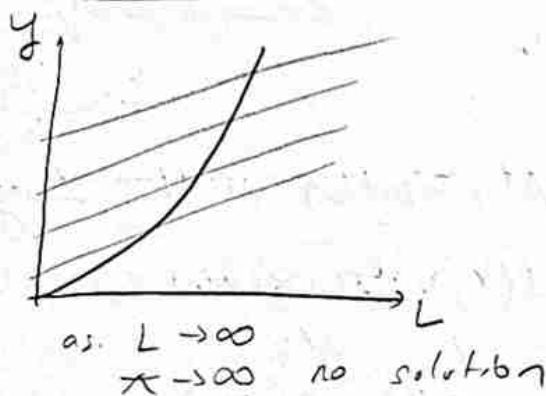
↓
catch (intercept)
all
constant terms
slope: $\frac{w_1}{p}$



CRTS



IRTS



12/12/2024 week 10

$$EX: f(x_1, x_2) = A x_1^a x_2^b$$

$$f(tx_1, tx_2) = t^{a+b} A x_1^a x_2^b = t^{a+b} f(x_1, x_2)$$

$$\text{if } a+b = 1 \quad \text{CRTS}$$

$$a+b < 1 \quad \text{DRTS}$$

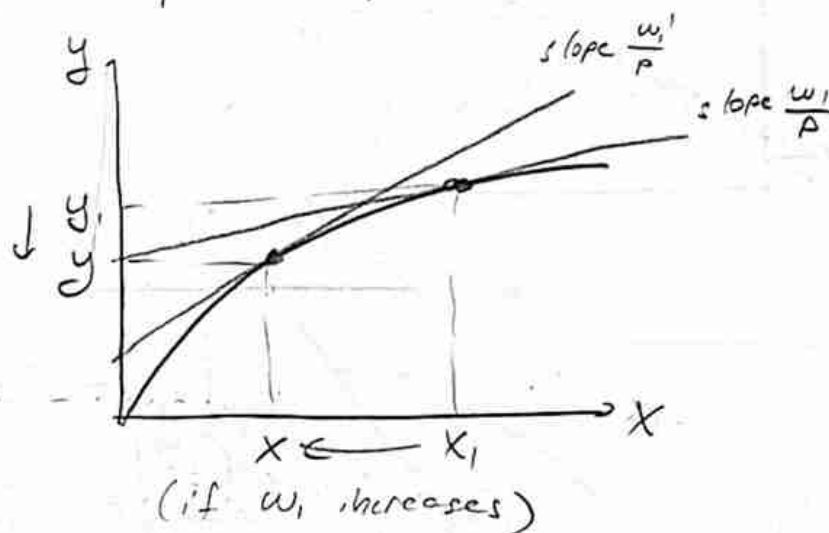
$$a+b > 1 \quad \text{IRTS}$$

$$EX: f(x_1, \bar{x}_2) = A x_1^a \bar{x}_2^b \quad \max p A x_1^a \bar{x}_2^b - w_1 x_1 - w_2 \bar{x}_2$$

$$\frac{\partial \pi}{\partial x_1} = 0 \Rightarrow a p A x_1^{a-1} \bar{x}_2^b - w_1 = 0$$

$$x_1 = \left(\frac{w_1}{a p A \bar{x}_2^b} \right)^{1/a-1} \quad \text{unconditional input demand function}$$

$$y = A \left(\frac{w_1}{a p A \bar{x}_2^b} \right)^{a/a-1} \bar{x}_2^b$$



Profit Maximization in the Long Run

$$\max_{x_1, x_2} p f(x_1, x_2) - w_1 x_1 - w_2 x_2$$

$$\frac{\partial \pi}{\partial x_1} = 0 \quad p \frac{\partial f}{\partial x_1} - w_1 = 0 \quad \frac{p \cdot MP_1}{MR_1} = MC_1$$

$$\frac{\partial \pi}{\partial x_2} = 0 \quad p \frac{\partial f}{\partial x_2} - w_2 = 0 \quad \frac{p \cdot MP_2}{MR_2} = MC_2$$

$$\text{Ex: } f(x_1, x_2) = x_1^a x_2^b$$

$$\left. \begin{aligned} \frac{\partial \pi}{\partial x_1} = 0 &= p \cdot a \cdot x_1^{a-1} \cdot x_2^b - w_1 = 0 \\ \frac{\partial \pi}{\partial x_2} = 0 &= p \cdot b \cdot x_1^a \cdot x_2^{b-1} - w_2 = 0 \end{aligned} \right\} \begin{aligned} \frac{a}{b} \frac{x_2}{x_1} &= \frac{w_1}{w_2} \\ x_2 &= \frac{b}{a} \frac{w_1}{w_2} x_1 \end{aligned}$$

$$p a x_1^{a-1} \left(\frac{b}{a} \frac{w_1}{w_2} x_1 \right)^b = w_1 \quad x_1 = p^{\frac{1}{1-a-b}} \left(\frac{w_1}{a} \right)^{\frac{1-b}{a+b-1}} \left(\frac{w_2}{b} \right)^{\frac{b}{a+b-1}}$$

Cost Minimization

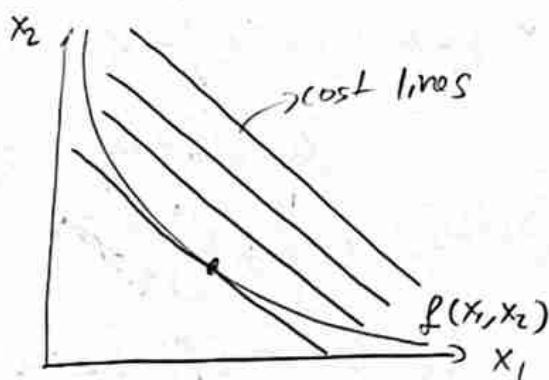
$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \quad \text{s.t. } f(x_1, x_2) = y$$

$$L = w_1 x_1 + w_2 x_2 + \lambda (y - f(x_1, x_2))$$

$$\frac{\partial L}{\partial x_1} = w_1 - \lambda \frac{\partial f}{\partial x_1} = 0$$

$$\frac{\partial L}{\partial x_2} = w_2 - \lambda \frac{\partial f}{\partial x_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = y - f(x_1, x_2) = 0$$



$$\text{Tangency condition} \rightarrow \underbrace{\frac{w_1}{w_2}}_{\text{slope}} = \underbrace{\frac{\partial f / \partial x_1}{\partial f / \partial x_2}}_{\text{TRS}}$$

$$\text{Ex: } f(x_1, x_2) = x_1^a x_2^b$$

$$L = w_1 x_1 + w_2 x_2 + \lambda (y - x_1^a x_2^b)$$

$$\frac{\partial L}{\partial x_1} = w_1 - a \lambda x_1^{a-1} x_2^b = 0$$

$$\frac{\partial L}{\partial x_2} = w_2 - b \lambda x_1^a x_2^{b-1} = 0$$

$$\frac{\partial L}{\partial \lambda} = y - x_1^a x_2^b = 0$$

$$\left. \begin{aligned} \frac{w_1}{w_2} &= \frac{a}{b} \frac{x_2}{x_1} \\ x_2 &= \frac{b}{a} \frac{w_1}{w_2} x_1 \end{aligned} \right\}$$

$$y = x_1^a \left(\frac{b}{a} \frac{w_1}{w_2} x_1 \right)^b$$

$$x_1 = \left(\frac{a}{b} \right)^{\frac{b}{a+b}} w_1^{\frac{-b}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$$

$$x_2 = \left(\frac{a}{b} \right)^{\frac{-a}{a+b}} w_1^{\frac{a}{a+b}} w_2^{\frac{-a}{a+b}} y^{\frac{1}{a+b}}$$

$$C(w_1, w_2, y) = w_1 x_1 + w_2 x_2$$

$$C = \left[\left(\frac{a}{b} \right)^{\frac{b}{a+b}} + \left(\frac{a}{b} \right)^{\frac{-a}{a+b}} \right] w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$$

$$EX: f(x_1, x_2) = \min \{ \alpha x_1, \beta x_2 \}$$

$$\alpha x_1 = \beta x_2 = y$$

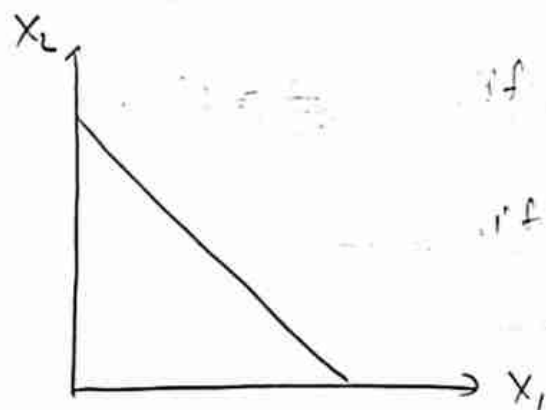
$$x_1 = \frac{y}{\alpha}$$

$$x_2 = \frac{y}{\beta}$$

not differentiable

$$C(w, y) = \left(\frac{w_1}{\alpha} + \frac{w_2}{\beta} \right) y$$

$$EX: f(x_1, x_2) = \alpha x_1 + \beta x_2$$



$$\text{if } \frac{w_1}{w_2} < \frac{\alpha}{\beta}$$

$$x_1 = \frac{y}{\alpha}$$

$$x_2 = 0$$

$$C(w, y) = \frac{w_1 y}{\alpha}$$

$$\text{if } \frac{w_1}{w_2} > \frac{\alpha}{\beta}$$

$$x_1 = 0$$

$$x_2 = \frac{y}{\beta}$$

$$C(w, y) = \frac{w_2 y}{\beta}$$

$$C(w, y) = \min \left\{ \frac{w_1}{\alpha}, \frac{w_2}{\beta} \right\} y$$

Gittens soru

$$EX: f(x_1, x_2, x_3) = \min \{ x_1, x_2 \} + x_3$$

$$x_4 \quad w_1 + w_2$$

$$C(w, y) = \left[\min \{ w_1 + w_2, w_3 \} \right] y$$

$$f(tx) = \min \{ tx_1, tx_2 \} + tx_3 = tf(x) \Rightarrow \text{CRTS}$$

To produce one unit:

x_1	x_2	x_3
1	1	0
0	0	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	1	1

$$EX: C(w_1, w_2, y) = \left(w_1 + \frac{w_2}{2} \right) y^{3/2} \quad \text{what is the production function?}$$

$$= w_1 y^{3/2} + \frac{w_2}{2} y^{3/2}$$

$$x_1 = y^{3/2} \quad \wedge \quad x_2 = \frac{y^{3/2}}{2}$$

$$x_1 = 2x_2 = y^{3/2}$$

$$y^{3/2} = \min \{ x_1, 2x_2 \}$$

$$y = \left[\min \{ x_1, 2x_2 \} \right]^{2/3} \quad \text{DRTS}$$

Ex: $C(w_1, w_2, y) = y^2 (\sqrt{w_1} + \sqrt{w_2})^2$ what is the production function?

$$\frac{\partial C}{\partial w_i} = X_i \quad \forall i=1,2$$

$$y^2 2(\sqrt{w_1} + \sqrt{w_2}) \cdot \frac{1}{2} w_1^{-1/2} = X_1 \quad X_1 = y^2 \left(1 + \frac{\sqrt{w_2}}{\sqrt{w_1}}\right) \quad \frac{\sqrt{w_2}}{\sqrt{w_1}} = \frac{X_1}{y^2} - 1$$

$$y^2 2(\sqrt{w_1} + \sqrt{w_2}) \cdot \frac{1}{2} w_2^{-1/2} = X_2 \quad X_2 = y^2 \left(1 + \frac{\sqrt{w_1}}{\sqrt{w_2}}\right) \quad \frac{\sqrt{w_1}}{\sqrt{w_2}} = \frac{X_2}{y^2} - 1$$

$$\frac{X_1 - y^2}{y^2} = \frac{y^2}{X_2 - y^2} \Rightarrow y^4 = X_1 X_2 - X_1 y^2 - X_2 y^2 + y^4$$

$$y^2 = \frac{X_1 X_2}{X_1 + X_2} \quad y = f(X_1, X_2) = \sqrt{\frac{X_1 X_2}{X_1 + X_2}}$$

$$f(tX_1, tX_2) = \sqrt{\frac{t^2 X_1 X_2}{t(X_1 + X_2)}} = \sqrt{t} f \Rightarrow \text{DRTS}$$

Ex: $f(K, L) = L^a + K^b$

$$f(tK, tL) = t^a L^a + t^b K^b$$

if $a=b=1$ CRTS

$a \geq 1, b \geq 1$ IRTS

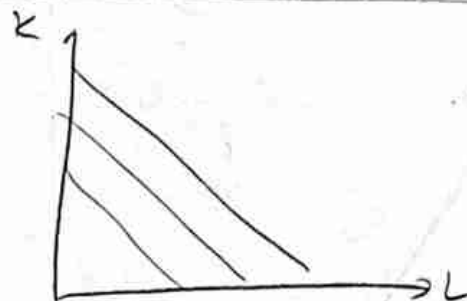
$a \leq 1, b \leq 1$ DRTS

Ex: $f(K, L) = \sqrt{4K + 3L}$

(given:)

$$\frac{w_L}{w_K} > \frac{3}{4} \quad L=0 \quad K=?$$

$$\max P\sqrt{4K} - w_K K$$



$$\frac{\partial \pi}{\partial K} \quad 4P \frac{1}{2} (4K)^{-1/2} - w_K = 0 \Rightarrow K = \frac{P^2}{(w_K)^2} \quad y = \frac{2 \cdot P}{w_K}$$

in case $\frac{w_L}{w_K} < \frac{3}{4} \Rightarrow K=0 \quad f(L) = \sqrt{3L}$

$$\max P\sqrt{3L} - w_L L$$

$$\frac{\partial \pi}{\partial L} = 3L \frac{1}{2} (3L)^{-1/2} - w_L = 0 \Rightarrow L = \dots \quad y = \dots$$

Problem Set 7

Q1) $U(F, C) = FC + F$ $m = 20$ $P_C = 4$

a) Optimal consumption bundle when $P_F = 1$

$$MRS_{FC} = \frac{P_F}{P_C} \text{ (tangency)} \quad \frac{C+1}{F} = \frac{1}{4} \quad F = 4C + 4$$

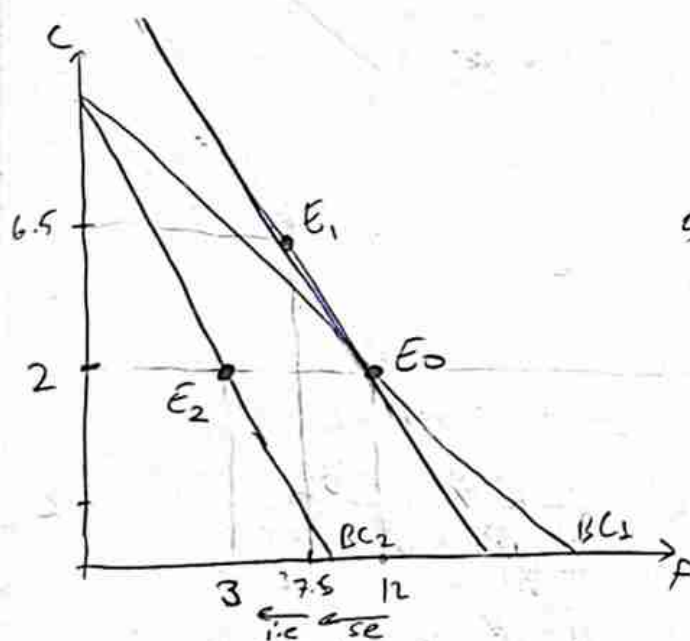
$$BC: 4C + F = 20 \quad 4C + 4C + 4 = 20 \quad \boxed{C^* = 2} \quad \boxed{F^* = 12}$$

b) Suppose $P_F' = 4$ find new optimal bundle

$$MRS = \frac{P_C}{P_C} \Rightarrow \frac{C+1}{F} = 1 \quad C+1 = F \quad BC: 4C + 4F = 20$$

$$4C + 4C + 4 = 20 \quad \boxed{C^{**} = 2} \quad \boxed{F^{**} = 3}$$

c) Slutsky



Blue budget: $C P_C' + F^* P_F' = 2 \cdot 4 + 12 \cdot 4 = 56$

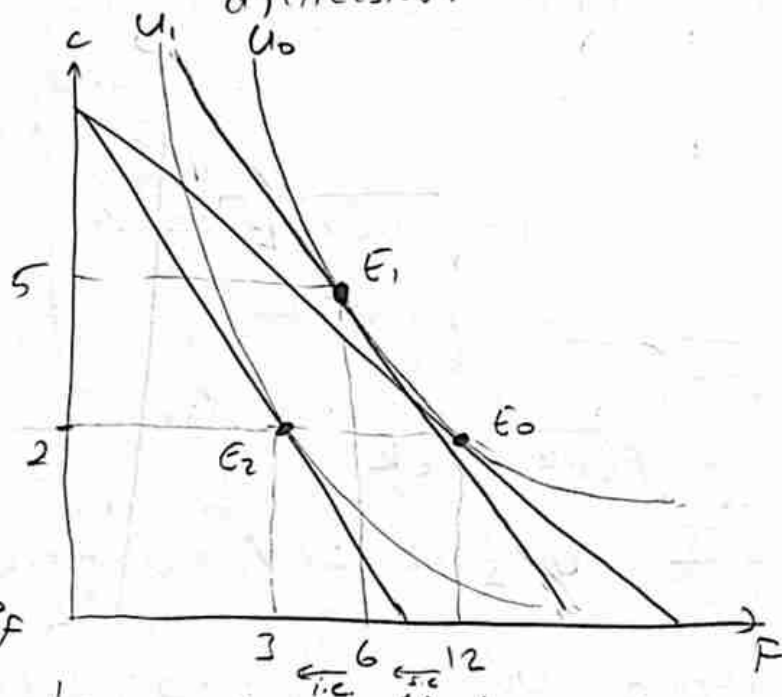
Blue budget line: $4C + 4F = 56$

Tangency condition: $F = C + 1$

$$\boxed{C = 6.5} \\ \boxed{F = 7.5}$$

$$3 \leftarrow 7.5 \leftarrow 12 \\ \text{i.e.} \quad \text{s.e.}$$

d) Hicksian



Blue budget: $U_0 = FC + F^* = 12 \cdot 2 + 12 = 36$

Tangency condition: $F = C + 1$

$$U_0 = C^2 + C + C + 1 = 36 = (C+1)^2 \quad \boxed{C = 5} \\ \boxed{F = 6}$$

$$3 \leftarrow 6 \leftarrow 7.5 \leftarrow 12 \\ \text{i.e.} \quad \text{s.e.}$$

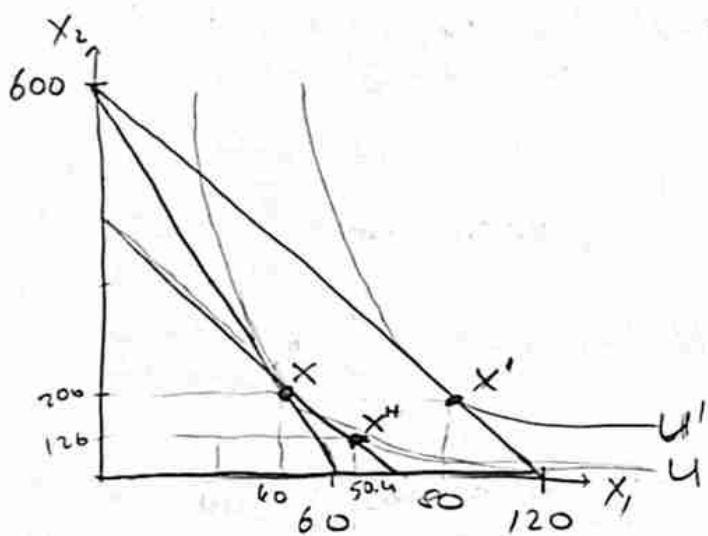
Q2 $U(x_1, x_2) = x_1^2 x_2$ $m = 600$ $P_1 = 10$ $P_2 = 1$
 $P_1' = 5$

a) Tangency condition $\rightarrow \frac{2x_1 x_2}{x_1^2} = \frac{10}{1}$ $x_1 = \frac{x_2}{5}$ $BC: 10x_1 + x_2 = 600$
 $x_2 = 5x_1$ $15x_1 = 600$
 $x_1 = 40$
 $x_2 = 200$

$\rightarrow \frac{2x_1' x_2'}{x_1'^2} = \frac{5}{1}$ $x_2' = \frac{5x_1'}{2}$ $BC: 5x_1' + x_2' = 600$
 $7.5x_1' = 600$
 $x_1' = 80$
 $x_2 = 200$

b) Hicksian compensation

Esti utility ogni olacık sekilde shift



$U = 40^2 \cdot 200 = 32 \cdot 10^4$

$x_1^{H2} \cdot x_2^H = 32 \cdot 10^4$

$\frac{2x_1^H x_2^H}{x_1^{H2}} = 5$ $x_2^H = \frac{5}{2} x_1^H$

$x_1^{H2} \cdot \frac{5}{2} x_1^H = 32 \cdot 10^4$

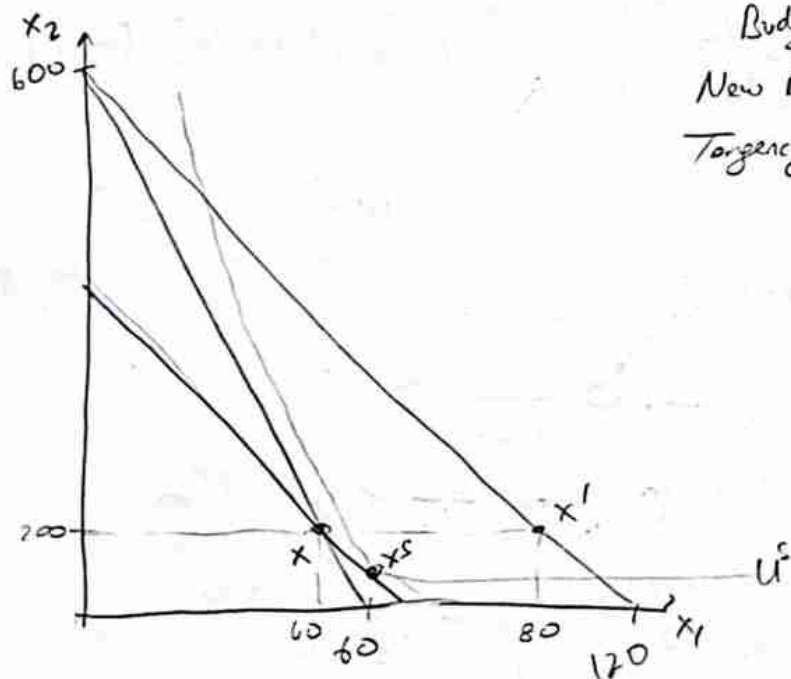
$x_1^{H3} = 128000$

$x_1^H \approx 50.4$ $x_2^H \approx 126$

$40 \xrightarrow{s.e.} 50.4 \xrightarrow{i.e.} 80$

c) Slutsky compensation

Esti bundle alınılabilecek sekilde shift



Budget^S = $40 \times 5 + 1 \times 200 = 400$

New BC: $5x_1^S + x_2^S = 400$

Tangency $\rightarrow \frac{2x_1^S x_2^S}{x_1^{S2}} = 5$ $x_2^S = \frac{5}{2} x_1^S$

$7.5x_1^S = 400$ $x_1^S = 53.33$
 $x_2^S = 133.33$

$40 \xrightarrow{s.e.} 53.33 \xrightarrow{i.e.} 80$

d) X_1 is an ordinary good $\frac{\partial X_1}{\partial P_1} < 0$

e) X_1 is a normal good $\frac{\partial X_1}{\partial I} > 0$

Q3 | Milk and strawberry ratio 1:5 $M=200$ $P_1=15$ $P_2=1$
 $\downarrow$ $\downarrow$
 X_1 X_2
 $P_1'=5$

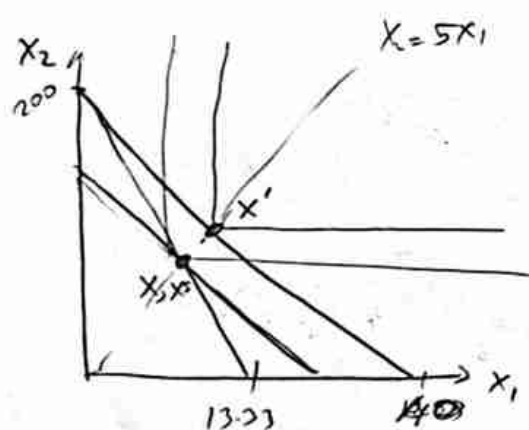
$$U = \min \{5X_1, X_2\} \rightarrow 5X_1 = X_2 \quad 15X_1 + X_2 = 200 \quad X_1 = 10$$

$$X_2 = 50$$

$$5X_1 + X_2 = 200 \rightarrow X_1' = 20$$

$$X_2' = 100$$

6) What's s.e.?



$X = X^S$ new budget line hits the same utility curve

$$M' = 5 \times 10 + 50 \times 1 = 100$$

$$\min \{5X_1, X_2\} \text{ s.t. } 5X_1^S + X_2^S = 100$$

$$10X_1^S = 100$$

$$X_1^S = 10$$

$$X_2^S = 50$$

No s.e.

c) What is i.e.? $10 \xrightarrow{s.e.} 10 \xrightarrow{i.e.} 20$

Q4 | $U(X_1, X_2) = 2X_1 + X_2$ $m = 5$ $P_1 = 1$ $P_2 = 1$

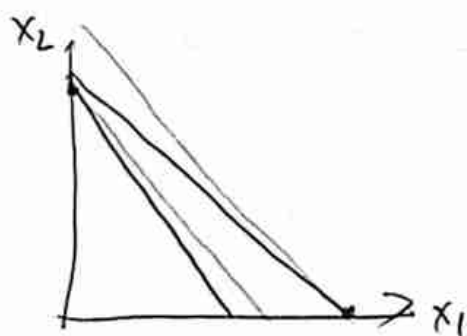
$$MRS_{12} = 2 \quad \frac{P_1}{P_2} = 1 \quad MRS_{12} > \frac{P_1}{P_2} \text{ all spend on } X_1 (5, 0)$$

$$b) P_1' = 3, \text{ now } MRS_{12} = 2 \quad \frac{P_1}{P_2} = 3 \quad MRS_{12} < \frac{P_1}{P_2} \text{ all spend on } X_2 (0, 5)$$

c) Find income e. and s.e. using hicksian

$$U_0 = 2 \times 5 + 1 \times 0 = 10$$

$$U_0' = 2 \times X_1^H + 1 \times X_2^H = 10 \quad X_1^H = 0 \quad X_2^H = 5$$



$$5 \xrightarrow{s.e.} 0 \xrightarrow{i.e.} 0$$

$$d) P_1' = 1.25 \quad MRS = 2 > \frac{P_1}{P_2} = 1.25$$

$$X_1' = 4$$

$$X_2' = 0$$

$$5 \xrightarrow{s.e.} 5 \xrightarrow{i.e.} 4$$

Q5] $U(x, y) = x + 10y - \frac{y^2}{2}$ $P_x = 1$ $P_y = 5$ $m = 200$

If SAM pays A amount of money, he can purchase y with 1 \$ $P_y^1 = 1$

Case 1: $P_x = 1$, $P_y = 5$, $m = 200$

Case 2: $P_x = 1$, $P_y = 1$, $m = 200 - A$

$$L = x + 10y - \frac{y^2}{2} + \lambda(200 - x - 5y)$$

$$\left. \begin{aligned} \frac{\partial L}{\partial x} &= 1 - \lambda = 0 \\ \frac{\partial L}{\partial y} &= 10 - y - 5\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= 200 - x - 5y = 0 \end{aligned} \right\} \begin{aligned} &\rightarrow \lambda = 1 \\ &\rightarrow y^* = 5 \\ &\rightarrow x^* = 175 \end{aligned}$$

Initially (175, 5)

$$L' = x + 10y - \frac{y^2}{2} + \lambda(200 - A - x - y)$$

$$\left. \begin{aligned} \frac{\partial L'}{\partial x} &= 1 - \lambda = 0 \\ \frac{\partial L'}{\partial y} &= 10 - y - \lambda = 0 \\ \frac{\partial L'}{\partial \lambda} &= 200 - A - x - y = 0 \end{aligned} \right\} \begin{aligned} &\rightarrow \lambda = 1 \\ &\rightarrow y^{**} = 9 \\ &\rightarrow x^{**} = 191 - A \end{aligned}$$

Now (191-A, 9)

$$U^* = 175 + 10 \cdot 5 - \frac{5^2}{2} = 212.5$$

$$U^* = U^{**} \quad \underline{\underline{A = 28}}$$

$$U^{**} = (191 - A) + 10 \cdot 9 - \frac{9^2}{2} = 240.5 - A$$

Q6) $U(X_1, X_2) = \sqrt{X_1 X_2}$ $M = 100$ $P_1 = 1$ $P_2 = 1$
 $P'_1 = 2$

What is CV? and EV?

$V(X_1, X_2) = X_1 X_2$ $MRS = \frac{P_1}{P_2}$ $\frac{X_2}{X_1} = 1$ $BC: X_1 + X_2 = 100$
 $X_1^* = 50$
 $X_2^* = 50$
 $U_0 = \sqrt{2500} \approx 50$
 $MRS = \frac{P_1}{P_2}$ $\frac{X_2}{X_1} = 2$ $BC: 2X_1 + X_2 = 100$
 $X_1^{**} = 25$
 $X_2^{**} = 75$
 $U_1 = \sqrt{1875} \approx 35$

Now, assume M'

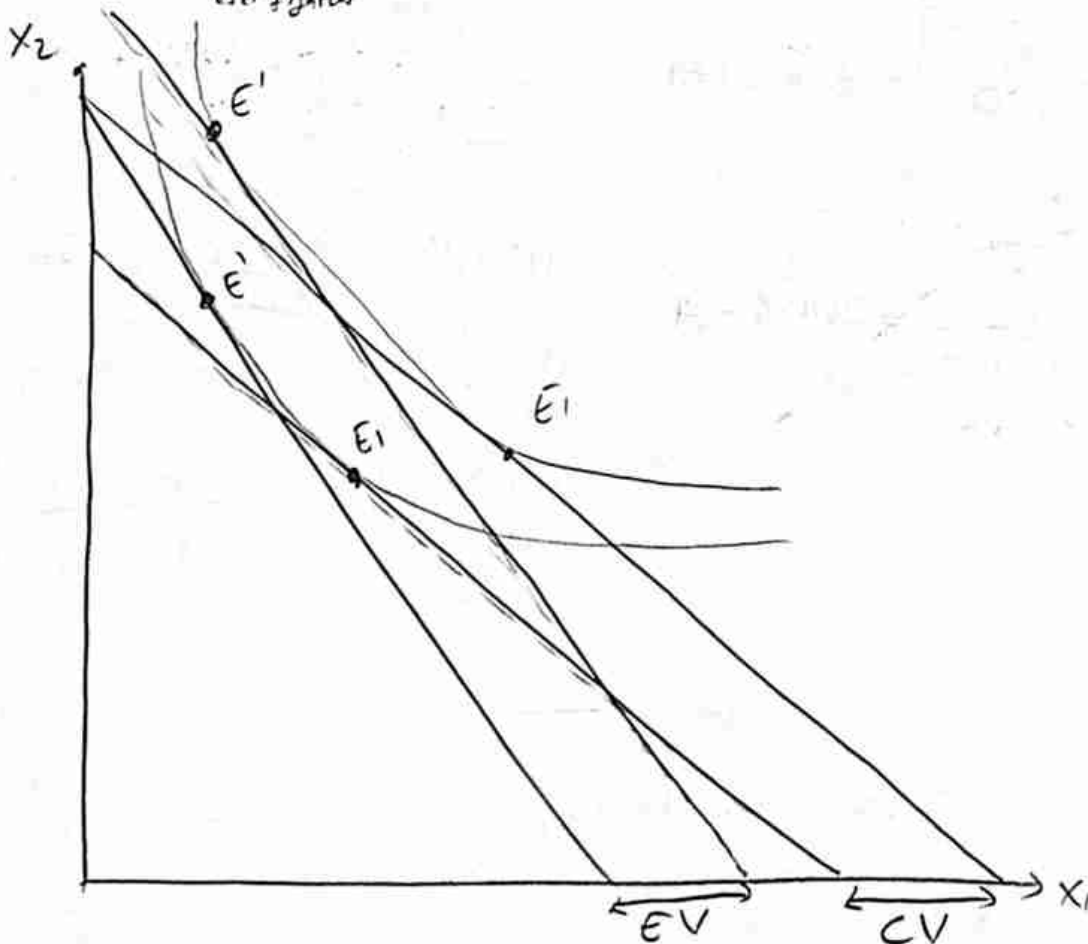
$U_0 = 50 = \sqrt{\frac{1}{2} \frac{M'}{P_1} \cdot \frac{1}{2} \frac{M'}{P_2}} = \frac{1}{2} \frac{M'}{\sqrt{2}}$ $M' = 100\sqrt{2}$
yani fiyatlar

$CV = 100\sqrt{2} - 100 \approx 41$

CV: Aynı utility'ye sahip olabilmek için yükselen fiyatı ne kadarlık bir income artışı ile compense edebiliriz?

EV: Fiyat artışı yapmak yerine ne kadarlık bir income eklenmesi yaparsam aynı hesarı veririz

$U_1 = 35 = \sqrt{\frac{1}{2} \frac{M'}{P_1} \cdot \frac{1}{2} \frac{M'}{P_2}} = \frac{1}{2} \frac{M'}{\sqrt{2}}$ $M' = 70$ $EV = 100 - 70 = \underline{\underline{30}}$
est. fiyatlar



Q7 $U = \min\{x, y\}$ $m = 150$ $P_x = 1$ $P_y = 1$
 $P'_y = 2$

EV, CV?

Initially

$x = y$ $U_0 = \min\{x, y\}$ $x + y = 150$ $x^* = 75$ $y^* = 75$ $U_0 = 75$

price increase

$x = y$ $x + 2y = 150$ $x^* = 50$ $y^* = 50$ $U_1 = 50$

$x'' + 2y'' = M'$ $x'' = \frac{M'}{3}$ $y'' = \frac{M'}{3}$ $\frac{M'}{3} = 75$ $M' = 225$

$CV = 225 - 150 = 75$

$U_1 = 50$ $P_x = 1$ $P_y = 1$

$x''' + y''' = M''$ $x''' = \frac{M''}{2} = 50$ $M'' = 100$ $EV = 150 - 100 = 50$

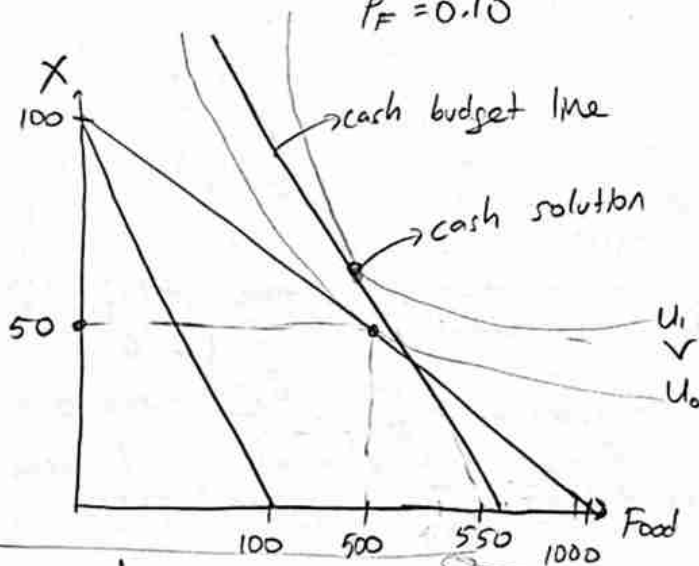
Q8

$m = 100$ $P_F = 1$ $P_x = 1$
 $P'_F = 0.10$

$X'_F = 500$

$X_x = 50$

Total Gov subsidy: 450\$



Cash verilirse utility daha yüksek olacaktır.

Q9

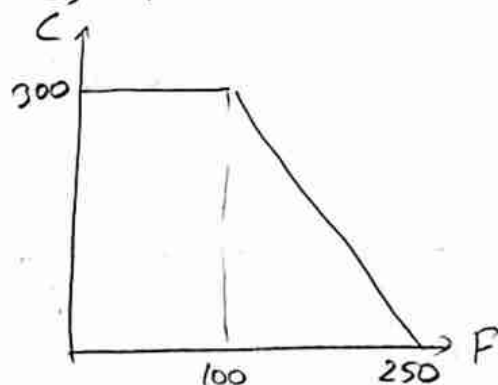
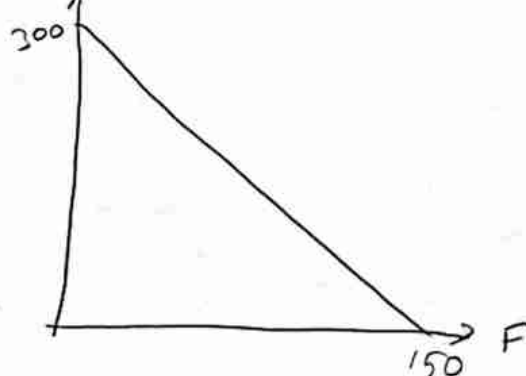
$M = 300$

$P_F = 2$

$P_C = 1$

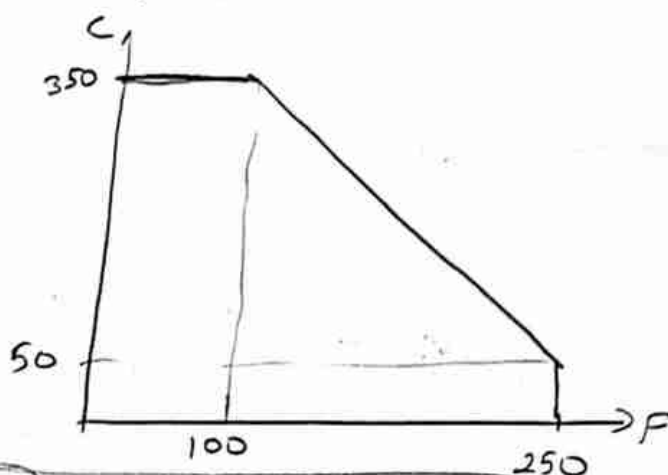
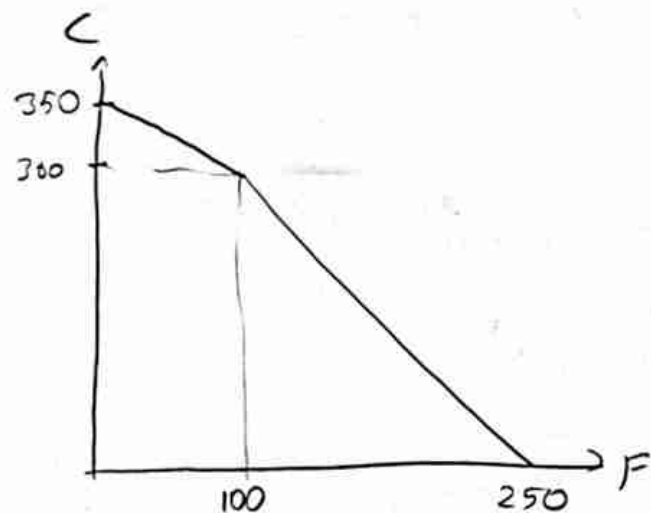
$2F + C = 300$

b) 100 free food stamps



c) Now food coupons can be sold for 0.50\$ in black market

d) 100 food stamp + 50 clothing coupon



Q10) $U(C, F) = C^{1/5} F^{4/5}$ $P_C = 2$ $P_F = 4$ $I = 20$

a) Find demands

Tangency condition $\rightarrow MRS_{CF} = \frac{P_C}{P_F}$

$$\frac{\frac{1}{5} C^{-4/5} F^{4/5}}{\frac{4}{5} C^{1/5} F^{-1/5}} = \frac{2}{4} \quad \frac{F}{C} = 2$$

$$\begin{aligned} 2C + 4F &= 20 \\ F + 4F &= 20 \end{aligned} \quad \boxed{\begin{aligned} F^* &= 4 \\ C^* &= 2 \end{aligned}}$$

$$\boxed{2C = 10}$$

put into budget C.

b) Income subsidy +10

$$\begin{aligned} 2C + 4F &= 30 \\ F + 4F &= 30 \end{aligned} \quad \boxed{\begin{aligned} F^* &= 6 \\ C^* &= 3 \end{aligned}}$$

c) +4 units coal
+0.5 units food

$$\begin{aligned} C^* : 2 &\rightarrow 6 \\ F^* : 4 &\rightarrow 4.5 \end{aligned}$$

$$\frac{6 + C^{**}}{4.5 + F^{**}} = \frac{1}{2} \quad \text{or} \quad \frac{1}{2}$$

consider those as income +10
the optimal bundle is (3, 6)
optimal cost was 3, now we have 6.
Ratio should be 1:2
Törvényesül f alsó ből 60 ar
törvényesül. Törvényesül f 4.112.

d) Sonra Bak!

Q11) $U(n, c) = n \cdot c$ n : leisure hours $w = 8$ $P_c = 1$
 c : consumption

$$P_c \cdot c = (24 - n)w$$

or

$$P_c \cdot c + w \cdot n = 24w \quad c + 8n = 24 \cdot 8 \quad \boxed{c + 8n = 192} \text{ BC}$$

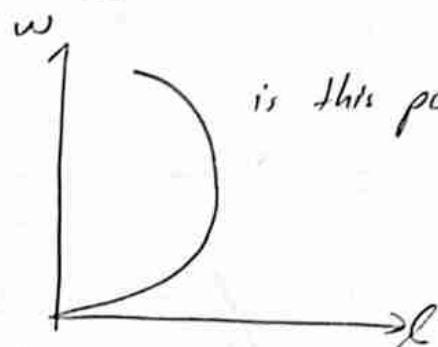
b) Tangency condition $MRS_{nc} = \frac{c}{n} = 8 \quad c = 8n$

$$16n = 192 \quad n^* = 12 \quad c^* = 96$$

c) $w' = 10$ $P_c \cdot c + wn = 24w$ $c + 10n = 24 \cdot 10 \quad \boxed{c + 10n = 240}$

$$MRS = \frac{c}{n} = 10 \quad c = 10n \quad 20n = 240 \quad n^{**} = 12 \quad c^{**} = 120$$

* Q12) $U(n, c) = n^{1/2} + c^{1/2}$ $P_c = 1$ $\bar{n}$



is this possible?

$$\frac{MU_n}{MU_c} = \frac{w}{P_c} \quad \frac{\frac{1}{2}n^{-1/2}}{\frac{1}{2}c^{-1/2}} = \frac{w}{1}$$

$$\left(\frac{c}{n}\right)^{1/2} = w \quad \frac{c}{n} = w^2 \quad c = n \cdot w^2$$

$$P_c \cdot c + nw = \bar{n}w \quad n^* = \frac{\bar{n}}{w+1}$$

$$l = \bar{n} - n^* = \bar{n} - \frac{\bar{n}}{w+1}$$

$$\frac{\partial l}{\partial w} = \frac{1}{(w+1)^2} > 0 \quad \text{always positive}$$

backward bending yes

Q13) $U(n, c) = c + nc$

$$\frac{MU_n}{MU_c} = \frac{c}{1+n} \neq \frac{w}{P_c}$$

$$(1+n)w = cP_c \quad \frac{P_c \cdot c + nw}{(1+n)w} = \bar{n}w$$

$$w(1+2n) = \bar{n}w \quad 2n = \bar{n} - 1 \quad n^* = \frac{\bar{n} - 1}{2} \quad l = \bar{n} - \frac{\bar{n} - 1}{2}$$

Q14] $u(n, c) = c - (12 - n)^2$ n : leisure $\bar{n} = 16$ hours $m = m$ from non-labor source
 $P_c = 1$

a) if $w = 0$ $n^* = ?$

$$MRS_{n,c} = \frac{2(12-n)}{1} = \frac{w}{P_c} \quad w = 0 \quad 24 - 2n = 0 \quad n = 12$$

$$h = 4$$

b) if $w = 10$

$$\frac{2(12-n)}{1} = 10 \quad 24 - 2n = 10 \quad 2n = 14 \quad n = 7$$

$$h = 9$$

c) No change

d) $C = [(16-n)w + m] \underbrace{(1-\tau)}_{0.8} = ((16-n)w + m) 0.8 = 144 - 8n$

$$\frac{2(12-n)}{1} = \frac{(1-\tau)w}{1} \quad 2(12-n) = 8 \quad n = 8$$

$$h = 8$$

$$C + 8n = 144$$

13/12/2024 week 10

$$\min w_1 x_1 + w_2 x_2 \quad \text{s.t.} \quad f(x_1, x_2) = y$$

$x_1(y, w_1, w_2)$ Conditional input demand functions

$x_2(y, w_1, w_2)$

$C(y, w_1, w_2)$

$$\max p f(x_1, x_2) - w_1 x_1 - w_2 x_2$$

$x_1(P, w_1, w_2)$ Unconditional input demand functions

$x_2(P, w_1, w_2)$

$y(P, w_1, w_2)$

EX: $f(x_1, x_2) = \ln(x_1 x_2) = \ln x_1 + \ln x_2$

what is the cost function?
cost minimization

$$L = w_1 x_1 + w_2 x_2 - \lambda (\ln x_1 + \ln x_2 - y)$$

$$\frac{\partial L}{\partial x_1} = w_1 - \frac{\lambda}{x_1} = 0 \quad \left\{ \begin{array}{l} \frac{w_1}{w_2} = \frac{x_2}{x_1} \rightarrow \ln \left(x_1 \cdot x_1 \frac{w_1}{w_2} \right) = y \\ e^y = x_1^2 \frac{w_1}{w_2} \end{array} \right.$$

$$\frac{\partial L}{\partial x_2} = w_2 - \frac{\lambda}{x_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = y - \ln x_1 - \ln x_2 = 0$$

$$x_1 = \sqrt{\frac{w_2 e^y}{w_1}}$$

$$x_2 = \sqrt{\frac{w_1 e^y}{w_2}}$$

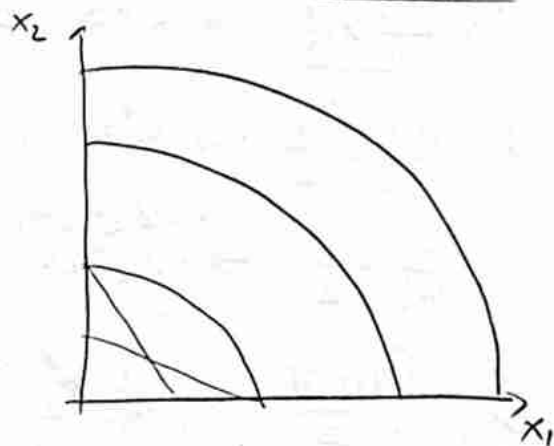
$$C = x_1 w_1 + x_2 w_2$$

$$C = \sqrt{w_1 w_2 e^y} + \sqrt{w_1 w_2 e^y} = 2\sqrt{w_1 w_2 e^y}$$

EX $f(x_1, x_2) = (x_1^2 + x_2^2)^{1/2}$

$$x_1 = \begin{cases} 0 & \text{if } w_1 > w_2 \\ y & \text{if } w_1 < w_2 \\ 0 \text{ or } y & \text{if } w_1 = w_2 \end{cases}$$

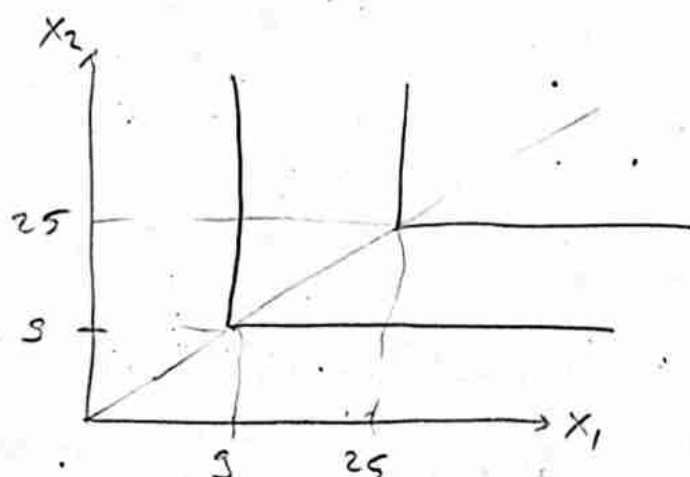
$$C = \begin{cases} w_1 y & \text{if } w_1 < w_2 \\ w_2 y & \text{if } w_1 > w_2 \\ w_1 y & \text{if } w_1 = w_2 = w \end{cases}$$



EX/ $f(x_1, x_2) = (\min\{x_1, x_2\})^{1/2}$

a) $y=3$ $x_1=x_2 \rightarrow y=\sqrt{x_1}$ or $\sqrt{x_2}$

b) $y=5$



$c(y, w_1, w_2) = w_1 y^2 + w_2 y^2 = \underline{\underline{y^2(w_1 + w_2)}}$

EX/ $f(x_1, x_2, x_3, x_4) = \min\{x_1, \frac{x_2}{2}\} + \max\{x_3, 3x_4\}$ $w = (1, 2, 3, 4)$

for $y=1$

x_1	x_2	x_3	x_4	cost
1	2	0	0	$1+2 \cdot 2=5$
0	0	1	0	$1 \cdot 3=3$
0	0	0	$1/3$	$4 \cdot \frac{1}{3} = 4/3 \rightarrow \text{minimum cost}$

$x_1 = x_2 = x_3 = 0$

$x_4 = y/3$

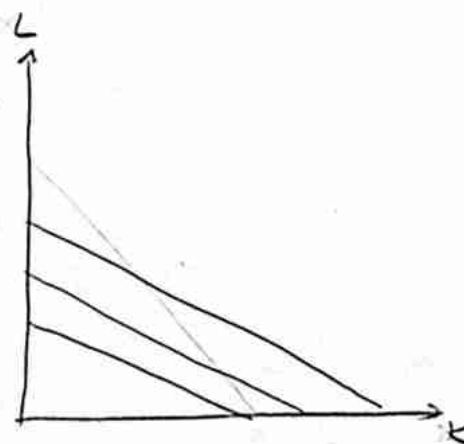
H/W/ $f(K, L) = K + 2L$ $K^*=?$ $L^*=?$

if $\frac{w_1}{w_2} > \frac{1}{2} \rightarrow K^*=0, L^*=y/2$

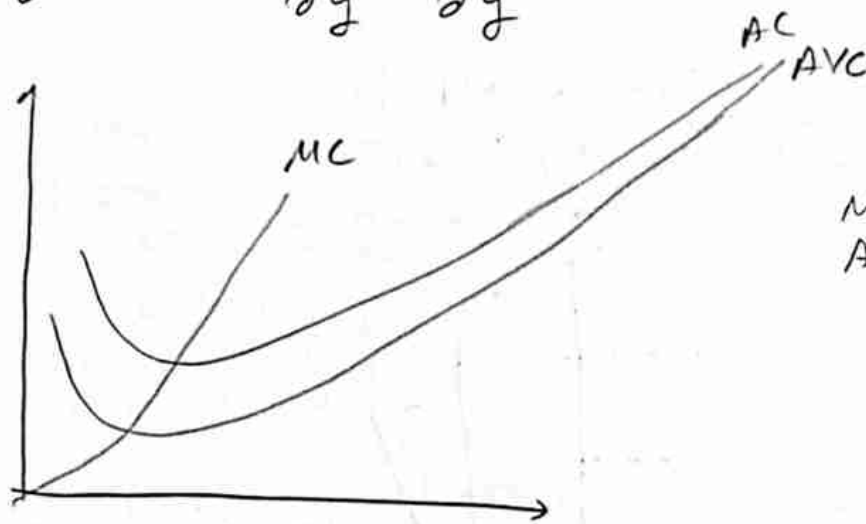
if $\frac{w_1}{w_2} < \frac{1}{2} \rightarrow K^*=y, L^*=0$

if $\frac{w_1}{w_2} = \frac{1}{2} \rightarrow 0 < K^* < y/3$
 $0 < L^* < y/3$

$C = \begin{cases} \frac{w_2 y}{2} & \text{if } \frac{w_1}{w_2} > \frac{1}{2} \\ w_1 y & \text{if } \frac{w_1}{w_2} < \frac{1}{2} \end{cases}$



$$\text{Marginal Cost} = \frac{\partial C}{\partial y} = \frac{\partial VC}{\partial y}$$



gap getting closer
because the average
fixed cost is decreasing

MC intersect AC and
AVC at minimum

$$\frac{\partial AL}{\partial q} = 0 = \frac{(C(q)/q)}{dq} = \left[\underbrace{\frac{dC}{dq}}_{\overline{MC}} - \underbrace{\frac{C(q)}{q}}_{\overline{AC}} \right] \frac{1}{q}$$

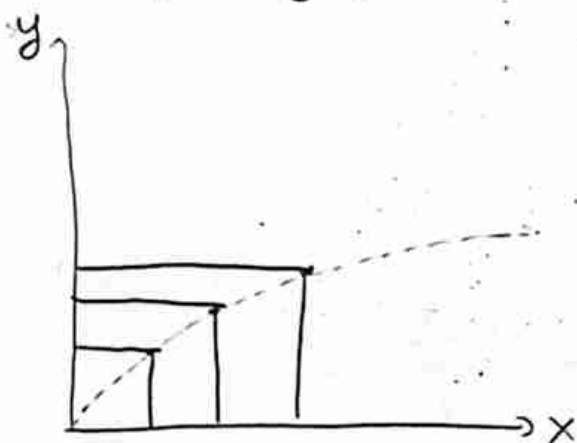
When MC is lower than AVC, AC is decreasing

Problem Set 8

Q1

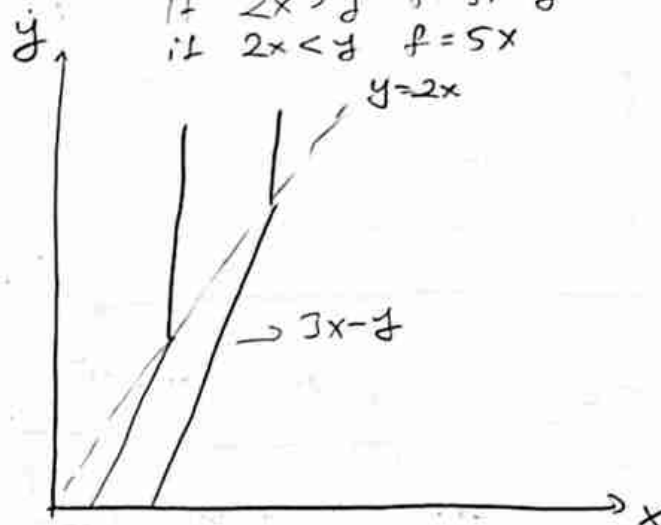
a) $f(x,y) = \max\{2x, y^2\}$

$2x = y^2 \quad y = \sqrt{2x}$



b) $f(x,y) = 3x - \min\{2x, y\}$

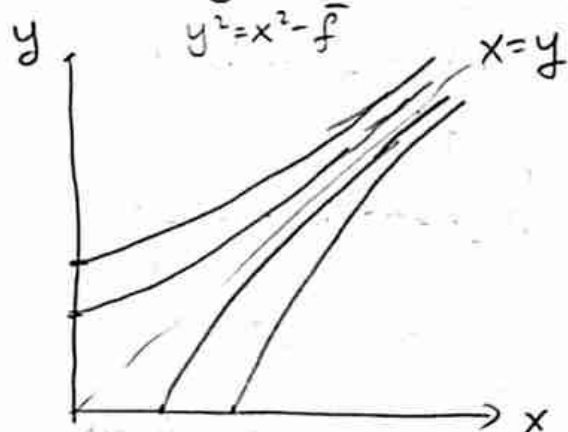
if $2x > y \quad f = 3x - y$
if $2x < y \quad f = 5x$



c) $f(x,y) = (\max\{x,y\})^2 - (\min\{x,y\})^2$

if $x > y \quad \bar{f} = x^2 - y^2$

if $x < y \quad \bar{f} = y^2 - x^2$



Q2) a) $f(L,K) = KL^2 + L \quad MP_L = 2KL + 1 \quad MP_K = L^2$

$MRTS = \frac{2KL+1}{L^2} = \frac{2K}{L} + \frac{1}{L^2}$

$f(tL, tK) = t^3 KL^2 + tL > tKL^2 + tL \quad IRTS$

b) $f(L,K) = 7L + 12K - 4 \quad MP_L = 7 \quad MP_K = 12 \quad MRTS_{LK} = \frac{7}{12}$

$f(tL, tK) = 7tL + 12tK - 4 > 7tL + 12tK - 4tK \quad IRTS$

c) $f(L,K) = 3L^{1/3}K \quad MP_L = \frac{1}{3}L^{-2/3}K \quad MP_K = \frac{1}{3}L^{1/3} \quad MRTS_{LK} = \frac{L^{-2/3}K}{\frac{1}{3}L^{1/3}} = \frac{K}{L}$

$f(tL, tK) = 3t^{4/3}L^{1/3}K > 3tL^{1/3}K \quad IRTS$

dimensi benar

d) $f(L, K) = \min \{3L, K\}$ if $3L > K$, $MP_L = 0$, $MP_K = 1$ MRTS = 0
 if $3L < K$, $MP_L = 3$, $MP_K = 0$ MRTS = ∞

$f(tL, tK) = \min \{3tL, tK\}$ CRTS

Q3 $f(x_1, x_2) = (x_1^b + x_2^b)^c$ $b > 0$, $c > 0$

if $b \cdot c > 1$ IRTS

if $b \cdot c = 1$ CRTS

if $b \cdot c < 1$ DRTS

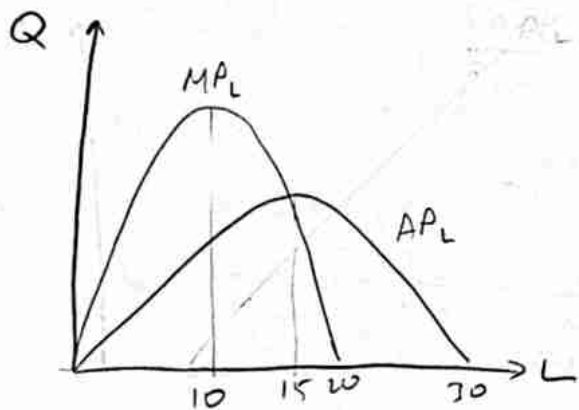
Q4 $Q(K, L) = 150K^2L^2 - K^3L^3$ $K = 5$ fixed

a) Total product of labor: $150 \cdot 5^2 \cdot L^2 - 5^3 L^3 = 3750L^2 - 125L^3$

Marginal product of labor: $\frac{\partial Q}{\partial L} = 7500L - 375L^2$

Average product of labor: $\frac{TP_L}{L} = 3750L - 125L^2$

b) Plot MPL and APL



c) Find L where MPL is max

$MPL = 7500L - 375L^2$

$\frac{\partial MPL}{\partial L} = 7500 - 750L \rightarrow L = 10$

d) Find $MPL = APL$

$7500L - 375L^2 = 3750L - 125L^2$
 $L = 15$

e) $\frac{\partial TP}{\partial L} = 0 = 7500L - 375L^2$

$L = 0$ or $L = 20$

$Q = 0$

$Q = 500000$

Q5) $f(x, y) = 6\sqrt{x} + 8\sqrt{y}$ $P_x = 1$ $P_y = 4$ $P = 8$

profit maximizing q?

$$\max_{\pi} \pi = f(x, y) - xP_x - yP_y = 8(6\sqrt{x} + 8\sqrt{y}) - x - 4y$$

$$\pi = 48\sqrt{x} + 64\sqrt{y} - x - 4y$$

FOCs:

$$\frac{\partial \pi}{\partial x} = 24 \cdot x^{-1/2} - 1 = 0 \quad x^* = 576$$

$$\frac{\partial \pi}{\partial y} = 32y^{-1/2} - 4 = 0 \quad y^* = 64$$

$$f(x, y) = 6\sqrt{576} + 8\sqrt{64} = 208 \quad \text{profit max output}$$

Q6) $f(K, L) = \sqrt{8L}$ $\bar{K} = 1$ $\pi = P\sqrt{8L} - r\bar{K} - wL$

Focs:

$$\frac{\partial \pi}{\partial L} = P \cdot \frac{1}{2} (8L)^{-1/2} - w = 0 \quad \boxed{L = \frac{2P^2}{w^2}}$$

$$f(K, L) = \sqrt{\frac{8 \cdot 2P^2}{w^2}} = \frac{4P}{w} \quad \text{profit max quantity}$$

$$\pi = P \cdot \frac{4P}{w} - \left[r + w \frac{2P^2}{w^2} \right] = \frac{4P^2}{w} - r - \frac{2P^2}{w} = \frac{2P^2}{w} - r$$

if $P \uparrow$, $Q \uparrow$
 $w \uparrow$, $Q \downarrow$

Q7) $y = x^{3/2}$ when $0 \leq x \leq 4$ $P_x = 3$
 $y = 4 + x$ when $x > 4$ $P_y = 1$

$$\pi = \begin{cases} x^{3/2} - 3x & 0 \leq x \leq 4 \rightarrow \text{always negative profit} \\ 4 + x - 3x = 4 - 2x & x > 4 \rightarrow \text{always negative profit} \end{cases}$$

most feasible solution: $x = 0$, $\pi = 0$

Q8

$$f(K, L) = K^2 L^2$$

$$f(K, L) = K + 0.5 L^4$$

$$f(K, L) = \sqrt{\min\{K, L\}}$$

a) $f(tK, tL) = t^4 K^2 L^2 > t K^2 L^2$ IRTS

$f(tK, tL) = tK + t^4 0.5 L^4 > tK + t 0.5 L^4$ IRTS

$f(tK, tL) = \sqrt{\min\{tK, tL\}}$ CRTS

b) find cost functions $w_L = 1, w_K = 1$

i $\rightarrow \min w_L L + w_K K$ s.t. $\bar{q} = K^2 L^2$

$$\mathcal{L} = L + K + \lambda(\bar{q} - K^2 L^2)$$

$$\frac{\partial \mathcal{L}}{\partial L} = 1 - 2\lambda K^2 L = 0 \quad \left\{ \frac{K}{L} = 1 \quad K = L \right.$$

$$\frac{\partial \mathcal{L}}{\partial K} = 1 - 2\lambda K L^2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{q} - K^2 L^2 = 0 \quad \bar{q} = K^4 \quad \bar{q} = L^4$$

$$K = \sqrt[4]{\bar{q}} \quad L = \sqrt[4]{\bar{q}}$$

$$C = 4\sqrt[4]{\bar{q}} + 4\sqrt[4]{\bar{q}} = 2\sqrt[4]{\bar{q}}$$

ii) $\rightarrow \min w_L L + w_K K$ s.t. $\bar{q} = K + 0.5 L^4$

$$\mathcal{L} = L + K + \lambda(\bar{q} - K - 0.5 L^4)$$

$$\frac{\partial \mathcal{L}}{\partial L} = 1 - 2\lambda L^3 = 0$$

$$\frac{\partial \mathcal{L}}{\partial K} = 1 - \lambda = 0 \rightarrow \lambda = 1 \rightarrow L^3 = \frac{1}{2} \quad L = 2^{-1/3}$$

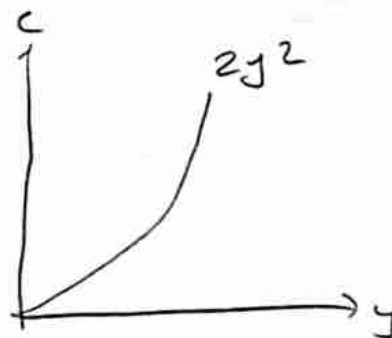
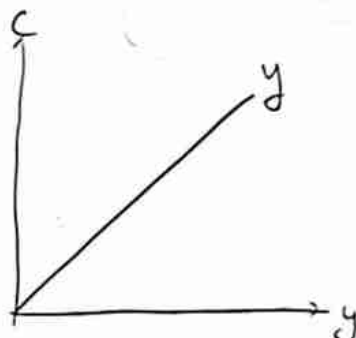
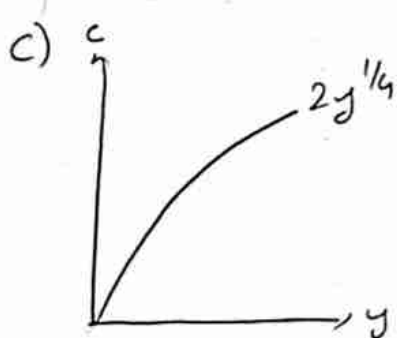
$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{q} - K - 0.5 L^4 = 0 \quad \bar{q} - K - 0.5 \cdot 2^{-4/3} = 0$$

iii) $\rightarrow \min w_L L + w_K K$ s.t. $\bar{q} = \sqrt{\min\{K, L\}}$

$$y = \sqrt{K} \quad y = \sqrt{L}$$

$$K = y^2 \quad L = y^2$$

$$C = 2y^2$$



Q9] $f(x, y) = x^{1/2} + y^{1/2}$

a) Find unconditional input demand (profit max)

$$\pi = P(x^{1/2} + y^{1/2}) - P_x x - P_y y$$

$$\frac{\partial \pi}{\partial x} = \frac{1}{2} P x^{-1/2} - P_x = 0 \rightarrow x = \left(\frac{P}{2P_x}\right)^2$$

$$\frac{\partial \pi}{\partial y} = \frac{1}{2} P y^{-1/2} - P_y = 0 \rightarrow y = \left(\frac{P}{2P_y}\right)^2$$

b) Find supply function

$$f(x, y) = q = \left(\left(\frac{P}{2P_x}\right)^2\right)^{1/2} + \left(\left(\frac{P}{2P_y}\right)^2\right)^{1/2} = \frac{P}{2P_x} + \frac{P}{2P_y}$$

c) Find profit function

$$\pi = P\left(\frac{P}{2P_x} + \frac{P}{2P_y}\right) - P_x\left(\frac{P}{2P_x}\right)^2 - P_y\left(\frac{P}{2P_y}\right)^2$$

d) Conditional (cost min)

min $P_x x + P_y y$ s.t. $\bar{q} = x^{1/2} + y^{1/2}$

$$\mathcal{L} = P_x x + P_y y + \lambda(\bar{q} - x^{1/2} - y^{1/2})$$

$$\frac{\partial \mathcal{L}}{\partial x} = P_x - \frac{1}{2} \lambda x^{-1/2} = 0 \quad \left\{ \begin{array}{l} \frac{P_x}{P_y} = \left(\frac{x}{y}\right)^{1/2} \\ y = x \frac{P_x^2}{P_y^2} \end{array} \right.$$

$$\frac{\partial \mathcal{L}}{\partial y} = P_y - \frac{1}{2} \lambda y^{-1/2} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{q} - x^{1/2} - y^{1/2} = 0 \quad \bar{q} = x^{1/2} + \left[x \frac{P_x^2}{P_y^2}\right]^{1/2} \quad x = \frac{\bar{q}^2 P_y^2}{(P_x + P_y)^2}$$

$$y = \frac{\bar{q}^2 P_x^2}{(P_x + P_y)^2}$$

$$C = P_x \frac{\bar{q}^2 P_y^2}{(P_x + P_y)^2} + P_y \frac{\bar{q}^2 P_x^2}{(P_x + P_y)^2}$$

Q10 $f(x_1, x_2, x_3, x_4) = \min \{ 2x_1 + x_2, x_3 + 2x_4 \}$

$w = (1, 2, 3, 4)$

$q = 1 = 2x_1 + x_2 = x_3 + 2x_4$

x_1	x_2	x_3	x_4	cost
$\frac{1}{2}$	0	0	$\frac{1}{2}$	2.5

b) $2x_1 + x_2 = x_3 + 2x_4 = q$
 $\text{MRTS} = 2 \quad \text{MRTS} = \frac{1}{2}$

$\frac{w_1}{w_2} > 2 \rightarrow x_1 = 0, x_2 = q$

$\frac{w_1}{w_2} < 2 \rightarrow x_1 = \frac{q}{2}, x_2 = 0$

$\frac{w_3}{w_4} > \frac{1}{2} \rightarrow x_3 = 0, x_4 = \frac{q}{2}$

$\frac{w_3}{w_4} < \frac{1}{2} \rightarrow x_3 = q, x_4 = 0$

$\frac{w_3}{w_4} > \frac{1}{2}$

$\frac{w_1}{w_2} > 2$

$\frac{w_3}{w_4} < \frac{1}{2}$

$\frac{w_1}{w_2} < 2$

$\frac{w_3}{w_4} < \frac{1}{2}$

$x_1 = 0$
 $x_2 = q$
 $x_3 = 0$
 $x_4 = q/2$

$x_1 = 0$
 $x_2 = q$
 $x_3 = q$
 $x_4 = 0$

$x_1 = q/2$
 $x_2 = 0$
 $x_3 = 0$
 $x_4 = q/2$

$x_1 = q/2$
 $x_2 = 0$
 $x_3 = q$
 $x_4 = q/2$

i $\rightarrow w_2 q + w_4 \frac{q}{2}$
 ii $\rightarrow w_2 q + w_3 q$
 iii $\rightarrow w_1 \frac{q}{2} + w_4 \frac{q}{2}$
 iv $\rightarrow w_1 \frac{q}{2} + w_3 q$

cost = $q \left[\min \left\{ \frac{w_1}{2}, w_2 \right\} + \min \left\{ w_3, \frac{w_4}{2} \right\} \right]$

Q11 $f(x_1, x_2, x_3, x_4) = 2 \min \{ x_1, x_2 \} + 3 \min \{ x_3, x_4 \}$

$w = (8, 5, 3, 4)$

At optimum $x_1 = x_2 = a$
 $x_3 = x_4 = b$

$f = 2a + 3b$

$\text{MRTS}_{ab} = \frac{2}{3}$

$\frac{P_a}{P_b} = \frac{P_1 + P_2}{P_3 + P_4} = \frac{13}{7}$

$\text{MRTS}_{ab} < \frac{P_a}{P_b}$ we should use b

$3x_3 = 3x_4 = 70 \quad x_3 = x_4 = \frac{70}{3}$

Total cost = $7 \cdot \frac{70}{3} = \frac{490}{3}$

$q = 70$

Q12) $f(K, L) = K^{0.5} L^{0.5} = \bar{q}$ $r=10$, $w=40$

a) cost min

$$\mathcal{L} = 10K + 40L + \lambda(\bar{q} - K^{0.5} L^{0.5})$$

$$\left. \begin{aligned} \frac{\partial \mathcal{L}}{\partial K} &= P_K - \lambda \frac{1}{2} K^{-1/2} L^{0.5} = 0 \\ \frac{\partial \mathcal{L}}{\partial L} &= P_L - \lambda \frac{1}{2} K^{0.5} L^{-1/2} = 0 \end{aligned} \right\} \frac{P_K}{P_L} = \frac{L}{K} \quad P_K K = P_L L$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{q} - K^{0.5} L^{0.5} = 0 \quad \bar{q} - \left(L \frac{P_L}{P_K} \right)^{1/2} L^{1/2} = 0$$

$$L = \left(\frac{P_K}{P_L} \right)^{1/2} \bar{q} = 500 \quad K = \left(\frac{P_L}{P_K} \right)^{1/2} \bar{q} = 2000$$

$$\text{cost} = 500 \cdot 40 + 2000 \cdot 10 = 40000$$

d) $\bar{K} = 2000$
 $\bar{q} = 2000$
 $L = ?$

$$f(L) = (2000)^{1/2} L^{1/2} = 2000 \quad \boxed{L = 2000}$$

$$C = 100000$$

e) $\left. \begin{aligned} L &= 1000 \\ K &= 4000 \end{aligned} \right\} C = 80000$ f) K is fixed in short run

Q14) $f(L, K, M) = 3L^{1/3} K^{1/3} + M^{1/3}$ $w=?$ $r=?$ $P_M = 1$

$$\mathcal{L} = wL + rK + M + \lambda[\bar{q} - 3L^{1/3} K^{1/3} - M]$$

$$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda L^{-2/3} K^{1/3} = 0 \quad \left. \begin{aligned} \frac{w}{r} &= \frac{K}{L} \\ wL &= rK \end{aligned} \right\} L = \frac{rK}{w}$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda L^{1/3} K^{-2/3} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{q} - 3L^{1/3} K^{1/3} - M = 0 \quad \text{Sonra devam}$$

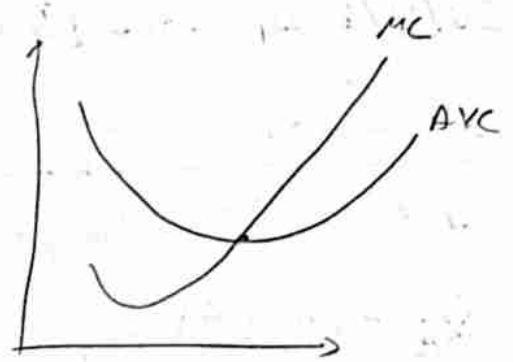
$$\frac{\partial \mathcal{L}}{\partial M} = 1 - \lambda = 0$$

Q15) a) $TC = 2q^2 - q + 4$

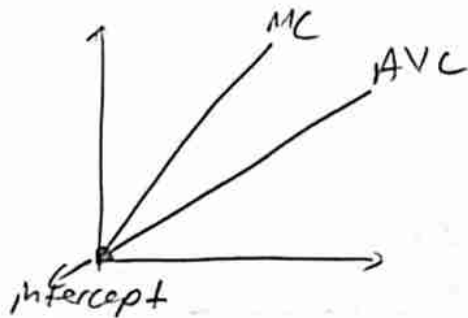
$AVC = \frac{TV C}{q}$ $\frac{dTC}{dq}$ olmaksın noktesi

$AC = 2q - 1 + \frac{4}{q}$ $AVC = 2q - 1$

$AFC = \frac{4}{q}$ $MC = 4q - 1$



b) $1000 + 50q^2$ $AC = \frac{1000}{q} + 50q$ $AVC = 50q$ $AFC = \frac{1000}{q}$
 $MC = 100q$



Edgeworth Box Economy

A, B agents (in final exam there might be 3 agents)

$X_A = (X_1^A, X_2^A) \rightarrow$ consumption of agent A

$X_B = (X_1^B, X_2^B) \rightarrow$ consumption of agent B

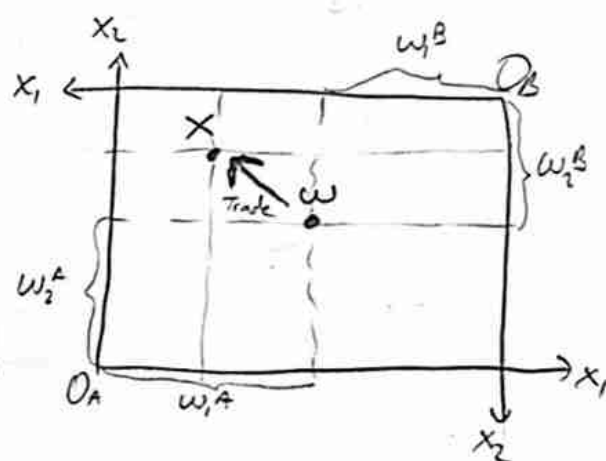
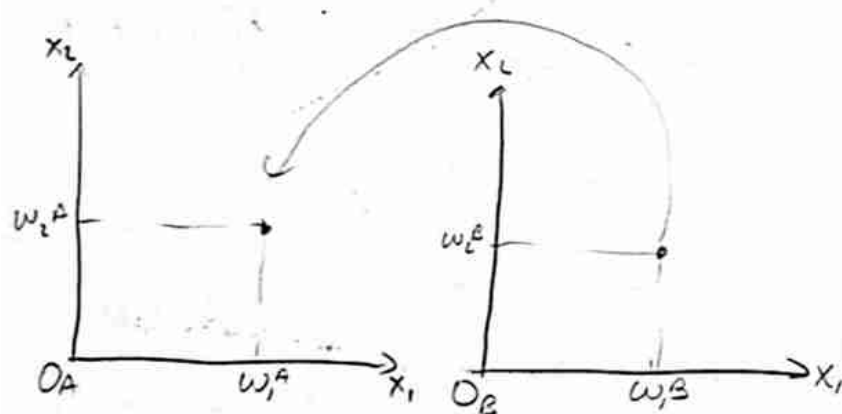
$(X_A, X_B) \rightarrow$ an allocation

$W_A = (W_1^A, W_2^A) \rightarrow$ initial endowment of A

$W_B = (W_1^B, W_2^B) \rightarrow$ initial endowment of B

$(W_A, W_B) \rightarrow$ initial allocation (initial endowment)

$U^A(X_1^A, X_2^A), U^B(X_1^B, X_2^B)$ potential gains from exchange

Feasibility Constraint

$$X_1^A + X_1^B = W_1^A + W_1^B \quad X_2^A + X_2^B = W_2^A + W_2^B$$

A is giving $W_1^A - X_1^A$ unit of good 1 to get $X_2^A - W_2^A$ units of good 2 from B

An allocation X is Pareto Efficient: One party cannot be made better off without harming to other

money (100£)
 $(100, 0, 0, \dots) \rightarrow$ Pareto efficient. You can not distribute 100 £ without harming the first agent

$(50, 10, 10, 30) \rightarrow$ Pareto efficient $\forall X$ s.t. $\sum X_i = 100$ is P.E.

$(50, 8, 2, 20, 19) \rightarrow$ Not P.E. $\sum X_i = 99$. We have still 1 to spare

$\downarrow$
 $(50, 8, 2, 20, 20) \rightarrow$ P.E.

Suppose there are 50 dark chocolate, 50 white chocolate

$(5DC, 1WC, 10DC, 5WC, \dots)$ not P.E

If their preferences are different, they can trade without harming each other.

Contract Curve

The set of all Pareto Efficient bundles (allocations)

$$\text{Max}_{x_1^A, x_2^A, \lambda} \mathcal{L} = U_A(x_1^A, x_2^A) + \lambda [U_B(w_1 - x_1^A, w_2 - x_2^A) - \bar{U}_B] \quad \text{where}$$

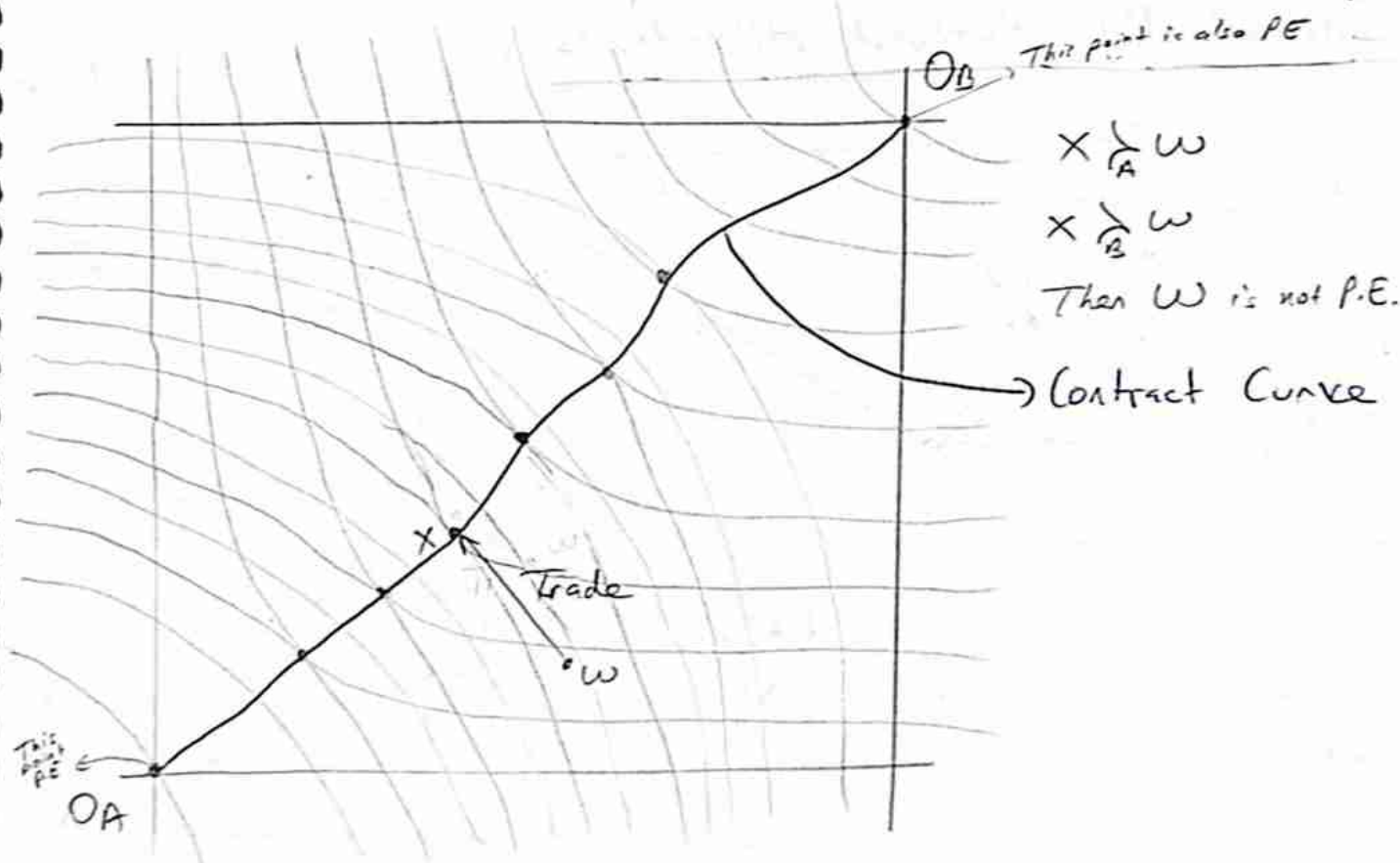
$$w_1 = w_1^A + w_1^B, \quad w_2 = w_2^A + w_2^B$$

$$\frac{\partial \mathcal{L}}{\partial x_1^A} = \frac{\partial U_A}{\partial x_1^A} - \lambda \frac{\partial U_B}{\partial x_1^B} = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2^A} = \frac{\partial U_A}{\partial x_2^A} - \lambda \frac{\partial U_B}{\partial x_2^B} = 0$$

$$MRS^A = \frac{\partial U^A / \partial x_1^A}{\partial U^A / \partial x_2^A} = \frac{\partial U^B / \partial x_1^B}{\partial U^B / \partial x_2^B} = MRS^B$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$



In PE allocations (interior), $MRS^A = MRS^B$ + feasibility constraint

* Can I make both of the agents better off?
If you can't, allocation is P.E.

EX

$$U_A = X_1^A X_2^A$$

$$W_A = (1, 1)$$

$$U_B = X_1^B (X_2^B)^2$$

$$W_B = (2, 1)$$

$$MRS^A = MRS^B$$

$$\frac{X_2^A}{X_1^A} = \frac{(X_2^B)^2}{2X_1^B X_2^B}$$

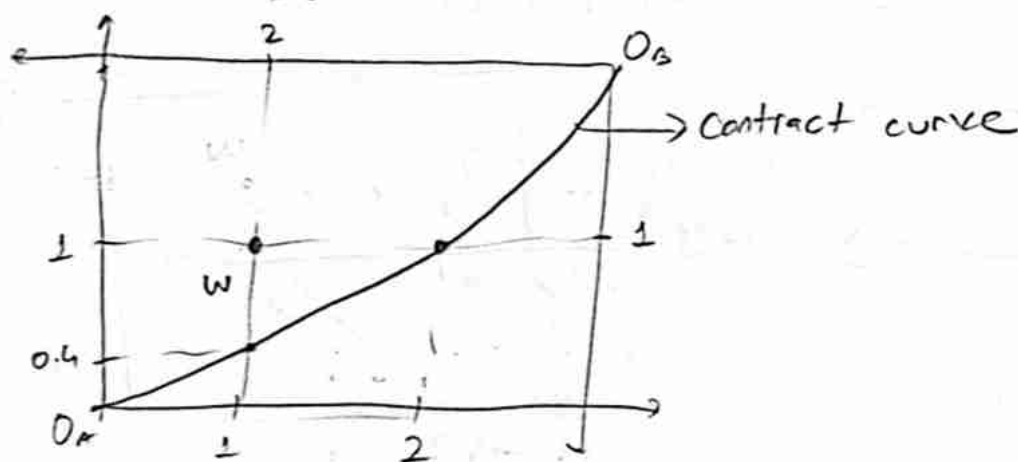
$$X_1^A + X_1^B = 3$$

$$X_2^A + X_2^B = 2$$

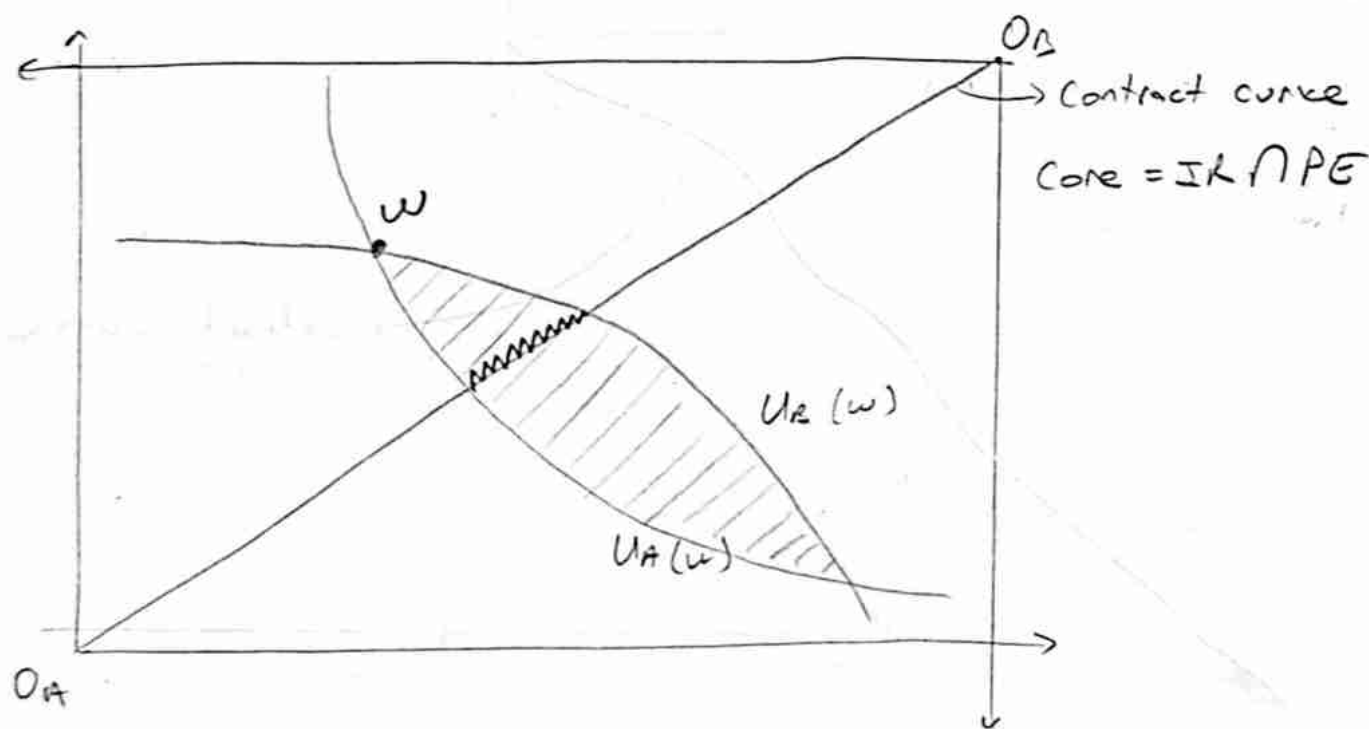
$$\frac{X_2^A}{X_1^A} = \frac{2 - X_1^A}{2(3 - X_1^A)}$$

$$\boxed{X_2^A = \frac{2X_1^A}{6 - X_1^A}}$$

Contract curve



Individually Rational Allocations



27/12/2024 week 13

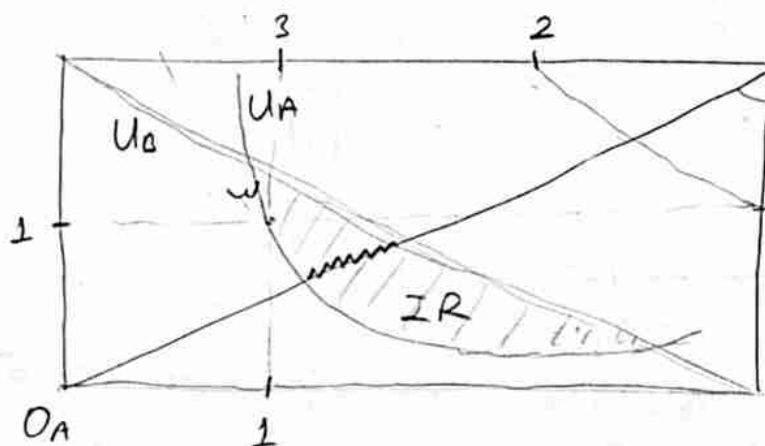
$$X \in PE \Leftrightarrow \nexists y \text{ s.t. } y \succ_i X \quad \forall i \in N$$

for some i $y \succ_i X$

For $N > 2 \Rightarrow \text{core} \neq IR \cap PE$

EX] $U^A = X_1^A X_2^A \quad U^B = X_1^B + 2X_2^B$

$$\begin{aligned} w_1 &= (1, 1) \\ w_2 &= (3, 1) \\ \hline w &= (4, 2) \end{aligned}$$



contract curve

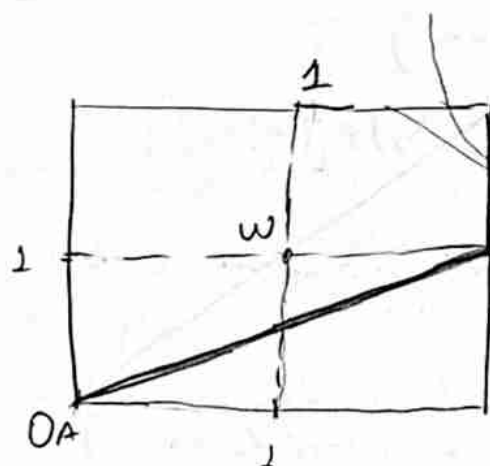
$$MRS^A = MRS^B$$

$$\frac{X_2^A}{X_1^A} = \frac{1}{2} \quad X_1^A = 2X_2^A$$

$w \notin PE$ so they will trade

EX] $U^A = X_1^A X_2^A \quad U^B = X_1^B + 2X_2^B$

$$\begin{aligned} w_1 &= (1, 1) \\ w_2 &= (1, 1) \\ \hline w &= (2, 2) \end{aligned}$$



Best bundles for A

corner solution possible

Best bundles for B

EX] $U^A = X_2^A \quad U^B = X_1^B$

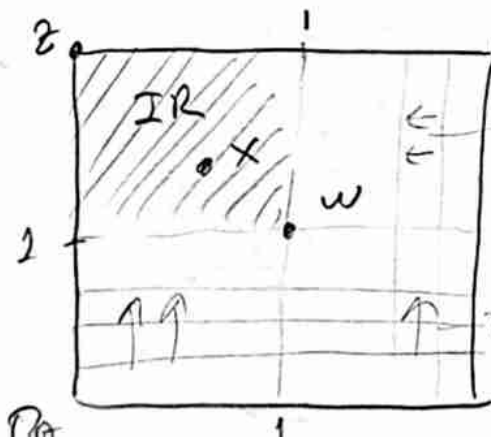
$$\begin{aligned} w_A &= (1, 1) \\ w_B &= (1, 1) \\ \hline w &= (1, 1) \end{aligned}$$

$$\left. \begin{aligned} X \succ_A w \\ X \succ_B w \end{aligned} \right\} w \notin PE$$

$$\left. \begin{aligned} z \succ_A x \\ z \succ_B x \end{aligned} \right\} x \notin PE$$

$z \in PE$ only

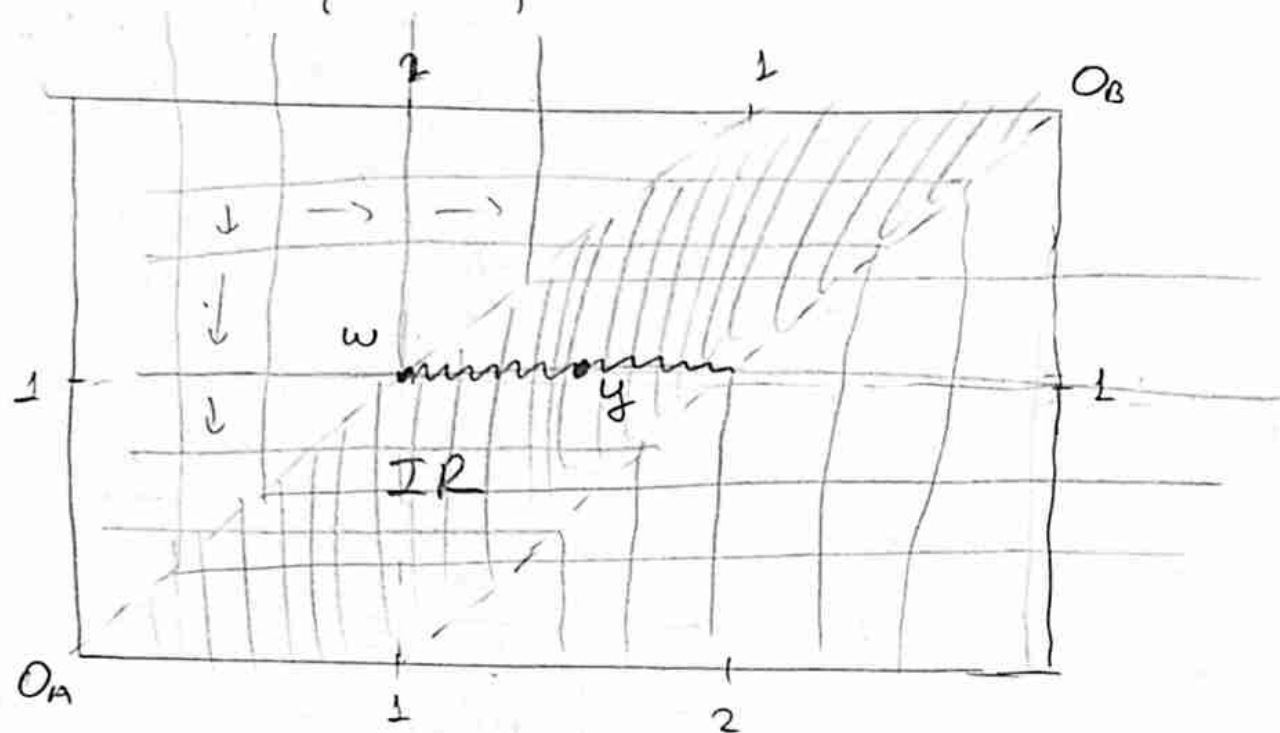
$PE \cap IR = z$ only point



Best bundles for B

Best bundles for A

EX) $U_A = \min \{x_1^A, x_2^A\}$ $w_A = (1, 1)$ PE? IR?
 $U_B = \min \{x_1^B, x_2^B\}$ $w_B = (2, 1)$



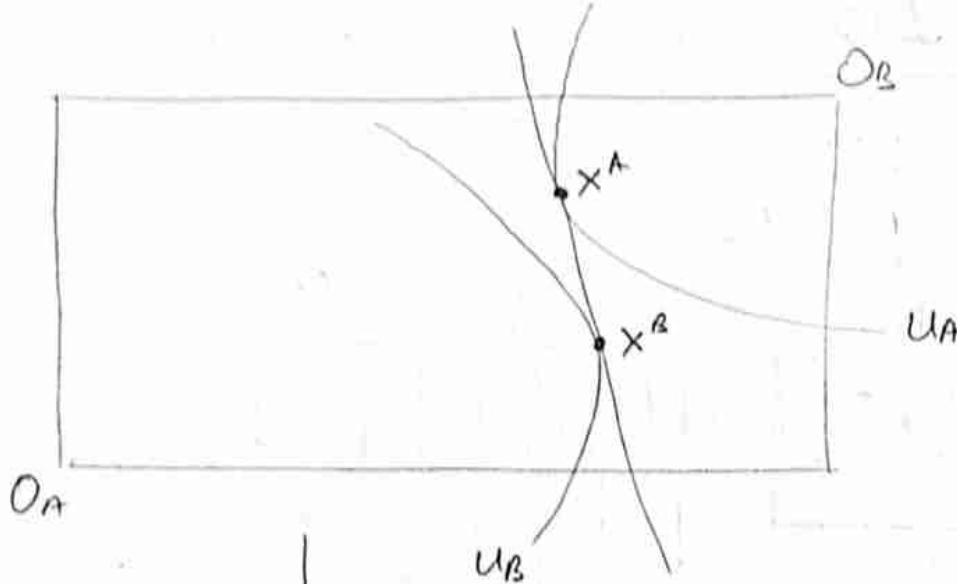
$w \sim y$ $w \in P.E$ Any allocations between offer curves are PE

Walrasian Equilibrium

(Competitive equilibrium) (Market Equilibrium)

A price vector (p_1, p_2) and an allocation $(x_A, x_B) = [(x_1^A, x_2^A), (x_1^B, x_2^B)]$ is a competitive equilibrium if

- 1) Each person is choosing the most preferred allocation in their budget set.
- 2) There should not be neither excess demand nor excess supply (markets clear)



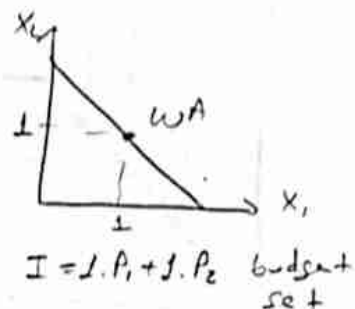
$$X_1^A + X_1^B < \text{total } X_1$$

excess supply

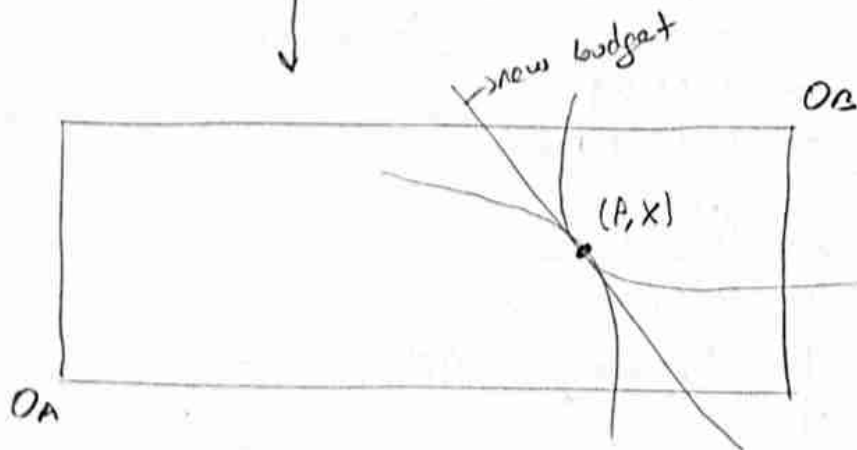
$$X_1^A + X_2^B > \text{total } X_2$$

excess demand

Not walrasian equilibrium



Prices change accordingly



Walrasian equilibrium ✓
 $(P, X) = WE$

EX) $U_A = X_1^A X_2^A$ $U_B = X_1^B (X_2^B)^2$
 $W^A = (1, 1)$ $W^B = (2, 1)$

Simultaneous 3 goods market
 $P_1 = 1, P_2 = P, P_3 = q$

Step 1: Finding the demand functions

Let $P_1 = 1$ $P_2 = P$, then $I^A = 1 + P$ $I^B = 2 + P$

$$X_1^A = \frac{1}{2} \frac{1+P}{1} \quad X_2^A = \frac{1}{2} \cdot \frac{1+P}{P} \quad X_1^B = \frac{1}{3} \frac{2+P}{1} \quad X_2^B = \frac{2}{3} \cdot \frac{2+P}{P}$$

Step 2: Clear the market

$$X_1^A + X_1^B = W_1^A + W_1^B \rightarrow \frac{1+P}{2} + \frac{2+P}{3} = 3 \quad P = \frac{11}{5}$$

Step 3: Find competitive allocation

$$X_1^A = \frac{1}{2} \left(1 + \frac{11}{5}\right) = \frac{8}{5} \quad X_1^B = \frac{1}{3} \left(2 + \frac{11}{5}\right) = \frac{7}{5}$$

$$X_2^A = \frac{1}{2} \left(\frac{1+11/5}{11/5}\right) = \frac{8}{11} \quad X_2^B = \frac{2}{3} \left(\frac{2+11/5}{11/5}\right) = \frac{14}{11}$$

Contract Curve

$$X_2^A = \frac{2 X_1^A}{6 - X_1^A}$$

$WE \in CC$

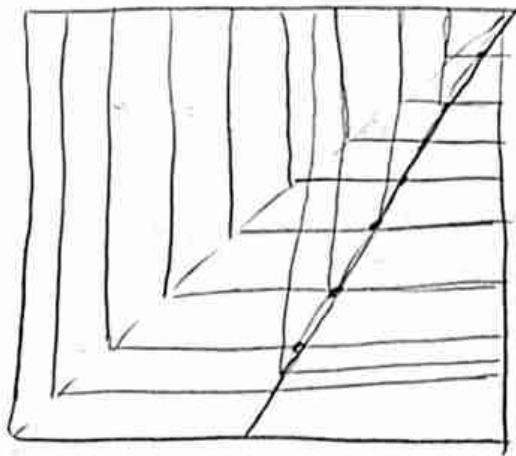
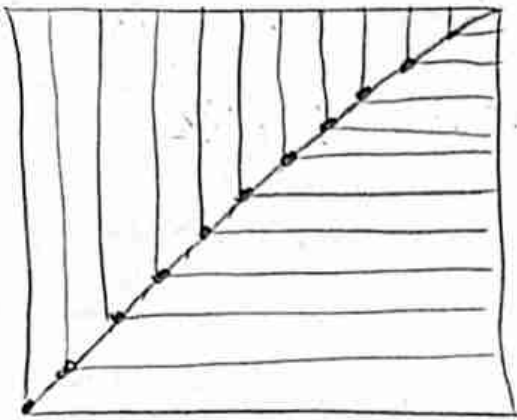
$$\Rightarrow \left[\left(1, \frac{11}{5}\right), \left(\left(\frac{8}{5}, \frac{8}{11}\right), \left(\frac{7}{5}, \frac{14}{11}\right)\right) \right] \text{ Walrasian Equilibrium}$$

Q1

$$U_A = \min\{X_A, Y_A\} = U_L$$

$$U_B = \max\{X_B, Y_B\} = U_L$$

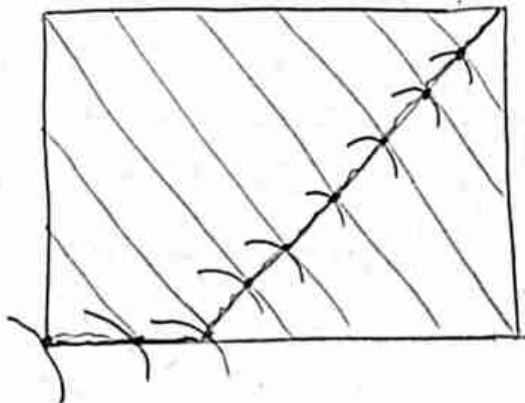
$$U_0 = \max\{2X_B, Y_B\}$$



Q2

$$U_{\text{get in}} = 3X + Y \rightarrow w = (1, 0)$$

$$U_{\text{emil}} = XY \rightarrow w = (0, 2)$$



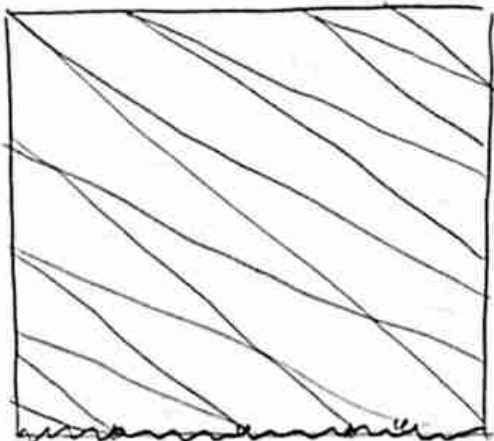
$$MRS_A = 3 = \frac{Y}{X} = MRS_B$$

$$3X = Y \text{ contract curve}$$

Q3

$$U_A = X_A + Y_A$$

$$U_B = X_B + 2Y_B$$



$$\text{Q4)} \quad U_A = X_A^\alpha Y_A^{1-\alpha} \quad U_B = X_B^\beta Y_B^{1-\beta} \quad W_A = (1, 2) \\ W_B = (2, 1)$$

Step 1: Individual max problem (Find Marshallian Demand)

Step 2: Clear the market (Total demand = total endowment)

Step 3: Plug P_x, P_y

$$I_A = P_x + 2P_y \quad X_A^* = \alpha \cdot \frac{(P_x + 2P_y)}{P_x} \quad Y_A^* = (1-\alpha) \frac{(P_x + 2P_y)}{P_y}$$

$$I_B = 2P_x + P_y \quad X_B^* = \beta \cdot \frac{(2P_x + P_y)}{P_x} \quad Y_B^* = (1-\beta) \frac{(2P_x + P_y)}{P_y}$$

$$\bar{X} = X_A + X_B = \alpha \frac{(P_x + 2P_y)}{P_x} + \beta \frac{(2P_x + P_y)}{P_x} = 3 \quad \text{set } P_x = 1$$

$$\alpha(1+2P_y) + \beta(2+P_y) = 3, \quad \alpha + 2P_y\alpha + 2\beta + \beta P_y = 3$$

$$P_y = \frac{3-\alpha-2\beta}{2\alpha+\beta} \quad (P_x, P_y) = \left(1, \frac{3-\alpha-2\beta}{2\alpha+\beta}\right)$$

$$X_A^* = \frac{\alpha(6-3\beta)}{2\alpha+\beta} \quad X_B^* = \frac{3\beta(\alpha+1)}{2\alpha+\beta}$$

$$Y_A^* = \frac{(1-\alpha)(6-3\beta)}{3-\alpha-2\beta} \quad Y_B^* = \frac{3(1-\beta)(\alpha+1)}{3-\alpha-2\beta}$$

$$\text{Q5)} \quad U_A = \sqrt{X_1^A} + X_2^A \quad U_B = X_1^B + \sqrt{X_2^B} \quad W_A = (2, 1) \\ W_B = (0, 2)$$

1) Find demand functions

$$\text{Max } \sqrt{X_1^A} + X_2^A \quad \text{s.t.} \quad P_1 X_1^A + P_2 X_2^A = 2P_1 + P_2$$

$$MRS_A = \frac{\frac{1}{2}(X_1^A)^{-1/2}}{1} = \frac{P_1}{P_2} \quad \frac{1}{2}(X_1^A)^{-1/2} \cdot P_2 = P_1 \quad \frac{1}{2\sqrt{X_1^A}} = \frac{P_1}{P_2} \quad X_1^A = \frac{P_2^2}{4P_1^2}$$

$$\text{set } P_2 = 1 \quad X_1^A = \frac{1}{4P_1^2}$$

$$X_2^A = \begin{cases} 2P_1 + 1 - P_1 \cdot \frac{1}{4P_1^2} & \text{if } 2P_1 + 1 > \frac{1}{4P_1} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{For B: Max } X_1^B + \sqrt{X_2^B} \quad \text{s.t.} \quad P_1 X_1^B + P_2 X_2^B = 2P_2 \quad P_1 X_1^B + X_2^B = 2$$

$$\frac{1}{2}(X_2^B)^{-1/2} = P_1 \quad X_2^B = \frac{P_1^2}{4}$$

$$X_1^B = \begin{cases} \frac{2 - \frac{P_1^2}{4}}{P_1} = \frac{2}{P_1} - \frac{P_1}{4} > 0 & \text{if } P_1 \leq 2\sqrt{2} \\ 0 & \text{otherwise} \end{cases}$$

For good 1: If $P_1 > 2\sqrt{2}$ $\frac{1}{4P_1^2} = 2$ $P_1 = \frac{1}{2\sqrt{2}}$ ~~$P_1 > 2\sqrt{2}$~~
contradiction

check $P_1 < 2\sqrt{2}$.

$$\frac{1}{4P_1^2} + \frac{2}{P_1} - \frac{P_1}{4} = 2$$

$$\cancel{P_1 = 1}, P_1 = -\frac{1}{2}\sqrt{37}, P_1 = \frac{1}{2}\sqrt{37}$$

$$X_1^A = \frac{1}{4} \quad X_2^A = \frac{11}{4} \quad X_1^B = \frac{7}{4} \quad X_2^B = \frac{1}{4} \quad \begin{matrix} P_1 = 1 \\ P_2 = 1 \end{matrix}$$

02/01/2025 week 15

Suppose that there are k goods. If $(k-1)$ markets clear, then also the k^{th} market clears

Let $z_i(p)$ is excess demand for good i at a given price vector $p = (p_1, p_2, \dots, p_k)$

$D_i(p)$ = Aggregate demand for good i at prices p

$S_i(p)$ = Aggregate supply for good i at prices p

$z_i(p) = D_i(p) - S_i(p)$ The market for good i is clear if $z_i(p) = 0$

Since all agents in economy satisfy budget constraints, the total value of excess demand across all goods must be zero

$$\sum_{i=1}^k p_i z_i(p) = 0$$

if $z_i(p) = 0$ for $i = 1, 2, \dots, (k-1)$ then $p_k z_k = 0$

since $p_k > 0$, $z_k = 0$

First fundamental theorem of Welfare Economics:

Any competitive allocation is PE (Adam Smith invisible hand)

$$WE \in PE \quad WE \in IR$$

$$\hookrightarrow (p, x) \quad u(x)$$

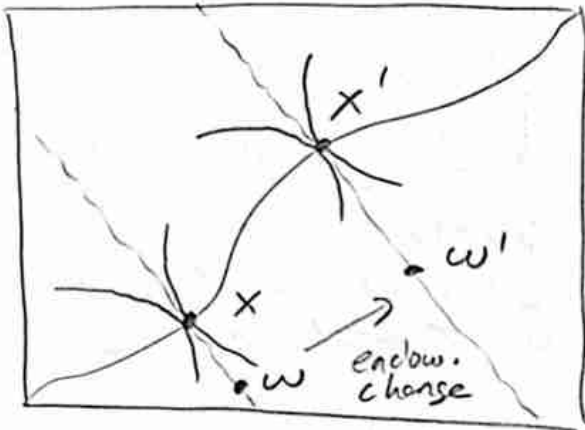
Assumptions:

- 1) Perfect competition
- 2) Complete markets
- 3) No externalities
- 4) Perfect Information
- 5) Convex preferences and production sets

Second Welfare Theorem:

Suppose the preferences are convex. Then, any interior PE allocation can be obtained as a competitive equilibrium from some initial endowment.

* Any Pareto Efficient allocation can be achieved as a competitive equilibrium, given appropriate endowment.



Lexicographic Preferences

Bundle $A = (x_1, x_2, \dots, x_k)$

Bundle $B = (y_1, y_2, \dots, y_k)$

Consumer gives absolute priority to the first good.

if $x_1 > y_1$ choose bundle A

if $x_1 = y_1$ continue compare

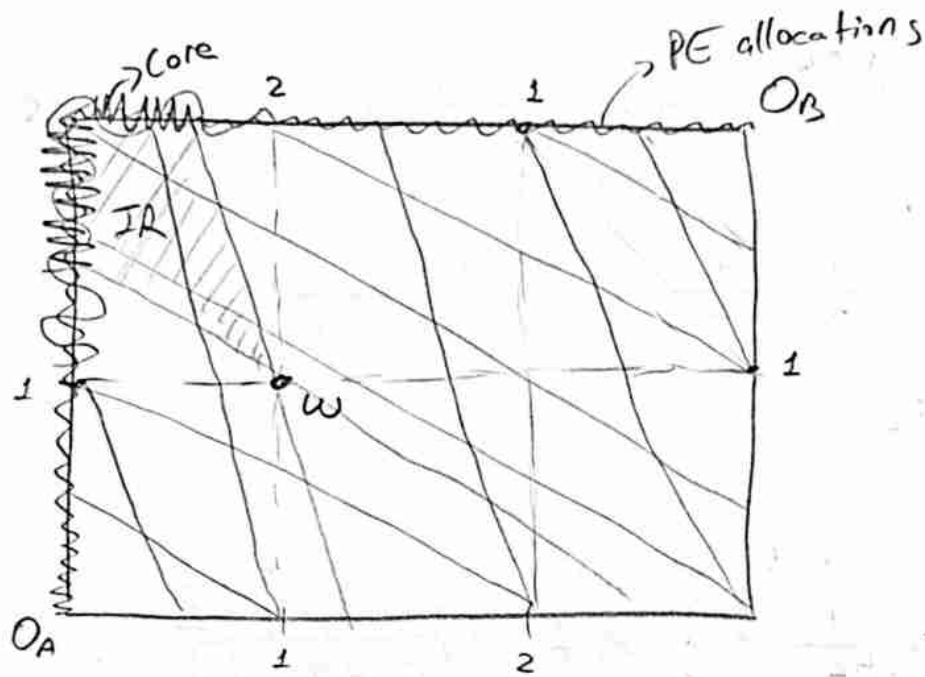
↳ if $x_2 > y_2$ choose bundle A

if $x_2 = y_2$, compare x_3, y_3

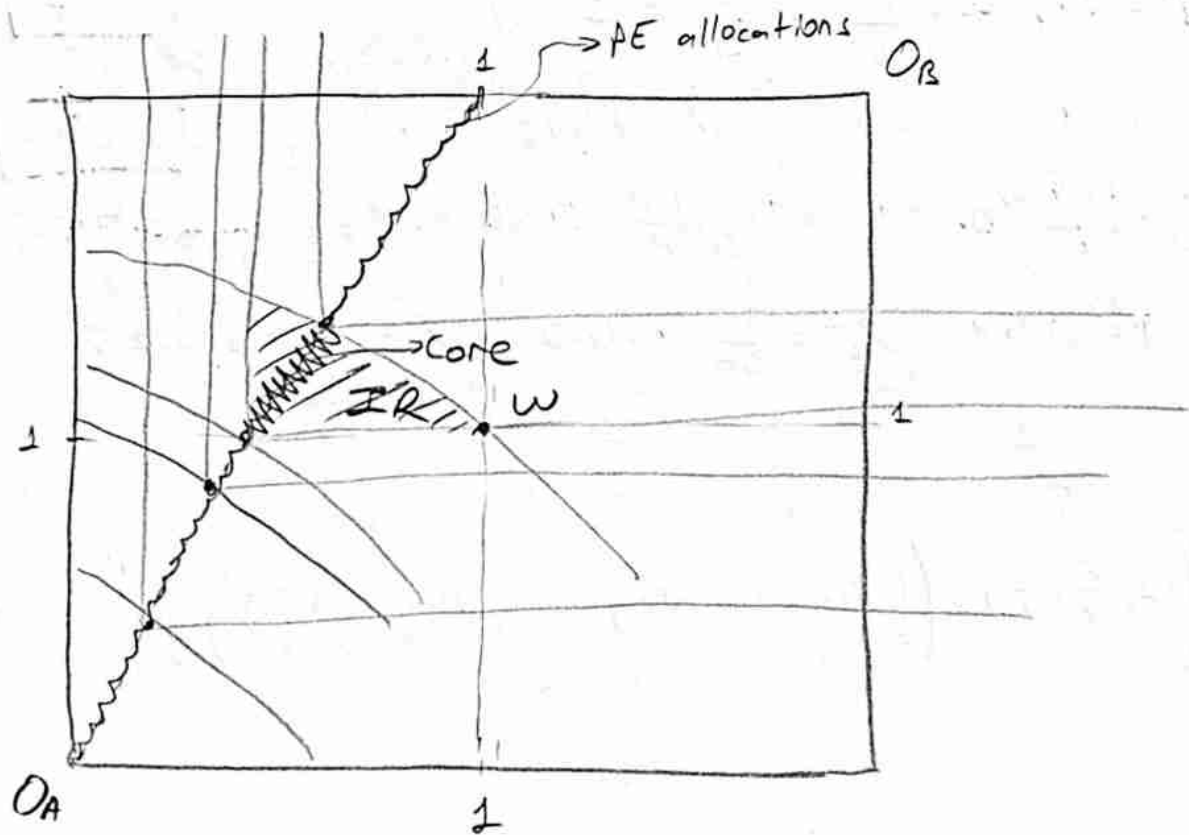
There is no utility function representation

$x \succeq_L y$ if $(x_1 > y_1)$ or $(x_1 = y_1 \text{ and } x_2 > y_2)$

EX)
$$\begin{aligned} U_A &= X_1^A + 2X_2^A & \omega_A &= (1, 1) \\ U_B &= 2X_1^B + X_2^B & \omega_B &= (2, 1) \end{aligned} \quad \left. \vphantom{\begin{aligned} U_A &= X_1^A + 2X_2^A \\ U_B &= 2X_1^B + X_2^B \end{aligned}} \right\} \omega = (3, 2)$$



EX)
$$\begin{aligned} U_A &= \min \{ 2X_1^A, X_2^A \} & \omega_A &= (1, 1) \\ U_B &= X_1^B X_2^B & \omega_B &= (1, 1) \end{aligned} \quad \left. \vphantom{\begin{aligned} U_A &= \min \{ 2X_1^A, X_2^A \} \\ U_B &= X_1^B X_2^B \end{aligned}} \right\} \omega = (2, 2)$$



$$EX) \quad U^A = X_1^A X_2^A (X_3^A)^2 \quad \omega^A = (1, 1, 0) \\ U^B = (X_1^B)^2 X_2^B X_3^B \quad \omega^B = (0, 0, 1)$$

WE?

Step 1: finding demand functions $P_1 = 1, P_2 = P, P_3 = q$

$$I^A = 1 + P \quad I^B = q$$

$$X_1^A = \frac{1}{4} \cdot \frac{1+P}{1} \quad X_2^A = \frac{1}{4} \cdot \frac{1+P}{P} \quad X_3^A = \frac{2}{4} \cdot \frac{1+P}{q}$$

$$X_1^B = \frac{2}{4} \cdot \frac{q}{1} \quad X_2^B = \frac{1}{4} \cdot \frac{q}{P} \quad X_3^B = \frac{1}{4} \cdot \frac{q}{q}$$

Step 2: clear the markets

$$\begin{aligned} X_1^A + X_1^B &= \omega_1 \rightarrow \frac{1+P}{4} + \frac{q}{2} = 1 \\ X_2^A + X_2^B &= \omega_2 \rightarrow \frac{1+P}{4P} + \frac{q}{4P} = 1 \\ X_3^A + X_3^B &= \omega_3 \rightarrow \frac{1+P}{2q} + \frac{1}{4} = 1 \end{aligned} \quad \left\{ \begin{aligned} 1+P+2q &= 4 \\ P+2q &= 3 \\ 1+P+q &= 4P \\ 3P-q &= 1 \\ \hline 7P &= 5 \end{aligned} \right. \quad \begin{aligned} P &= \frac{5}{7} \\ q &= \frac{8}{7} \end{aligned}$$

Step 3: find competitive allocations

$$\begin{aligned} X_1^A &= \frac{1}{4} \cdot \frac{1+\frac{5}{7}}{1} \approx 0.43 & X_2^A &= \frac{1}{4} \cdot \frac{1+P}{P} = 0.60 & X_3^A &= \frac{1}{2} \cdot \frac{1+P}{q} = 0.75 \\ X_1^B &\approx \frac{1}{2} \cdot q \approx 0.57 & X_2^B &= \frac{q}{4P} = 0.40 & X_3^B &= \frac{1}{4} \cdot \frac{q}{q} = 0.25 \\ \hline &1 &1 &1 \end{aligned}$$

Markets clear

$$WE = \left[\left(1, \frac{5}{7}, \frac{8}{7} \right), \left((0.43, 0.60, 0.75), (0.57, 0.40, 0.25) \right) \right]$$

$$\text{EX)} \quad \left. \begin{aligned} U^A &= \sqrt{X_1^A} + \sqrt{X_2^A} & \omega_A &= (4, 0) \\ U^B &= \sqrt{X_1^B} + \sqrt{X_2^B} & \omega_B &= (0, 4) \end{aligned} \right\} \omega = (4, 4) \quad WE = ?$$

$$\text{Let } P_1 = 1, P_2 = P \Rightarrow I^A = 4 \quad I^B = 4P$$

1st step) Find demand functions

$$\text{Max } U^A \text{ s.t. } X_1^A + P X_2^A = 4$$

$$\mathcal{L} = \sqrt{X_1} + \sqrt{X_2} + \lambda (4 - X_1 - P X_2)$$

$$\frac{\partial \mathcal{L}}{\partial X_1} = \frac{1}{2\sqrt{X_1}} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_2} = \frac{1}{2\sqrt{X_2}} - P\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 4 - X_1 - P X_2 = 0$$

$$\left\{ \begin{aligned} \frac{1}{2\sqrt{X_1}} &= P = \frac{\sqrt{X_2}}{\sqrt{X_1}} & X_2 &= P^2 X_1 \\ \frac{1}{2\sqrt{X_2}} &= P\lambda & & \\ 4 - X_1 - P^3 X_1 &= 0 & X_1(1 + P^3) &= 4 \\ X_1^A &= \frac{4}{1 + P^3} & X_2^A &= \frac{4P^2}{1 + P^3} \end{aligned} \right.$$

$$\text{Max } U^B \text{ s.t. } X_1^B + P X_2^B = 4P$$

$$X_1^B = X_2^B$$

$$X_1^B + P X_1^B = 4P$$

$$X_1^B = \frac{4P}{1 + P}$$

$$X_2^B = \frac{4P}{1 + P}$$

2) Clear markets

$$X_1^A + X_1^B = 4 \rightarrow \frac{4}{1 + P^3} + \frac{4P}{1 + P} = 4$$

$$X_2^A + X_2^B = 4 \rightarrow \frac{4P^2}{1 + P^3} + \frac{4P}{1 + P} = 4$$

$$\left\{ \begin{aligned} \frac{4P^2}{1 + P^3} &= \frac{4}{1 + P} & \boxed{P = 1} \end{aligned} \right.$$

3) Find Walrasian eqm

$$X_1^A = 2 \quad X_2^A = 2 \quad X_1^B = 2 \quad X_2^B = 2$$

$$WE = \left[(1, 1), ((2, 2), (2, 2)) \right]$$

$$\begin{array}{ll}
 \text{EX)} & U_A = X_1^A (X_2^A)^3 \quad W_A = (0, 8) \\
 & U_B = X_1^B X_2^B \quad W_B = (0, 4) \\
 & U_C = \min\{2X_1^C, X_2^C\} \quad W_C = (8, 0) \\
 & U_D = 2X_1^D + X_2^D \quad W_D = (4, 0)
 \end{array}
 \left. \vphantom{\begin{array}{l} U_A \\ U_B \\ U_C \\ U_D \end{array}} \right\} W = (12, 12)$$

Let $P_1 = 1$ $P_2 = P$ $I^A = 8P$ $I^B = 4P$ $I^C = 8$ $I^D = 4$

1) Find demand functions

$$X_1^A = \frac{1}{4} \cdot \frac{8P}{1} \quad X_2^A = \frac{3}{4} \cdot \frac{8P}{P}$$

$$X_2^A = \frac{1}{2} \cdot \frac{4P}{1} \quad X_2^B = \frac{1}{2} \cdot \frac{4P}{P}$$

Max U_C s.t.o $X_1 + PX_2 = 8$

$$2X_1 = X_2 \quad X_1 + \frac{P}{2}X_1 = 8 \quad X_1^C = \frac{8}{1 + \frac{P}{2}} = \frac{16}{2+P} \quad X_2^C = \frac{32}{2+P}$$

Max U_D s.t.o $X_1 + PX_2 = 4$

if $\frac{1}{P} < 2$ $X_1^D = 4$ $X_2^D = 0$ if $\frac{1}{P} > 2$ $X_1^D = 0$ $X_2^D = 4/P$

2) Clear the markets

Case 1 ($\frac{1}{P} < 2$)

$$X_1^A + X_1^B + X_1^C + X_1^D = 12 \rightarrow 2P + 2P + \frac{16}{2+P} + 4 = 12$$

$$4P + \frac{16}{2+P} = 8 \rightarrow P = 0?$$

$$X_2^A + X_2^B + X_2^C + X_2^D = 12 \rightarrow 6 + 2 + \frac{32}{2+P} + 0 = 12$$

$$\frac{32}{2+P} = 4 \rightarrow P = 6 \quad P = 6$$

Case 2 ($\frac{1}{P} > 2$)

$$X_1^A + X_1^B + X_1^C + X_1^D = 12 \rightarrow 2P + 2P + \frac{16}{2+P} + 0 = 12 \Rightarrow 4P + \frac{16}{2+P} = 12$$

$$8P + 4P^2 + 16 = 12$$

$$P^2 + 2P + 1 = 0$$

$$P = -1?$$

$$X_2^A + X_2^B + X_2^C + X_2^D = 12 \rightarrow 6 + 2 + \frac{32}{2+P} + \frac{4}{P} = 12$$

$$\frac{32}{2+P} + \frac{4}{P} = 0 \quad P < 0?$$

Market equilibrium not possible

Sample Final

Q1) Find demand functions for each goods

$$U(x_1, x_2, x_3) = \sqrt{x_1} + 2\sqrt{x_2} + 3\sqrt{x_3}$$

Assume interior solution

$$\mathcal{L} = \sqrt{x_1} + 2\sqrt{x_2} + 3\sqrt{x_3} + \lambda (I - P_1 x_1 - P_2 x_2 - P_3 x_3)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{1}{2\sqrt{x_1}} - P_1 \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{1}{\sqrt{x_2}} - P_2 \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_3} = \frac{3}{2} \frac{1}{\sqrt{x_3}} - P_3 \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_1 x_1 - P_2 x_2 - P_3 x_3 = 0$$

$$\left\{ \begin{array}{l} \frac{\frac{1}{2\sqrt{x_1}}}{\frac{1}{\sqrt{x_2}}} = \frac{P_1}{P_2} = \frac{\sqrt{x_2}}{2\sqrt{x_1}} \quad \frac{P_1^2}{P_2^2} = \frac{x_2}{4x_1} \quad x_2 = \frac{4P_1^2}{P_2^2} x_1 \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\frac{1}{2\sqrt{x_1}}}{\frac{3}{2} \frac{1}{\sqrt{x_3}}} = \frac{P_1}{P_3} = \frac{\sqrt{x_3}}{3\sqrt{x_1}} \quad \frac{P_1^2}{P_3^2} = \frac{x_3}{9x_1} \quad x_3 = \frac{9P_1^2}{P_3^2} x_1 \end{array} \right.$$

$$I - P_1 x_1 - P_2 x_1 \frac{4P_1^2}{P_2^2} - P_3 x_1 \frac{9P_1^2}{P_3^2} = 0 \quad I = x_1 \left(1 + \frac{4P_1^2}{P_2^2} + \frac{9P_1^2}{P_3^2} \right)$$

$$x_1 = \frac{I}{1 + \frac{4P_1^2}{P_2^2} + \frac{9P_1^2}{P_3^2}} \quad x_2 = \frac{4P_1^2}{P_2^2} \cdot \frac{I}{1 + \frac{4P_1^2}{P_2^2} + \frac{9P_1^2}{P_3^2}} \quad x_3 = \frac{9P_1^2}{P_3^2} \cdot \frac{I}{1 + \frac{4P_1^2}{P_2^2} + \frac{9P_1^2}{P_3^2}}$$

Q3) In an exchange economy, assume that the budget constraints for each of the I consumers and the market clearing constraints for the first $N-1$ goods are satisfied. Show that the last market-clearing constraints for good N is also satisfied.

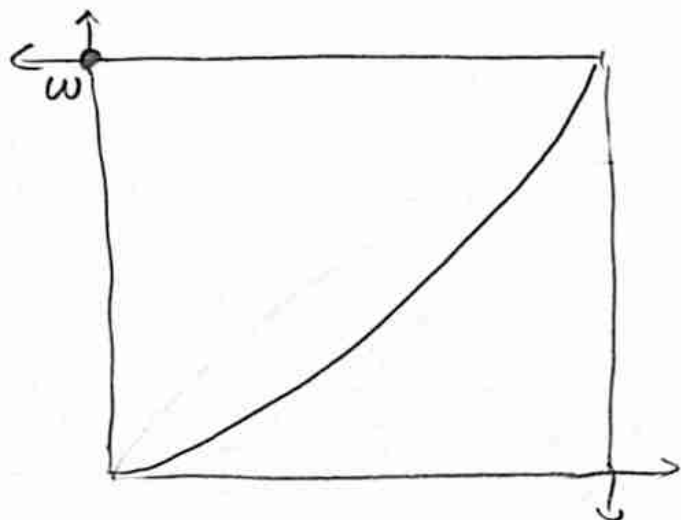
Let $z_i(p)$ is the excess demand for good i

$$z_i = x_i - w_i \quad \text{we know that } \sum z_i = 0 \text{ for } i=1, 2, \dots, N-1$$

then z_N must be 0

Q4) $U_A = (X_1^A)^{2/3} (X_2^A)^{1/3}$ $W_A = (0, 30)$ P_1, P_2
 $U_B = (X_1^B)^{1/3} (X_2^B)^{2/3}$ $W_B = (30, 0)$

a) Draw edgeworth box, and mark endowment point



b) Derive contract curve

$$MRS_A = \frac{\frac{2}{3} \cdot (X_2^A)^{1/3}}{\frac{1}{3} \cdot (X_1^A)^{2/3}} = 2 \cdot \frac{X_2^A}{X_1^A}$$

$$MRS_B = \frac{\frac{1}{3} \cdot (X_2^B)^{1/3}}{\frac{2}{3} \cdot (X_1^B)^{2/3}} = \frac{1}{2} \frac{X_2^B}{X_1^B}$$

$$X_1^A + X_1^B = 30$$

$$X_1^B = 30 - X_1^A$$

$$X_2^A + X_2^B = 30$$

$$X_2^B = 30 - X_2^A$$

$$2 \frac{X_2^A}{X_1^A} = \frac{1}{2} \cdot \frac{30 - X_2^A}{30 - X_1^A}$$

$$4 \cdot (30 - X_1^A) X_2^A = X_1^A (30 - X_2^A)$$

$$(120 - 4X_1^A) X_2^A = 30X_1^A - X_1^A X_2^A$$

$$120X_2^A - 4X_1^A X_2^A = 30X_1^A - X_1^A X_2^A$$

$$120X_2^A - 3X_1^A X_2^A = 30X_1^A$$

$$X_2^A (120 - 3X_1^A) = 30X_1^A$$

$$X_2^A = \frac{30X_1^A}{120 - 3X_1^A}$$

c) Solve for competitive eqm.

Let $P_1 = 1$, $P_2 = P$ $I^A = 30P$ $I^B = 30$

1) Find demand functions

$$X_1^A = \frac{2}{3} \cdot \frac{30P}{1}$$

$$X_2^A = \frac{1}{3} \cdot \frac{30P}{P}$$

$$X_1^B = \frac{1}{3} \cdot \frac{30}{1}$$

$$X_2^B = \frac{2}{3} \cdot \frac{30}{P}$$

2) Clear markets

$$X_1^A + X_1^B = 30 \rightarrow 20P + 10 = 30 \rightarrow P = 1$$

$$X_2^A + X_2^B = 30 \rightarrow 10 + \frac{20}{P} = 30 \rightarrow P = 1$$

3) Solve

$$X_1^A = 20 \quad X_2^A = 10 \quad X_1^B = 10 \quad X_2^B = 20$$

$$WE = ((1, 1), ((20, 10), (10, 20)))$$

satisfies PE $\rightarrow X_2^A = \frac{30X_1^A}{120 - 3X_1^A}$ ✓

Q5 $U_i = \max \{X_{i1}, X_{i2}\}$ $W_i = (1, \alpha)$

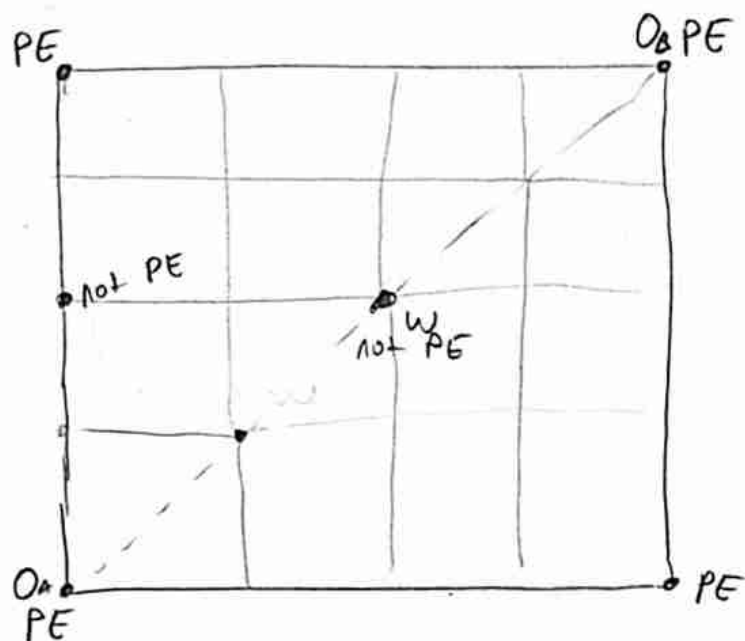
Find contract curve and edgeworth box

per capita

$$\left. \begin{aligned} U_A &= \max \{X_1^A, X_2^A\} \cdot W_A = (1, \alpha) \\ U_B &= \max \{X_1^B, X_2^B\} \cdot W_B = (1, \alpha) \end{aligned} \right\} W = (2, 2\alpha)$$

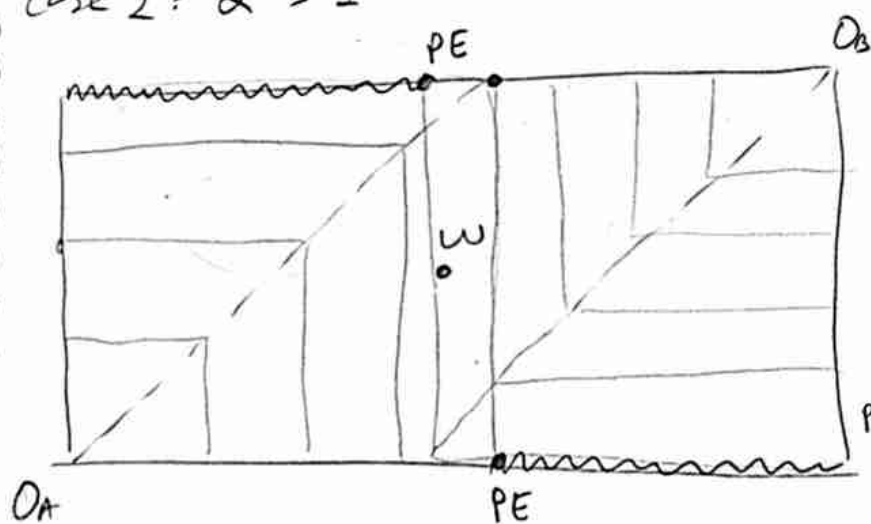
Let $P_1 = 1, P_2 = P$ $I^A = 1 + \alpha P$ $I^B = 1 + \alpha P$

Case 1: $\alpha = 1$



Only PE points are the corners

Case 2: $\alpha > 1$



Only PE points are the corners

Case 3: $\alpha < 1$

