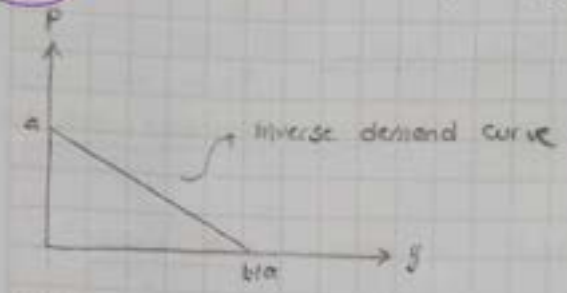
Ex: (Linear Demand Curve) $P(y) = a - by$, $a, b > 0$



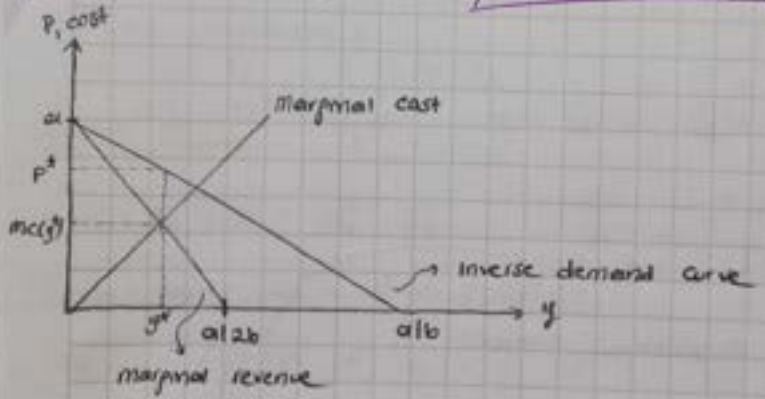
$$\max_{y \geq 0} \underbrace{(a-by)\cdot y}_{\text{Revenue}} - \underbrace{c(y)}_{\text{cost}}$$

$$\underbrace{R(y)}_{\text{Revenue}} = (a-by)\cdot y = a\cdot y - b\cdot y^2$$

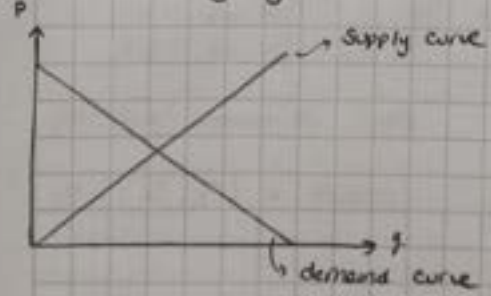
$$\underbrace{MR(y)}_{\text{marginal revenue}} = a - 2by$$

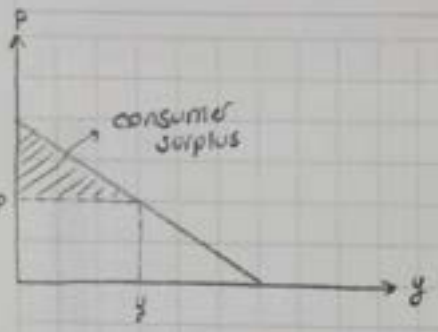
$$\text{marginal revenue} = R'(y)$$

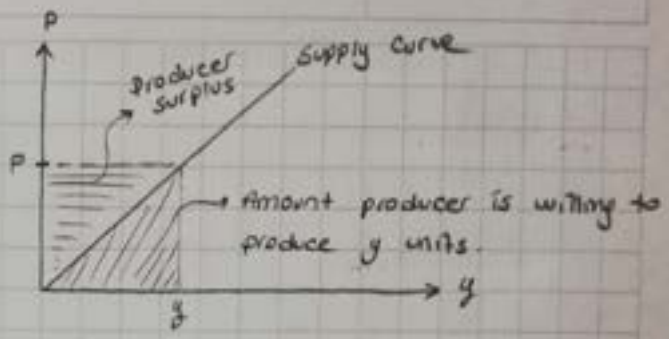
Profit maximizing output: $\boxed{MR(y^*) = Mc(y^*)}$

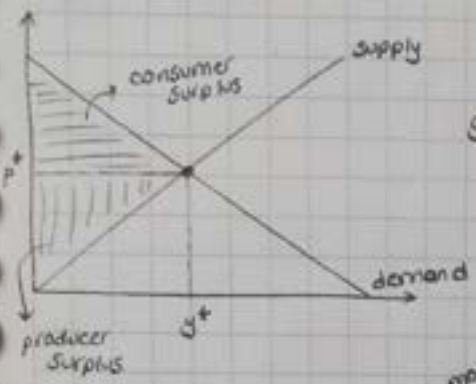


- Is monopoly good for social welfare?

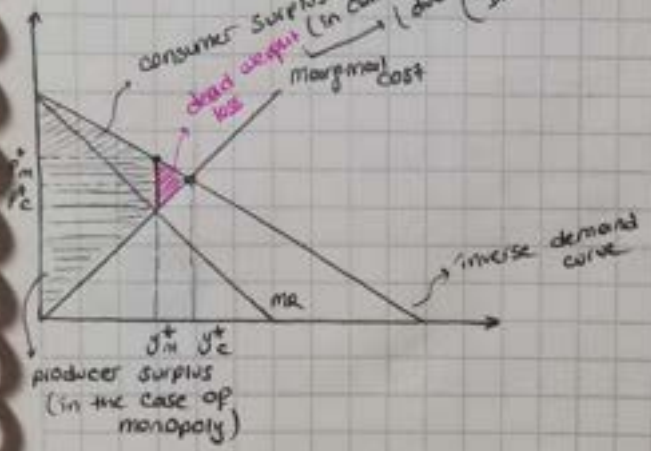








Social welfare = CS + PS



Profit Maximization

$$\max_{x_1, x_2, y} \; p \cdot y - (w_1 x_1 + w_2 x_2)$$

s.to $f(x_1, x_2) \geq y$

$$\Downarrow$$

$$(x_1^*, x_2^*, y^*)$$

Step 1 : For a given output level $\bar{y}$, what is the least costly way to produce $\bar{y}$?

$$\left.\begin{array}{l} \min_{x_1, x_2} \; w_1 x_1 + w_2 x_2 \\ \text{s.to} \; f(x_1, x_2) = \bar{y} \end{array}\right\} \text{cost minimization problem}$$

Step 2 what is the optimal y^*?

Short-run cost minimization : $x_2 = \bar{x}_2$ (variable cost / fixed cost)

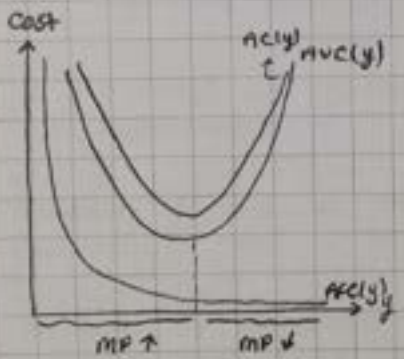
$$\left.\begin{array}{l} \min_{x_1} \; (w_1 x_1) + (w_2 \bar{x}_2) \\ \text{s.to} \; f(x_1, x_2) = \bar{y} \end{array}\right\}$$

$\underbrace{c(w_1, w_2, \bar{y})}_{\text{value of the obj func at optimal point}}$

Cost = variable cost + fixed cost

Fix $\bar{w}_1, \bar{w}_2$: $C(y) = \underbrace{C_v(y)}_{\text{variable cost}} + \underbrace{F}_{\text{fixed cost}}$

Average cost : $AC(y) = \frac{C(y)}{y} = \underbrace{\frac{C_v(y)}{y}}_{AVC(y)} + \underbrace{\frac{F}{y}}_{AFC(y)}$



★ "Diminishing Marginal Product"

L	y	
0	0	MP(1) = 10
1	10	
2	25	MP(2) = 15
3	50	MP(3) = 25
⋮	⋮	

L ↑ MP ↓ (but increasing)

$f(x_1, x_2) = x_1^{1/2} \cdot x_2^{1/2}$

$MP_1 = \frac{1}{2} \cdot x_1^{-1/2} \cdot x_2^{1/2}$

If $x_1 \uparrow$, $MP_1 \downarrow$

$y = 10 \rightarrow$ Cost = (wage of 1 Labor) ω

$AVC = \frac{\omega}{10}$

$y = 50 \rightarrow AVC = \frac{3\omega}{50}$

$\Rightarrow \max_{x_1 \geq 0} \; p \cdot f(x_1, \bar{x}_2) - (w_1 \cdot x_1 + w_2 \cdot \bar{x}_2)$

If $x_1^* > 0$, $p \cdot \frac{df}{dx_1} - w_1 = 0$, $w_1 = p \cdot \frac{df}{dx_1}$

long-run profit maximation problem $\max_{x_1, x_2, y} \; p \cdot y - (w_1 \cdot x_1 + w_2 x_2)$

s.to $f(x_1, x_2) = y$ ①

$\Rightarrow \max_{x_1, x_2} \; p \cdot f(x_1, x_2) - (w_1 x_1 + w_2 x_2)$

If $x_1^*, x_2^* > 0$: $p \cdot \frac{df}{dx_1} - w_1 = 0$ ②

$p \cdot \frac{df}{dx_2} - w_2 = 0$ ③

① ② and ③ give us the optimal plan.

② $p \cdot \frac{df}{dx_1} = w_1$

③ $p \cdot \frac{df}{dx_2} = w_2$

$\Rightarrow \frac{df/dx_1}{df/dx_2} = \frac{w_1}{w_2}$

$\Rightarrow \underbrace{\frac{df/dx_1}{w_1}}_{\text{marginal product of } x_1 \text{ \$ spent on input 1}} = \frac{df/dx_2}{w_2}$ } marginal product of x_2 \$ spent on input 2

$\hookrightarrow \frac{MP_1}{w_1} = \frac{MP_2}{w_2}$ 시

Suppose profit-maximizing plan (x_1^*, x_2^*, y^*)

$\Rightarrow$ to produce y^*, the "cost-minimizing" input levels are (x_1^*, x_2^*)

$f(x_1, x_2) = y^*$

Decompose PMP into two steps :

Step 1 : for any output level $\bar{y}$, find cost-minimizing input levels and the minimum cost required for $\bar{y}$

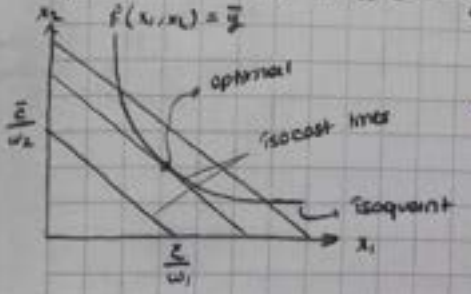
Step 2 : find optimal y^*

Step 3 : for a given output level $\bar{y}$, input prices w_1, w_2 :

$\min_{x_1, x_2} \; w_1 x_1 + w_2 x_2$

s.to $f(x_1, x_2) = \bar{y}$

If x_1^*, x_2^* solve this problem, then the value of the objective function : $w_1 x_1^* + w_2 x_2^* \equiv \underbrace{C(w_1, w_2, \bar{y})}_{\text{cost function}}$



isocost line : for a given cost level $\bar{c}$,

$w_1 x_1 + w_2 x_2 = \bar{c}$

$x_2 = \frac{\bar{c}}{w_2} - \frac{w_1 x_1}{w_2}$ 시

At optimal point

Slope of isoquant = Slope of isocost

$$\underbrace{-\frac{df/dx_1}{df/dx_2}}_{} = \underbrace{-\frac{w_1}{w_2}}_{}$$

★ TRS = technical rate of substitution

$$= \frac{df/dx_1}{df/dx_2} = \frac{w_1}{w_2} \Rightarrow \underbrace{\frac{w_1}{df/dx_1}}_{} = \underbrace{\frac{w_2}{df/dx_2}}_{}$$

cost of increasing production by 1 unit by only using input 1

$$\min_{x_1, x_2} \; w_1 x_1 + w_2 x_2 \quad \text{s.to} \quad f(x_1, x_2) = \bar{y}$$

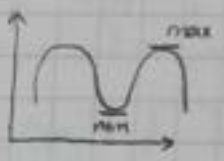
Alternative method : Lagrange method

$$\mathcal{L}(x_1, x_2, \lambda) = (w_1 x_1 + w_2 x_2) - \lambda\left(f(x_1, x_2) - \bar{y}\right)$$

$$\frac{d\mathcal{L}}{dx_1} = w_1 - \lambda \cdot \frac{df}{dx_1} = 0 \quad \cdots ①$$

$$\frac{d\mathcal{L}}{dx_2} = w_2 - \lambda \cdot \frac{df}{dx_2} = 0 \quad \cdots ②$$

$$\frac{d\mathcal{L}}{d\lambda} = -\left(f(x_1, x_2) - \bar{y}\right) = 0 \quad \cdots ③$$



$$\Rightarrow \; w_1 = \lambda \cdot \frac{df}{dx_1} \quad \text{by } ①$$

$$w_2 = \lambda \cdot \frac{df}{dx_2} \quad \text{by } ②$$

$$\frac{w_1}{w_2} = \frac{df/dx_1}{df/dx_2}$$

(If solution is interior, $x_1^* , x_2^* > 0$)

ex. $f(x_1, x_2) = x_1^{1/2} \cdot x_2^{1/2}$

cost function?

$$\begin{cases} \min & w_1 x_1 + w_2 x_2 \quad \text{for given } w_1, w_2, \bar{y} > 0 \\ \text{s.to} & x_1^{1/2} \cdot x_2^{1/2} = \bar{y} \end{cases}$$

$$\mathcal{L}(x_1, x_2, \lambda) = w_1 x_1 + w_2 x_2 - \lambda\left(x_1^{1/2} \cdot x_2^{1/2} - \bar{y}\right)$$

$$\frac{d\mathcal{L}}{dx_1} = w_1 - \lambda \cdot \frac{1}{2} \cdot x_1^{-1/2} \cdot x_2^{1/2} = 0 \quad ①$$

$$\frac{d\mathcal{L}}{dx_2} = w_2 - \lambda \cdot \frac{1}{2} \cdot x_2^{-1/2} \cdot x_1^{1/2} = 0 \quad ②$$

$$\frac{d\mathcal{L}}{d\lambda} = x_1^{1/2} \cdot x_2^{1/2} - \bar{y} = 0 \quad ③$$

Second fundamental Theorem of Welfare Economics:

- If preferences of each consumer is convex and w is Pareto efficient, there are prices $p^* = (p_1^*, p_2^*)$ such that (p^*, w) is a CE.

w = initial endowment

PE ⇒ CE

Mostly: market outcome ⇔ socially optimal outcome

EQUILIBRIUM WITH PRODUCTION

- Two agents: A, B
- Two goods: x, y
- Each consumer's initial endowment: w_A, w_B
- There is one firm which produces y by using x by a technology $y = f(x)$
- Firm is owned by agent A.

Feasible allocation:

x_A, x_B, x_f ; y_A, y_B

A consumes, B consumes, ↳ Firm's usage of x

$$x_A + x_B + x_f = w_A^x + w_B^x$$

$$y_A + y_B = w_A^y + w_B^y + f(x_f)$$

CE: (p^*, x^*, y^*) is a CE if

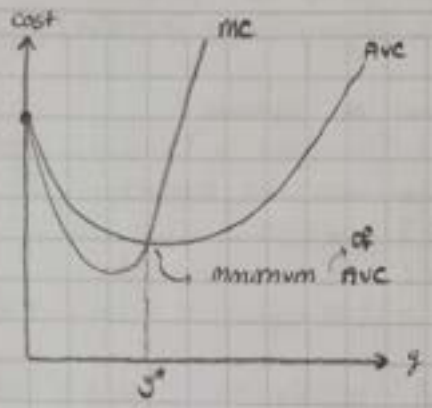
(1) Each consumer maximizes his/her utility given budgets:

$m_A = p_x \cdot w_A^x + p_y \cdot w_A^y + \pi$ ↳ profit of firm (owner = A)

$m_B = p_x \cdot w_B^x + p_y \cdot w_B^y$

(2) Firm maximizes profit

(3) (x^*, y^*) is feasible



Step 2 : what is the optimal y^* ?

Given prices p, w_1, w_2,

$\max_{y \geq 0} \; p \cdot y - c(y)$

If $y^* > 0$, $p - mc(y^*) = 0$

$\Rightarrow$ $p = mc(y^*)$ (→ (= additional extra money))

$\frac{T}{10}$	y	mc
	0	
	1	3
	2	4
	3	5
	⋮	⋮
	9	9
	10	11

$p < mc$ → decrease production

$p > mc$ → increase "

* If $p = mc(y)$, is y necessarily optimal?

// NOT NECESSARILY !

Producing 0 units ($-F$) vs. Producing y units ($p \cdot y - C_v(y) - F$)

In order for the producer to choose y.

$-F \leq p \cdot y - C_v(y) - F$

$\Rightarrow C_v(y) \leq p \cdot y$

$\Rightarrow \frac{C_v(y)}{y} \leq p$ → $AVC(y) \leq p$

* Producing a positive amount can be optimal if

Proof: Let (p, x) is an equilibrium
(x_A, x_B)

We claim x is PE. Suppose that x is not PE. Then, there is some feasible allocation $y = (y_A, y_B)$ such that

$$y_A \succsim^A x^A$$
and $$y_B \succsim^B x^B$$

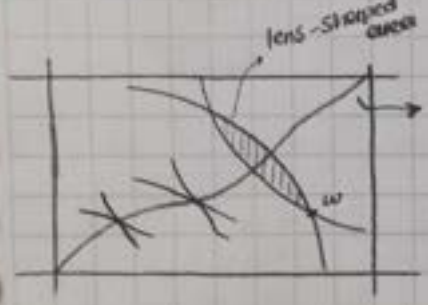
Since x_A maximises A's utility in the budget set:

$p \cdot y_A > p \cdot x_A$
(y_A's cost for A; x_A's cost for A)

Similarly, $p \cdot y_B > p \cdot x_B$

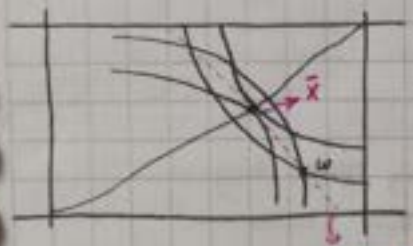
$$p(y_A + y_B) > p \cdot (x_A + x_B)$$
$$= w_A + w_B \qquad = w_A + w_B$$
(since y is feasible; since x is feasible)

this is not possible ↯
→ x is PE.

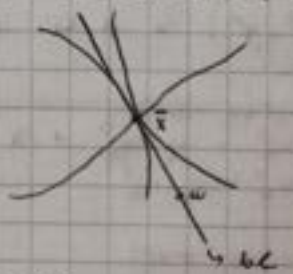


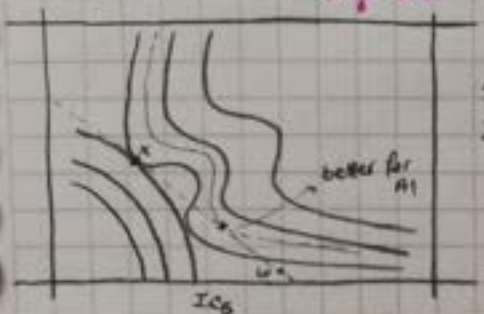
→ PE allocations

- CE cannot be outside the lens-shaped area.



✓ If prices are as shown above, $\bar{x}$ is CE.





✓ X is PE.
✗ X cannot be a CE.

Is there an equi. in which $p_x > p_y$?

$x \rightarrow \quad 2 + \frac{2p_y}{p_x+p_y} = 4$

$2p_y = 4p_x + 4p_y$

} not possible

Is there an equi. in which $p_x < p_y$?

$x \rightarrow$ A var olan bütün X'i istiyor. 3'ye talep edebileceği bir x kalmadı.

} not possible

Is there an equi. in which $p_x = p_y$?

$x \rightarrow \quad x_A + \frac{2p_y}{p_x+p_y} = 4$

$y_A + \frac{2p_y}{p_x+p_y} = 2$

} $x_A + y_A = 4$

$\frac{2p_y}{p_x+p_y} = 4 - x_A = 2 - y_A$

$\frac{2p_y}{p_x+p_y} = 1$

$2p_y = p_x + p_y$

$\boxed{p_y = p_x}$

} $2 = x_A - y_A$

$4 = x_A + y_A$

$\boxed{x_A = 3 \quad y_A = 1}$

$p^* = (p_x^*, p_y^*)$ where $p_x^* = p_y^*$ and $x_A^* = 3, \; y_A = 1$

$x_B = 1, \; y_B = 1$

is an equilibrium

FIRST FUNDAMENTAL THEOREM OF WELFARE ECONOMICS

- Any competitive equi. allocation is Pareto Efficient.
- Pareto efficiency ≠ "social welfare" maximization

Ex: $f(x_1, x_2) = x_1^a x_2^b$

If $a+b = 1$ → CRS
$a+b > 1$ → IRS
$a+b < 1$ → DRS

$$f(tx_1, tx_2) = (tx_1)^a \cdot (tx_2)^b = t^{a+b} \cdot \underbrace{x_1^a \cdot x_2^b}_{f(x_1, x_2)}$$

Assumption: Producer only cares about "profit".

Profit = Revenue − Cost

Given prices: p - price of outputs
w_1, w_2 - input prices

If I use (x_1, x_2) and produce y units,
profit $(\pi) = p \cdot y - (w_1 \cdot x_1 + w_2 \cdot x_2)$

Producer's Problem: $\max_{x_1, x_2, y} p \cdot y - (w_1 \cdot x_1 + w_2 \cdot x_2)$

subject to $f(x_1, x_2) \geq y$

Short-Run vs. Long-Run:

- Some inputs may require time to adjust (size of a factory, f.e.)
Long-Run: All inputs can be varied.
Short-Run: Some inputs cannot be varied.

Short-Run Profit Maximization:
- Suppose in the short-run: $x_2 = \bar{x}_2$

$\max_{x_1, y} p \cdot y - (w_1 \cdot x_1 + w_2 \cdot \bar{x}_2)$

subject to $f(x_1, \bar{x}_2) \geq y$

Short Run Cost Minimization :

- Suppose $x_2 = \bar{x}_2$ (fixed cost)

$$\min_{x_1} \; w_1 x_1 + w_2 \bar{x}_2 \quad \text{s.t.} \quad f(x_1, \bar{x}_2) = \bar{y}$$

} Short-run cost-minimization problem

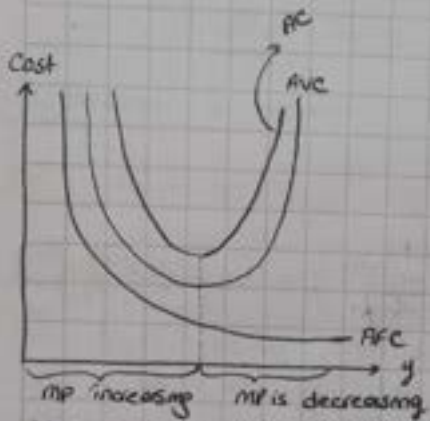
In general, we can write cost function in two components :

★ $C(y) = \underbrace{C_v(y)}_{\text{variable cost}} + \underbrace{F}_{\text{fixed cost (does not depend on } y)}$

★ Average Cost : $AC(y) = \dfrac{C(y)}{y}$

$$= \frac{C_v(y) + F}{y}$$

$$= \underbrace{\frac{C_v(y)}{y}}_{AVC} + \underbrace{\frac{F}{y}}_{AFC}$$



$1 \to C_v(1) \to AVC \to \dfrac{C(1)}{1}$

$2 \to C_v(2) \to AVC \hookrightarrow \dfrac{C(2)}{2}$

$\vdots$

Diminishing Marginal Product :

Labor (L) : $0 \to 0$, $1 \to 10$, $2 \to 25$, $3 \to 45$, $\vdots$, 100

$MP_1 = 10$, $MP_2 = 15$, $MP_3 = 20$

$10 \to w_1 \quad AVC = w_1/10$

$25 \to 2w_1 \quad AVC = 2w_1/25$

At least after some input level, marginal product starts to diminish.

Cobb-Douglas : $f(x_1, x_2) = x_1^{1/2} \cdot x_2^{1/2}$

$$MP_1 = \frac{1}{2} \cdot x_1^{-1/2} \cdot x_2^{1/2}$$

$$\frac{dMP_1}{dx_1} = \frac{-1}{4} \cdot x^{-3/2} \cdot x_2^{1/2} < 0$$

* At a P.E allocation : $x_B = y_B$

$\{((4-x, 2-x), (x, x))$ are P.E $\quad 0 \le x \le 2$

* Is there an "equilibrium price" ?

Defn. A price $p^* = (p_1^*, p_2^*)$ is a (walrasian) Competitive equilibrium price if

$x_A^1(p_1^*, p_2^*, m_A) + x_B^1(p_1^*, p_2^*, m_B) = \omega_A^1 + \omega_B^1$ and

$x_A^2(p_1^*, p_2^*, m_A) + x_B^2(p_1^*, p_2^*, m_B) = \omega_A^2 + \omega_B^2$

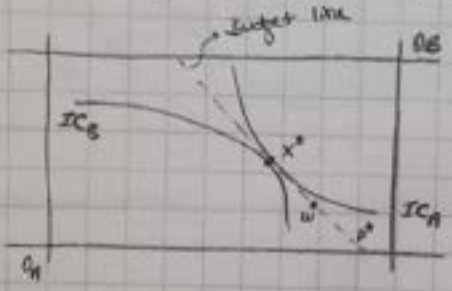
$(x_A^1)^*$ $(x_B^1)^*$ $(x_B^2)^*$

(m_A: income of A)

Demand of consumer A of good 2 $(x_A^2)^*$

$m_A = p_1^* \omega_A^1 + p_2^* \omega_A^2$, $m_B = p_1^* \omega_B^1 + p_2^* \omega_B^2$

$x^* = (x_A^*, x_B^*)$ is an equilibrium allocation.



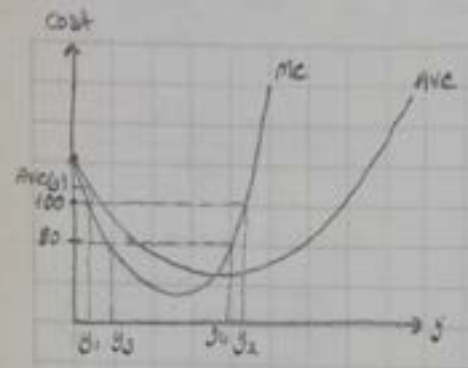
(Eğim = fiyatlar oranıdır) widen param ile tam olarak ω'yı almaya yetecek kadar

$m_A = p_1 \omega_A^1 + p_2 \omega_A^2$ $\quad m_B = p_1 \omega_B^1 + p_2 \omega_B^2$

- x^* equilibrium allocation
- p^* equilibrium price

*) At x^* :

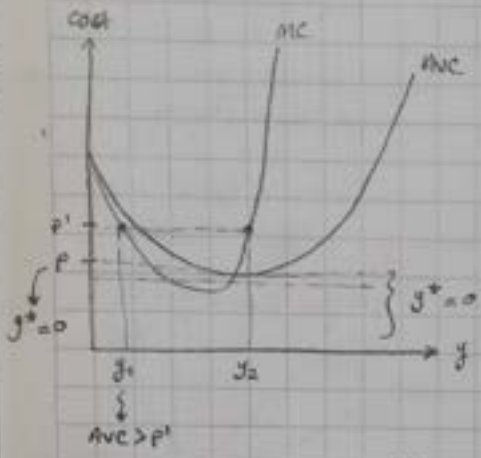
$MRS_A(x^*) = MRS_B(x^*) = \frac{-p_1}{p_2}$



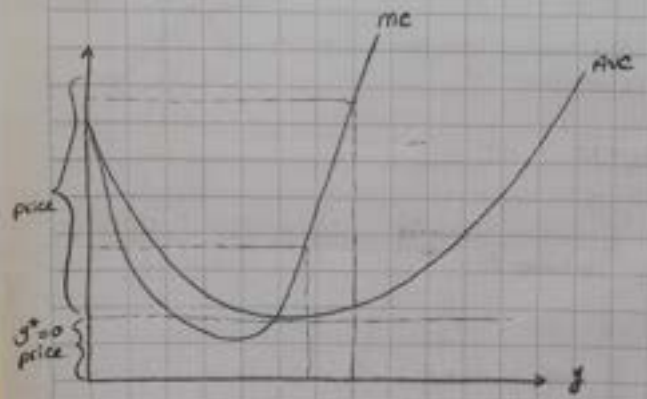
- At y_1, $mc(y_1) = p$ but producing nothing is better. (since $p < AVC(y)$)
- y_1 cannot be optimal.

* Diminishing marginal Product

$$\frac{dF}{dx_i}\left(\frac{dF}{dx_i}\right) \leq 0$$



- for p' : y_2 is optimal.



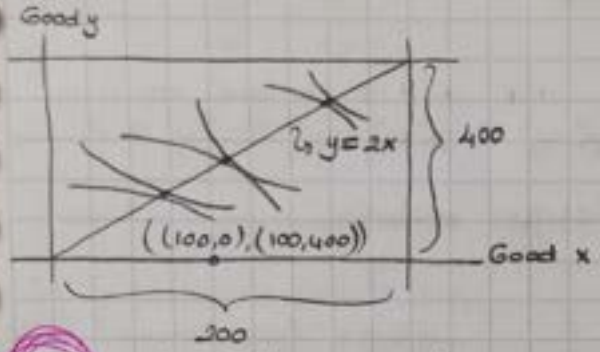
$$\frac{y_A}{x_A} = \frac{y_B}{x_B} = \frac{400 - y_A}{200 - x_A}$$

(1) (2)

$$\Rightarrow \frac{y_A}{x_A} = \frac{400 - y_A}{200 - x_A} \Rightarrow 200 \cdot y_A - x_A y_A = 400 \cdot x_A - x_A y_A$$

$$\Rightarrow \boxed{y_A = 2x_A}$$

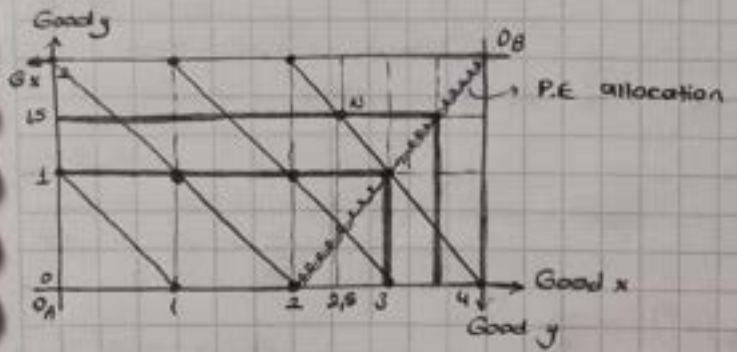
$$\left\{ \left((x, 2x), (200 - x, 400 - 2x) \right) : 0 \leq x \leq 200 \right\} \text{ are P.E}$$

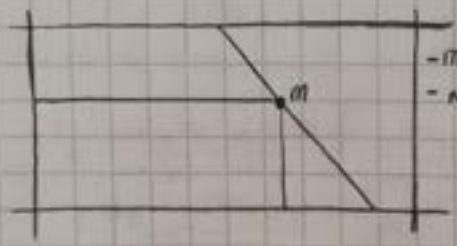


Ex: $U_A(x_A, y_A) = x_A + y_A$

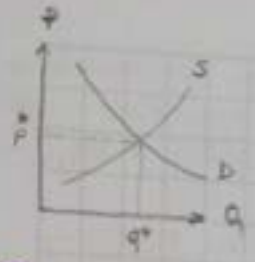
$U_B(x_B, y_B) = \min\{x_B, y_B\}$

$w_A = (4,0)$, $w_B = (0,2)$



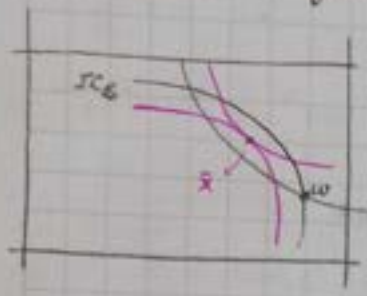


"Partial Equilibrium Analysis"



(C) Competitive Equilibrium :
p^* and x^* constitude a CE if (1) Consumers maximize their utilities by consuming x^*.
(2) x^* is feasible.

(C) Pareto Efficiency :
Q : Is there a "good" allocation?



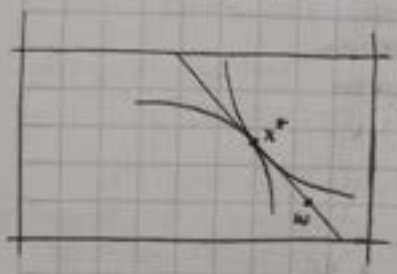
$\bar{x}$: pareto eff. (+,-)
(-,+)

★ Equi. all. ⇔ pareto eff.

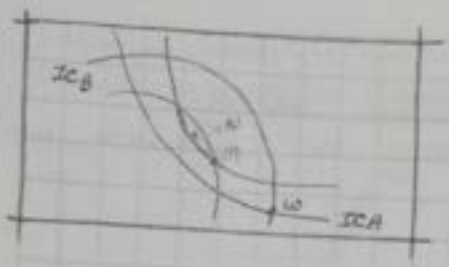
★ First Fundamental Theorem of Welfare Economics
→ Every CE allocation (-Piyasa dengesi-) is Pareto efficient. → When preferences are monotone.

→ Pareto efficiency ⇔ "Social welfare" maximization
$W(x_1, x_2) = \min\{x_1, x_2\}$

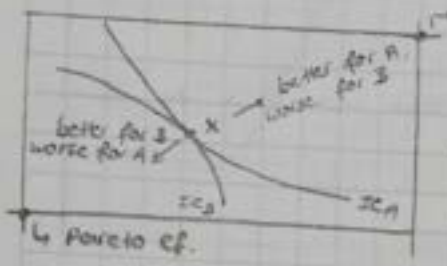
CE : market outcome
PE : Socially good allocation



x^* : CE ⇒ PE



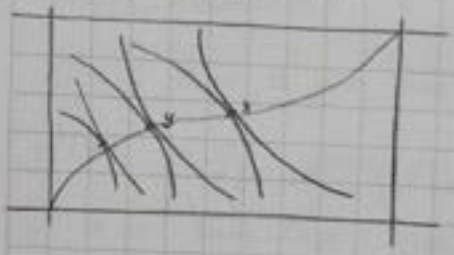
- M is "better" than W.
- N is "better" than M.



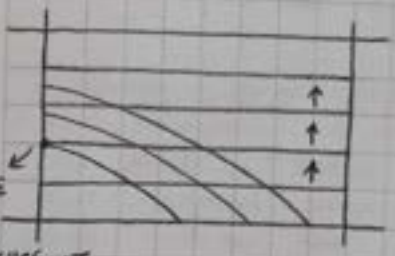
- It is not possible to increase one consumer's well-being without making other consumer worse off. (At X)

→ X is "Pareto Efficient"

Defn: A feasible allocation { X (optimal) } is not Pareto efficient if we can make some consumer better off without making other consumer worse off.



* At an interior P.E allocation : Slope of IC_A = Slope of IC_B → MRS_A = MRS_B



MRS per unit dept.

Ex: 2 goods : x, y

$u_A(x_A, y_A) = x_A y_A$ $\quad w_A = (100, 350)$

$u_B(x_B, y_B) = x_B y_B$ $\quad w_B = (100, 50)$

P.E allocations?

$MRS_A(x_A, y_A) = \frac{-y_A}{x_A}$ ①

$MRS_B(x_B, y_B) = \frac{-y_B}{x_B}$

$\{ ((x_A, y_A), (x_B, y_B))$ is feasible

$x_A + x_B = 200$ and $y_A + y_B = 400$ } ②

* Equilibrium :

x : $x_A + x_B = 100 + 100$

$50 + \frac{175p_y}{p_x} + 50 + \frac{25p_y}{p_x} = 200$

$\frac{200p_y}{p_x} = 100 \rightarrow \frac{p_y}{p_x} = \frac{1}{2}$,

y : $y_A + y_B = 350 + 50$

$\frac{50p_x}{p_y} + 175 + \frac{50p_x}{p_y} + 25 = 400$

$\frac{100p_x}{p_y} = 200$

$\frac{p_x}{p_y} = 2$

(Sadece x üzerinden talebinin arzına eşit olması yeterli (y de aynı oranı veriyor)

for p_x^*, p_y^* such that $\frac{p_x^*}{p_y^*} = 2$, (p_x^*, p_y^*) is an equilibrium price.

$x_A^* = 87.5 + 50 = 137.5$ $\quad y_A^* = 100 + 175 = 275$

$x_B^* = 50 + 12.5 = 62.5$ $\quad y_B^* = 100 + 25 = 125$

equilibrium allocation

Ex

$u_A(x_A, y_A) = x_A + y_A$

$u_B(x_B, y_B) = \min\{x_B, y_B\}$

$w_A(4, 0)$

$w_B(0, 2)$

} equilibrium = ?

* Consumer A's problem :

$m_A = 4p_x$

$x_A(p_x, p_y, m_A)$ {

$(0, \frac{4p_x}{p_y})$ $\quad p_x > p_y$

$(4, 0)$ $\quad p_y > p_x$

$x_A + y_A = 4$ $\quad p_x = p_y$

* Consumer B's problem :

$m_B = 2p_y$

$x_B(p_x, p_y, m_B) = \left(\frac{2p_y}{p_x+p_y}, \frac{2p_y}{p_x+p_y}\right)$

$p_x = p_y$

$p_x \cdot x + p_y \cdot y = w_x \cdot p_x + w_y \cdot p_y$

$(x+y)p_x = (w_x + w_y)p_x$

$x + y = 4$

(Marginal) Technical Rate of Substitution :

- Suppose we are at (x_1, x_2). If we change x_1 by Δx_1, in order to stay on the same isoquant, how much does x_2 change?

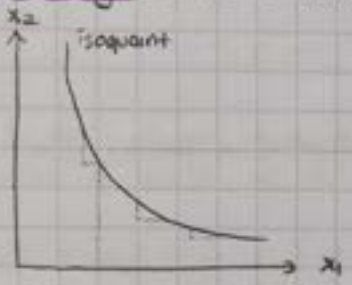
- If x_1 changes by Δx_1, how much does production change?
  x_2 " " Δx_2

$\Delta y = MP_1 \cdot \Delta x_1 + MP_2 \cdot \Delta x_2$

$\Rightarrow \quad 0 = MP_1 \cdot \Delta x_1 + MP_2 \cdot \Delta x_2$

$\Rightarrow \quad \frac{\Delta x_2}{\Delta x_1} = \frac{-MP_1}{MP_2}$ $\quad$ ✱ $TRS(x_1, x_2) = \frac{-MP_1}{MP_2}$

Convexity $\Rightarrow$ TRS is diminishing.



Returns to Scale :

$(x_1, x_2) \to y$

(i) f satisfies Constant returns to Scale if $f(tx_1, tx_2) = t f(x_1, x_2)$ $(t > 0)$

$f(x_1, x_2) = x_1 + x_2$ ✓

(ii) f satisfies Increasing Returns to Scale if $f(tx_1, tx_2) > t f(x_1, x_2)$ for $t > 1$

(iii) f " Decreasing " " if $f(tx_1, tx_2) < t f(x_1, x_2)$ for $t > 1$

Ex: Two goods: x, y

$u_A(x_A, y_A) = x_A \cdot y_A$

$u_B(x_B, y_B) = x_B \cdot y_B$

$w_A = (100, 350)$

$w_B = (100, 50)$

} equilibrium allocation = ?

for p_x and p_y:

a) Consumer A's problem:

$$\max_{x_A, y_A} \; x_A \cdot y_A$$

s.to $p_x \cdot x_A + p_y \cdot y_A = 100 \cdot p_x + 350 \cdot p_y$

$$MRS_A = \frac{y_A}{x_A} = \frac{p_x}{p_y}$$

$$y_A = \frac{p_x \cdot x_A}{p_y}$$

$$p_x \cdot x_A + p_y \cdot \frac{p_x \cdot x_A}{p_y} = 100p_x + 350p_y$$

$$2p_x \cdot x_A = 100p_x + 350p_y$$

$$x_A = 50 + 175 p_y / p_x$$

$$y_A = \frac{p_x}{p_y} \cdot (50 + 175 p_y / p_x)$$

$$y_A = 50 p_x / p_y + 175$$

b) Consumer B's problem:

$$\max_{x_B, y_B} \; x_B \cdot y_B$$

s.to $p_x \cdot x_B + p_y \cdot y_B = 100p_x + 50p_y$

$$MRS_B = \frac{y_B}{x_B} = \frac{p_x}{p_y}$$

$$y_B = x_B \cdot \frac{p_x}{p_y}$$

$$p_x \cdot x_B + p_y \cdot x_B \cdot \frac{p_x}{p_y} = 100p_x + 50p_y$$

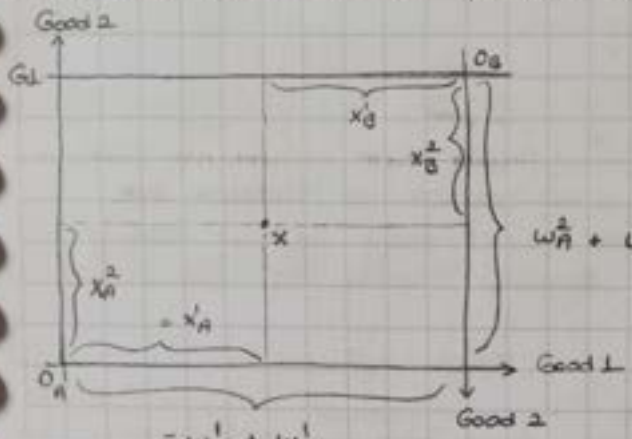
$$2p_x x_B = 100p_x + 50p_y$$

$$x_B = 50 + 25 p_y / p_x$$

$$y_B = 50 p_x / p_y + 25$$

Q: Is there a "good" or "equilibrium" feasible allocation?

- We can represent all feasible allocations via "Edgeworth box":



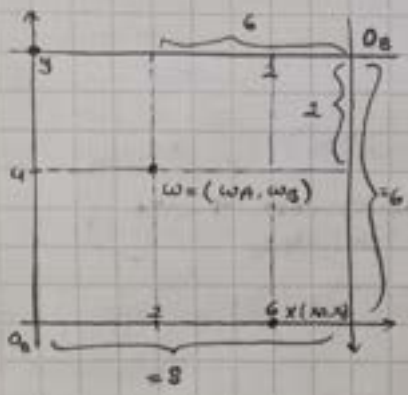
$\omega_A^2 + \omega_B^2$ (Total good 2 in the econ)

$= \omega_A^1 + \omega_B^1$

(Total good 1 in the econ.)

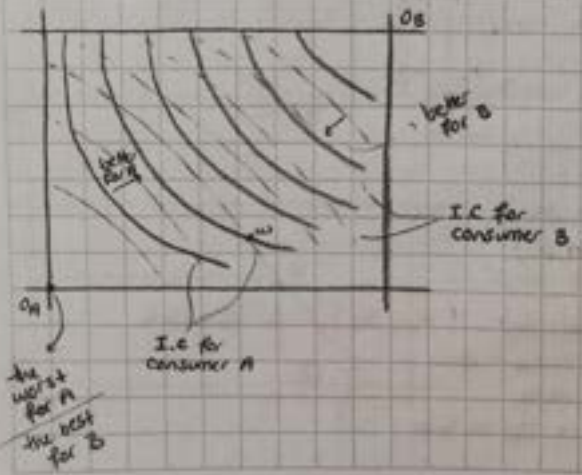
Ex: $\omega_A = (2,4)$

$\omega_B = (6,2)$



$x = (x_A, x_B)$, $x_A = (6,0)$, $x_B = (2,6)$

$y = (y_A, y_B)$, $y_A = (0,6)$, $y_B = (8,0)$



Profit Maximization Problem :

2 inputs, 1 output

$f(x_1, x_2) = y$

Given prices $\underbrace{p}_{\text{output price}}, \underbrace{w_1, w_2}_{\text{input prices}}$

$$\left.\begin{array}{l} \underset{x_1, x_2, y}{\text{MAX}} \; p \cdot y - (w_1 \cdot x_1 + w_2 \cdot x_2) \\ \text{s.to} \quad f(x_1, x_2) \geq y \end{array}\right\} \text{PMP}$$

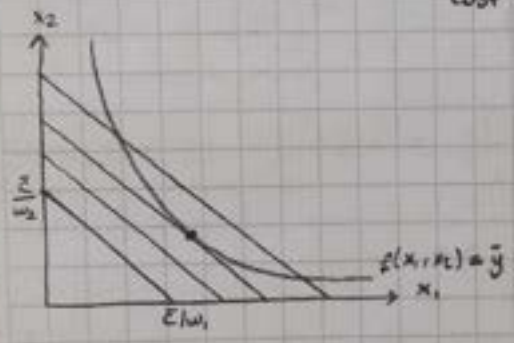
We can think of PMP in 2 steps : (PMP ⇒ cost minimization)

Step 1 : For a given output level $\bar{y}$.

$$\left.\begin{array}{l} \underset{x_1, x_2}{\text{MIN}} \; w_1 x_1 + w_2 x_2 \\ \text{Subject to} \quad f(x_1, x_2) \geq \bar{y} \end{array}\right\} \text{cost minimization problem}$$

If $f()$ is increasing, at optimal point $f(x_1, x_2) = \bar{y}$

We will denote the minimum cost required to produce $\bar{y}$ at prices (w_1, w_2) by $\underbrace{c(w_1, w_2, \bar{y})}_{\text{cost function}}$



- isocost line

$\bar{C} = w_1 x_1 + w_2 x_2$

$\Rightarrow x_2 = \frac{-w_1}{w_2} x_1 + \frac{\bar{C}}{w_2}$

Optimal : slope of isoquant = slope of isocost

$$\frac{-\partial f / \partial x_1}{\partial f / \partial x_2} = -\frac{w_1}{w_2}$$

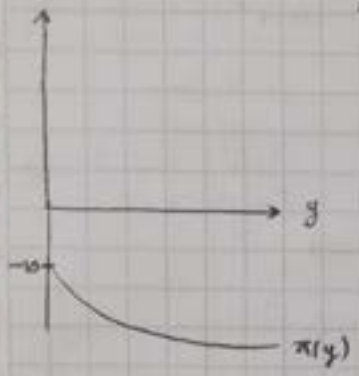
Ex: $c(y) = \underbrace{y^2 + 10y}_{Cv(y)} + \underbrace{10}_{F}$

$$\max_{y \geq 0} \underbrace{p \cdot y - (y^2 + 10y + 10)}_{\equiv \pi(y)}$$

$$\pi'(y) = p - (2y + 10) = (p - 10) - 2y$$

If $p \leq 10$, $\pi'(y) < 0$ for any $y \geq 0$

$\Rightarrow$ maximum of π is achieved at $\boxed{y^* = 0}$



If $p > 10$, $\pi'(y)$

> 0 when $y < \frac{p-10}{2}$

$= 0$ when $y = \frac{p-10}{2}$

< 0 when $y > \frac{p-10}{2}$

$\Rightarrow \boxed{y^* = \frac{p-10}{2}}$ ✓

$AVC(y) = y + 10$

If $p \leq 10$, $y^* = 0$

If $p > 10$, $\underbrace{mc(\hat{y})}_{2\hat{y}+10} = p$

$\Rightarrow y^* = \frac{p-10}{2}$ ✓

* Consumers and producers are "price takers"
- Many consumers, producers
- Producers produce "homogenous" product. (Perfectly competitive market)

MONOPOLY:
- There is a single firm that produces a good. "price maker"

How many goods firm will sell depends on the price of the good. (on the demand curve of consumers)

Demand curve: $y = q^d(p)$
Inverse demand curve: $P(y) = p$
$(q^d(p) = y)$

Monopoly's problem:

$$\max_{y \geq 0} \underbrace{P(y) \cdot y}_{\text{Total revenue}} - \underbrace{c(y)}_{\text{cost}}$$

If $y^* > 0$: $P'(y^*) \cdot y^* + P(y^*) - c'(y^*) = 0$

$$\Rightarrow \frac{dP}{dy} \cdot y + P - MC = 0$$

$$\Rightarrow P\left[1 + \underbrace{\frac{dP}{dy}}_{<0} \cdot \frac{y}{P}\right] = MC$$

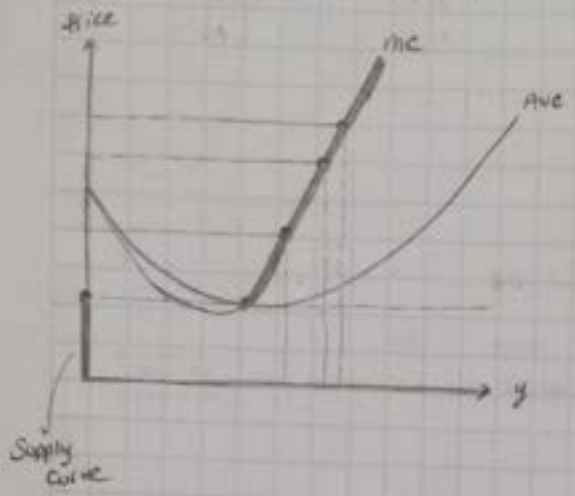
(elasticity)

Recall: In a perfectly competitive market:

$\boxed{P = MC}$

$1 + \frac{dP}{dy} \cdot \frac{y}{P} < 1 \rightarrow \boxed{P > MC}$ (for a monopoly) **

Consumer choices → demand
Producer choices → "Supply" (or 2)



$p = mc(y^*)$
Supply curve ≡ Marginal cost curve (Above AVC)

Market Equilibrium :

$q^d(\vec{p})$ ⇒ At a given price p, quantity demanded
=
$q^s(\vec{p})$ ⇒ At a given price p, quantity supplied

p^* is an equilibrium price

"PARTIAL EQUILIBRIUM" (All parameters (except the own price of the good) are fixed.)

"GENERAL EQUILIBRIUM"
- Two consumers : A and B
- Two goods : 1 and 2
- "Pure exchange economy" : No production
- Each consumer has "initial endowment"

$w_A = (w_A^1, w_A^2)$, $w_B = (w_B^1, w_B^2)$

w_A^1 ↓ Initial endowment of A of good 1

Defn : An allocation $x = (x_A, x_B)$ is feasible if
$x_A = (x_A^1, x_A^2)$, $x_B = (x_B^1, x_B^2)$
Total good 1 in the econ.

$x_A^1 + x_B^1 = w_A^1 + w_B^1$

and

$x_A^2 + x_B^2 = w_A^2 + w_B^2$
Total good 2

$\{(x_1, x_2, y)\} : f(x_1, x_2) \geq y$

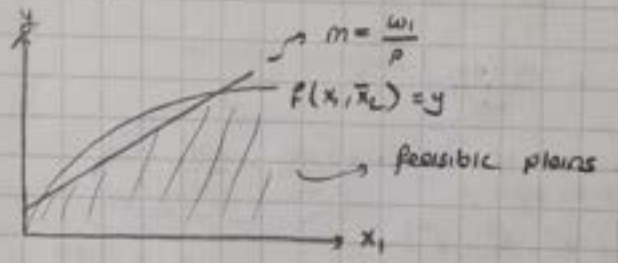
given prices p, w_1, w_2

MAX $p \cdot y - (w_1 \cdot x_1 + w_2 \cdot x_2)$
Subject to $f(x_1, x_2) \geq y$

Short-run vs long-run → all input levels can be varied
↳ some input levels are fixed

Suppose $x_2 = \bar{x}_2$ is fixed

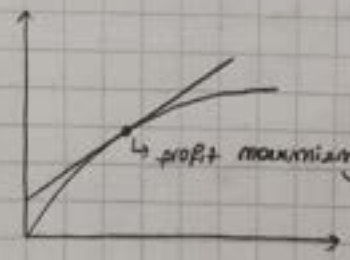
$$\max_{x_1, y} \left\{ \begin{array}{l} p \cdot y - (w_1 \cdot x_1 + w_2 \cdot \bar{x}_2) \\ \text{Subject to } f(x_1, \bar{x}_2) \geq y \end{array} \right\}$$



Isoprofit curve: for a given profit level $\bar{\pi}$, find (x, y) which yields a profit of $\bar{\pi}$

$$\bar{\pi} = p \cdot y - w_1 \cdot x_1 - w_2 \bar{x}_2$$

$$\Rightarrow y = \frac{w_1}{p} \cdot x_1 + \left(\frac{\bar{\pi}}{p} + \frac{w_2}{p} \cdot \bar{x}_2 \right)$$



slope of isoprofit line = slope of production func.

$$\underbrace{\frac{w_1}{p}}_{\text{slope of isoprofit line}} = \underbrace{\frac{df}{dx_1}}_{\text{slope of production func.}} \quad \left\{ w_1 = p \cdot \frac{df}{dx_1} \right.$$

w_1: price of input 1 (marginal cost); $p \cdot \frac{df}{dx_1}$: marginal revenue of input 1

$$\max_{x, y} \; p \cdot y - (w_1 \cdot x_1 + w_2 \cdot \bar{x}_2)$$

s.t. $f(x_1, \bar{x}_2) \geq y$

At optimal point : $f(x_1^*, \bar{x}_2) = y^*$

① $\Rightarrow \omega_1 = \lambda \cdot \frac{1}{2} \cdot x_1^{-1/2} \cdot x_2^{1/2}$

② $\Rightarrow \omega_2 = \lambda \cdot \frac{1}{2} \cdot x_1^{1/2} \cdot x_2^{-1/2}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{x_1^{-1/2} \cdot x_2^{1/2}}{x_1^{1/2} \cdot x_2^{-1/2}}$

$\frac{\omega_1}{\omega_2} = \frac{x_2}{x_1} \longrightarrow \boxed{x_2 = \frac{\omega_1}{\omega_2} \cdot x_1}$

By ③

$\hookrightarrow x_1^{1/2} \left(\frac{\omega_1}{\omega_2} \cdot x_1 \right)^{1/2} = \bar{y}$

$\hookrightarrow x_1 = \bar{y} \cdot \left(\frac{\omega_2}{\omega_1} \right)^{1/2}$

$\boxed{\omega_1 \uparrow \; x_1 \downarrow}$

$x_2 = \bar{y} \cdot \left(\frac{\omega_1}{\omega_2} \right)^{1/2}$

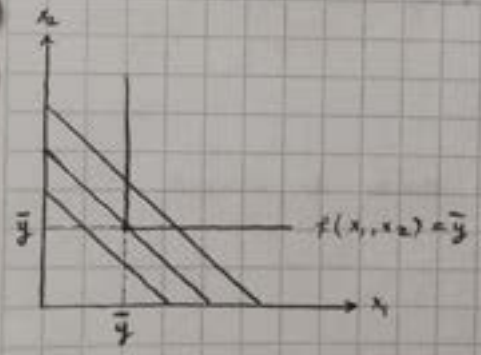
Cost function : $c(x_1, x_2, \bar{y}) = \omega_1 x_1 + \omega_2 x_2$

$= \omega_1 \left(\bar{y} \cdot \left(\frac{\omega_2}{\omega_1} \right)^{1/2} \right) + \omega_2 \left(\bar{y} \cdot \left(\frac{\omega_1}{\omega_2} \right)^{1/2} \right)$

$= \bar{y} \cdot \omega_1^{1/2} \cdot \omega_2^{1/2} + \bar{y} \cdot \omega_1^{1/2} \cdot \omega_2^{1/2}$

$= 2 \bar{y} \cdot \omega_1^{1/2} \cdot \omega_2^{1/2}$ //

Ex : $f(x_1, x_2) = \min \{ x_1, x_2 \}$



Optimal inputs :

$x_1 = x_2 = \bar{y}$

$c(\omega_1, \omega_2, \bar{y}) = \omega_1 \bar{y} + \omega_2 \bar{y}$

$= (\omega_1 + \omega_2) \cdot \bar{y}$

Marginal Cost : If y changes by Δy, how much does cost change per unit ?

$$\frac{C(y+\Delta y) - C(y)}{\Delta y}$$

If $\Delta y \to 0$: $C'(y)$

$$MC(y) = C'(y)$$

$C(y)$ 10 → (x_1, x_2) : (8, 2) → 100 TL
20 → (20, 5) → 250 TL

$C(10) = 100$ TL
$C(20) = 250$ TL

Ex : $C(y) = a + by + cy^2$, $a, b, c \geq 0$

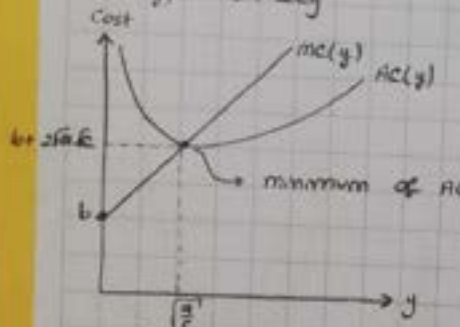
$AC(y) = \frac{C(y)}{y} = \frac{a}{y} + b + cy$

$AC\left(\sqrt{\frac{a}{c}}\right) = \frac{a}{\sqrt{\frac{a}{c}}} + b + c \cdot \sqrt{\frac{a}{c}}$

$= b + 2\sqrt{a} \cdot \sqrt{c}$

$MC(y) = b + 2cy$

$MC\left(\sqrt{\frac{a}{c}}\right) = b + 2c \cdot \sqrt{\frac{a}{c}} = b + 2\sqrt{a} \cdot \sqrt{c}$



$[AC(y)]' = \frac{-a}{y^2} + c$

< 0 $y < \sqrt{\frac{a}{c}}$
0 $y = \sqrt{\frac{a}{c}}$
> 0 $y > \sqrt{\frac{a}{c}}$

Observations : In general,
(1) If MC < AVC, AVC is decreasing.
(2) If MC > AVC, AVC is increasing.
(3) For the first unit of production, AVC = MC

y	cost
0	F
1	F+10

son yapılan üretim maliyeti

$MC = F + 10 - F = 10$
$AVC = \frac{10}{1} = 10$

1. ünit. üretim maliyeti

If $\Delta p_1 \to 0$:

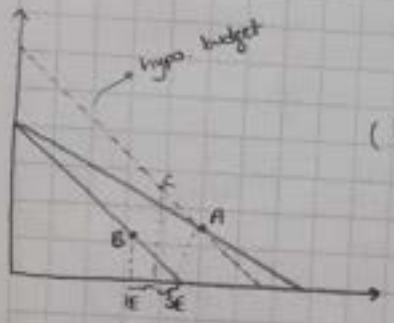
$$\frac{dx_1}{dp_1} = \frac{dx_1^S}{dp_1} - \frac{dx_1}{dm} \cdot x_1$$

x_1^S : Compensated demand function

$$\rightarrow \frac{dx_1^S}{dp_1} = \frac{dx_1}{dp_1} + \frac{dx_1}{dm} \cdot x_1$$

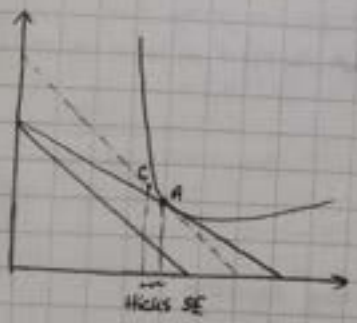
- If p_1 changes : $p_1 \rightarrow p_1'$

(i) PPY : Income effect
(ii) Substitution effect



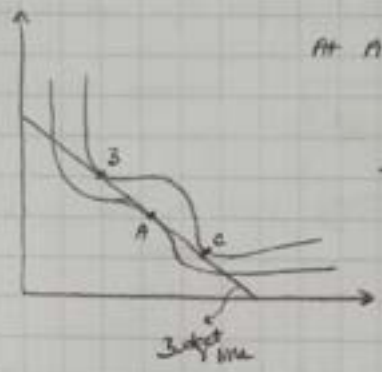
(Slutsky SE)

★ Another Substitution Effect :
(Hicks Substitution Effect)



→ compensate consumer so that he can afford some bundle that is indifferent to A of new points.
- for small price changes, Hicks SE = Slutsky SE

Ex:

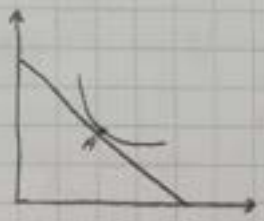


At A : $|MRS| = \frac{P_1}{P_2}$ but not optimal

- In both examples, preferences are not convex.

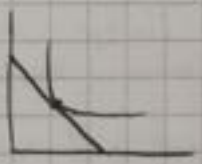
* If preferences are convex, if $|MRS(A)| = \frac{P_1}{P_2}$, A is optimal.

⇒ Slope of IC(A) = slope of budget line



How to find optimal bundle(s)?

- If there are "flat parts" in IC, there can be multiple optimal bundles even when preferences are convex.



- If preferences are "strictly convex" (convex and no "flat parts"), then there is a unique solution

→ ① If preferences are monotone : $p_1 x_1^* + p_2 x_2^* = m$

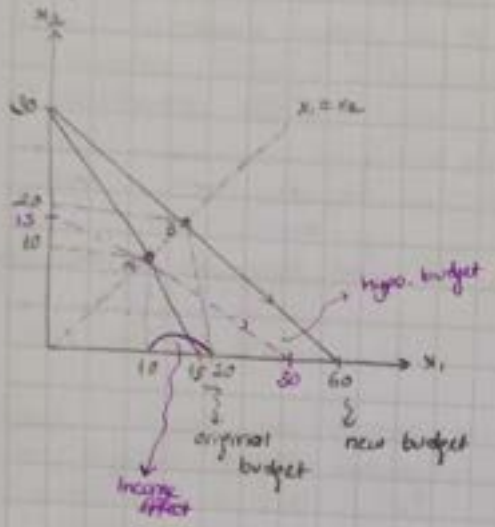
② If preferences are "smooth" and convex use $|MRS| = \frac{P_1}{P_2}$ and the solution is interior

Ex: $u(x_1, x_2) = \min\{x_1, x_2\}$

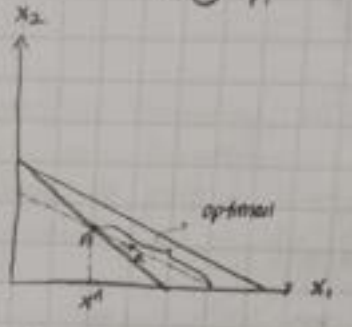
$P_1 = 4$, $P_2 = 1$, $m = 60$

$\downarrow$

$P_1' = 1$

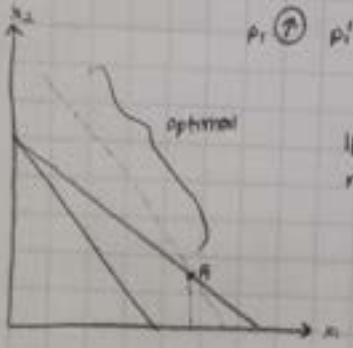


$P_1 \downarrow P_1'$



If P_1 decreases, demand change due to SE must be positive

$P_1 \uparrow P_1'$



If P_1 increases, demand for good 1 (due to SE) must be negative

$(P_1, P_2, m) \rightarrow (P_1', P_2, m)$

Let $\underbrace{\Delta x_1}_{\text{change in quantity demanded of good 1}} = x_1(P_1', P_2, m) - x_1(P_1, P_2, m)$

$m = P_1 x_1(P_1, P_2, m) + P_2 x_2(P_1, P_2, m)$

Let $m' = P_1' \cdot x_1(P_1, P_2, m) + P_2 \cdot x_2(P_1, P_2, m)$

$\Delta x_1 = x_1(P_1', P_2, m) - x_1(P_1, P_2, m) + x_1(P_1', P_2, m') - x_1(P_1', P_2, m')$

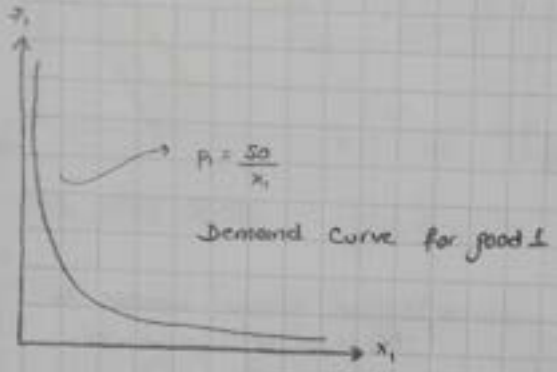
$= \underbrace{\left[x_1(P_1', P_2, m') - x_1(P_1, P_2, m)\right]}_{\text{Substitution Effect} = \Delta x_1^S} + \underbrace{\left[x_1(P_1', P_2, m) - x_1(P_1', P_2, m')\right]}_{\text{Income Effect} = \Delta x_1^I}$

Ex : $U(x_1, x_2) = x_1 \cdot x_2$

$x_1(p_1, p_2, m) = \left(\frac{m}{2p_1}, \frac{m}{2p_2} \right)$

$\bar{p}_2 = 4$, $\bar{m} = 100$

Demand curve for good 1



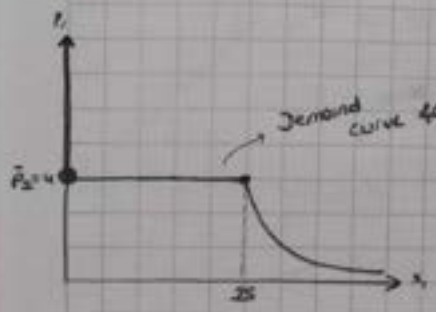
$x_1 = \frac{100}{2p_1} = \frac{50}{p_1}$

$\Rightarrow \boxed{p_1 = \frac{50}{x_1}}$

Ex : $U(x_1, x_2) = x_1 + x_2$

$\bar{p}_2 = 4$, $\bar{m} = 100$

- If $p_1 > \bar{p}_2 = 4$, $x_1 = 0$
- If $p_1 < \bar{p}_2 = 4$, $x_1 = \frac{m}{p_1} = \frac{100}{p_1}$
- If $p_1 = \bar{p}_2 = 4$, $x_1 \in [0, 25]$



Ex: $u(x_1, x_2) = x_1 + x_2$

$p_1 = 4, \quad p_2 = 2, \quad m = 20$

$4x_1 + 2x_2 = 20 \Rightarrow \boxed{x_2 = 10 - 2x_1}$

max

$0 \le x_1 \le 5 \qquad x_1 + (10 - 2x_1)$

max $\quad 10 - x_1$

$0 \le x_1 \le 5$

$\Rightarrow x_1^* = 0, \quad x_2^* = 10$

- we know if $x_1^*, x_2^* > 0, \quad \frac{mu_1}{mu_2} = \frac{p_1}{p_2}$.

$\max u\left(x_1, \frac{m}{p_2} - \frac{p_1}{p_2} x_1\right)$

$0 \le x_1 \le m/p_1$

If $0 < x_1^* < m/p_1 \Rightarrow \boxed{f'(x_1^*) = 0}$

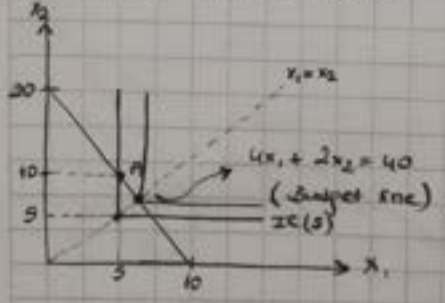
$\frac{du}{dx_1} \cdot 1 + \frac{du}{dx_2} \cdot \left(-\frac{p_1}{p_2}\right) = 0$

$\Rightarrow \frac{du}{dx_1} = \frac{du}{dx_2} \cdot \frac{p_1}{p_2}$

$$\frac{\frac{du}{dx_1}}{\frac{du}{dx_2}} = \frac{p_1}{p_2} \Rightarrow \frac{mu_1}{mu_2} = \frac{p_1}{p_2}$$

Ex: $u(x_1, x_2) = \min\{x_1, x_2\}$

$p_1 = 4 \quad p_2 = 2 \quad m = 40$



① $x_1 = x_2$

② $4x_1 + 2x_2 = 40$

$\Rightarrow x_1 = x_2 = 20/3$

(8,6)
(6,6)
(6.1, 6.1)

$x_1 \le 5, \quad x_2 \ge 5 \rightarrow u = 5$

$x_2 = 5, \quad x_1 \ge 5 \rightarrow u = 5$

→ Cross-price elasticity

$$\varepsilon_{1,2} = \frac{\Delta x_1 / x_1}{\Delta p_2 / p_2} = \frac{\Delta x_1}{\Delta p_2} \cdot \frac{p_2}{x_1}$$

If substitutes, $\varepsilon_{1,2} > 0$
complements, $\varepsilon_{1,2} < 0$

If I'm interested in marginal change

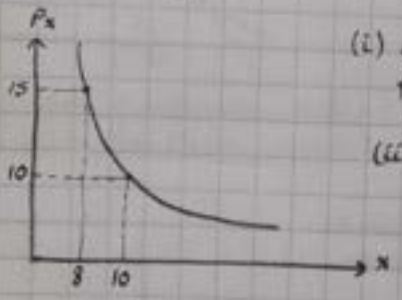
$$\text{Income elasticity} = \frac{dx_1}{dm} \cdot \frac{m}{x_1}$$

$\Delta m \to 0$

(*) Another measure: compensating variation (CV)
How much should the consumer be compensated for to keep him at the same utility level if P_1 increases?

* Alternative measure: Equivalent variation (EV)
How much should the consumer be compensated for before the price change to keep him at the same utility if P_1 decreases?

Q: How much does the "welfare" of a consumer change due to a price change?



(i) Demand curve shows the quantity demand for each price level.
Alternatively:
(ii) It shows the "max-willingness-to-pay" for each possible unit

→ Consumer surplus:

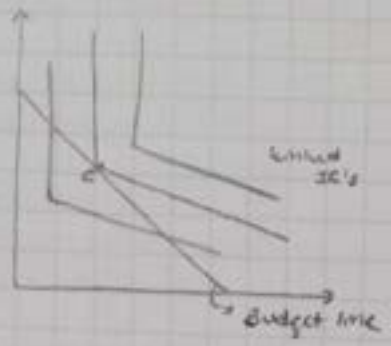
Ex: If; $P_x = 15$, $g_x = 0$
$= 8$ $= 1$
$= 7$ $= 2$
$= 5$ $= 3$
$= 2$ $= 4$

Consumer is willing to pay 8 for the 1st
7 2nd
5 3rd
2 4th

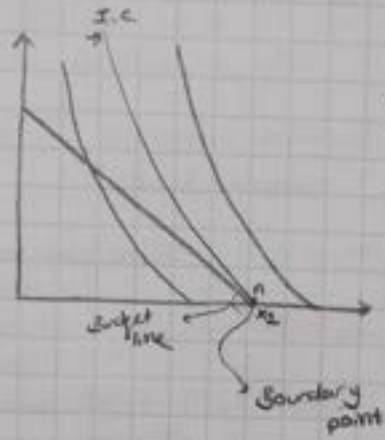
Q: Does it always the case at optimal solution?

$$|MRS| = \frac{P_1}{P_2}$$

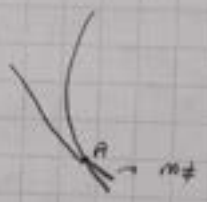
Ex:



Ex:



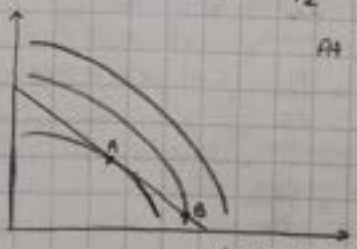
A optimal but $|MRS(A)| \neq \frac{P_1}{P_2}$



Remark: If indifference curves are "smooth" and the solution is "interior" (both goods are consumed in positive amounts), then at optimal point

$$|MRS| = \frac{P_1}{P_2}$$

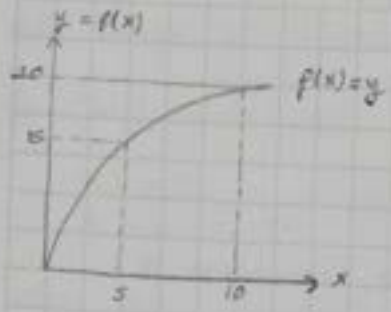
Q: If at A (on the budget line) $|MRS(A)| = \frac{P_1}{P_2}$ then is A necessarily optimal?

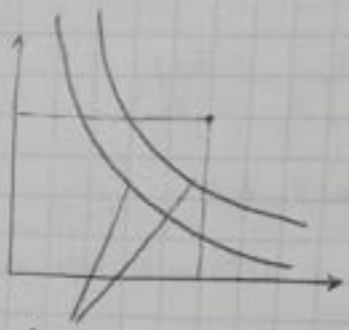


At A, $|MRS| = \frac{P_1}{P_2}$

but A is not optimal

Ex: one input, one output





Isoquant: At a given production level y, combinations of input levels that can achieve at most y units of production.

Definition: Production technology is monotone if more inputs → more outp.

Definition: production technology is convex if "balanced inputs → more outp."

Ex: A student produces "homework"
two inputs: blue pen, black pen
He can produce 1 hw by using 1 pen
$f(x_{blue}, x_{black}) = x_{blue} + x_{black}$

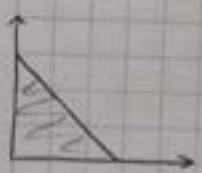
Consumer decisions / choices ⇓ Demand curve

Producer decisions / choices ⇓ Supply curve

Production function:
2 inputs, 1 output

$f(x_1, x_2) = y$
$f(5,3) = 10$ ✓ max
$(5,3)$, 12 (X) feasible
$(5,3)$, 8 (V) feasible

Set of feasible production plans:

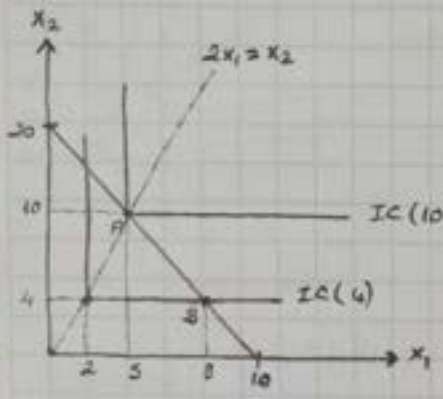


$p_1x_1 + p_2x_2 \leq m$

✓ convex

Ex: $u(x_1, x_2) = \min\{2x_1, x_2\}$

$p_1 = 4$, $p_2 = 2$, $m = 40$



$A(5,10) \rightarrow$ Optimal

$2x_1 = x_2$

$4x_1 + 2x_2 = 40 \Rightarrow 8x_1 = 40$

$x_1 = 5$

$x_2 = 10$

$2x_1 = 10, \quad x_2 \geq 10$

$x_2 = 10, \quad 2x_1 \geq 10$

$\rightarrow$

$x(p_1, p_2, m) = (x_1(p_1, p_2, m), x_2(p_1, p_2, m))$

Demand function — quantity demanded of good 1 when prices are p_1, p_2 income is m

Ex: $u(x_1, x_2) = \min\{x_1, x_2\}$

$\Rightarrow x_1 = x_2$

$\Rightarrow p_1 x_1 + p_2 x_2 = m$

$\Rightarrow x_1 = \frac{m}{p_1 + p_2}$ & $x_2 = \frac{m}{p_1 + p_2}$

$\Rightarrow x(p_1, p_2, m) = \left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}\right)$

$m = p_1 \cdot x_1(p_1, p_2, m) + p_2 \cdot x_2(p_1, p_2, m)$

$$(p_1 - p_1') \cdot \Delta x_1 = \underbrace{(p_1 - p_1') \cdot \Delta x_1^S}_{\geq 0} + \underbrace{(p_1 - p_1') \cdot \Delta x_1^I}_{\geq 0 \text{ if good 1 is normal}}$$

normal? inferior?

≤ 0 if good 1 is inferior

$p_1 > p_1' : m > m'$

$p_1 < p_1' : m < m'$

If good 1 is normal, $\underbrace{(p_1 - p_1')\Delta x_1}_{\text{If } p_1 > p_1',\ \Delta x_1 > 0} \geq 0$ [If p_1 decreases, quantity demanded of good 1 increases ⇒ good 1 is ordinary]

If good 1 is Giffen, it must be inferior

for convenience, $\Delta x_1^m = -\Delta x_1^I$

$$\Delta x_1 = \Delta x_1^S - \Delta x_1^m$$

$$\Delta p_1 = p_1' - p_1$$

$$\underbrace{\frac{\Delta x_1}{\Delta p_1}}_{\text{average rate of change in quantity demanded of good 1 due to price change}} = \frac{\Delta x_1^S}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

$$\Delta m = m' - m$$

$$= [p_1' x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m)] - [p_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m)]$$

$$= (p_1' - p_1) \cdot x_1(p_1, p_2, m) = \Delta p_1 \cdot x_1(p_1, p_2, m)$$

$$\Rightarrow \Delta p_1 = \frac{\Delta m}{x_1(p_1, p_2, m)}$$

$$\Rightarrow \underbrace{\frac{\Delta x_1}{\Delta p_1}}_{\text{average rate of change in quantity demanded of good 1}} = \underbrace{\frac{\Delta x_1^S}{\Delta p_1}}_{\text{average rate of change in quantity demanded due to SE}} - \underbrace{\frac{\Delta x_1^m}{\Delta m} \cdot x_1(p_1, p_2, m)}_{\text{average rate of change in quantity demanded due to increase in wealth / income}}$$

SE

Income effect

→ Slutsky equation

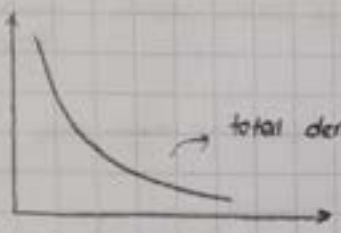
Market Demand

Given $\bar{p}_2, (\bar{m}_1, \bar{m}_2, \ldots, \bar{m}_n)$

$$X_1(p_1, p_2, \bar{m}_1, \ldots, \bar{m}_n) = \sum_{i=1}^{n} x_i(p_1, \bar{p}_2, \bar{m})$$

for some types of preferences total demand is independent of income distribution.

$$x_1(p_1, \bar{p}_2, \underbrace{\bar{m}}_{\text{total income}}) \qquad \bar{m} = \sum_{i=1}^{n} \bar{m}_i$$



PRODUCTION THEORY : A producer faces "technological constrains" there are only certain feasible combinations of input and output levels.

Inputs : labor, land, capital .... (goods and services that are usable in production (factors in product.)

physical capital : factory, computer, machine..)

financial capital

Production function : input levels $\xrightarrow{f}$ output levels

$f(x_1, x_2, \ldots x_n) = y$: I can produce at most y units of output by using x_1 units of input 1

x_2 units of input 2

Production set : set of all "feasible" input output levels

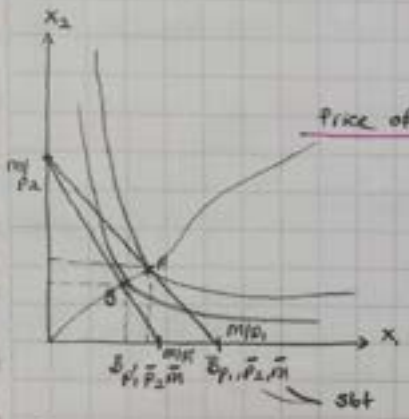
Inferior → toplu taşıma
gösterir → (bütçe bir şeyi sevme sırasıdır
Price normal şöyle inferior)

② Price Changes :

- Suppose $p_1 \uparrow$

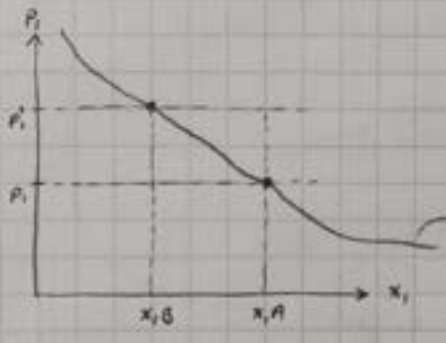
→ ordinary good ($p_1 \uparrow$, $x_1 \downarrow$)

→ giffen good ($p_1 \uparrow$, $x_1 \uparrow$)



Price offer curve (for p_1) (gives optimal bundles for each possible price of good 1 , p_1)

$p_1 \to p_1'$



- Given $\bar{p}_2$, $\bar{m}$

→ Demand curve (for good 1)

$\frac{dx_1}{dp_1} < 0$ → ordinary good

Elasticity: Alternative way to represent the effect of income / price changes on quantity demands

Q: How much percentage does the quantity demanded of good 1 if income / prices changes by 1 percent?

$p_1 \to p_1'$: $\frac{p_1' - p_1}{p_1} \cdot 100$

Income elasticity:

gelir $m \to m'$ $\quad \Delta m' = m' - m$

Percentage change : $\frac{\Delta m}{m} \times 100$

$x_1 \to x_1' \Rightarrow \Delta x_1 = x' - x_1$

Percentage change : $\frac{\Delta x_1}{x_1} \times 100$

Income elasticity = $\frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta m}{m}}$

$= \frac{\Delta x_1}{\Delta m} \cdot \frac{m}{x_1}$

Percentage change x_1 if m changes by 1%.

If good 1 is normal income elasticity < 0
" inferior " " < 0

if $|$income elasticity$| < 1$, demand is inelastic
" $|$ " $| > 1$, demand is elastic

Ex: $u(x_1, x_2) = x_1 \cdot x_2$
$p_1 = p_2 = 1$, $m = 100$, $p_1 = 2$
orginal demand = $(50, 50)$
utility = 2500
new demand = $(25, 50)$
CV = Find m' such that
$\frac{m'}{4} \cdot \frac{m'}{2} = 2500 \Rightarrow m' \approx 141.4$

(Own) price elasticity

$PE = \frac{\%\Delta x_1}{\%\Delta p_1} \cdot \frac{p_1}{x_1}$

< 0 if good 1 ordinary
> 0 if good 1 giffen

MAX $u(x_1, x_2)$

subject to $p_1x_1 + p_2x_2 = m$ $\quad (p_1x_1 + p_2x_2 - m = 0)$

Lagrangian method:

Construct Lagrange function:

$$\mathcal{L}(x_1, x_2, \lambda) = u(x_1, x_2) - \lambda \cdot (p_1x_1 + p_2x_2 - m)$$

(λ: Lagrange multiplier) → (max değer bulmak için türevi = 0 eşitlemek)

$$\frac{\partial \mathcal{L}}{\partial x_1}(x_1, x_2, \lambda) = \frac{\partial u}{\partial x_1}(x_1, x_2) - \lambda p_1 = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial x_2}(x_1, x_2, \lambda) = \frac{\partial u}{\partial x_2}(x_1, x_2) - \lambda \cdot p_2 = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda}(x_1, x_2, \lambda) = -(p_1x_1 + p_2x_2 - m) = 0 \quad (3)$$

(1) $\frac{\partial u}{\partial x_1} = \lambda p_1$

(2) $\frac{\partial u}{\partial x_2} = \lambda p_2$

$$\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{p_1}{p_2} \Rightarrow \frac{mu_1}{mu_2} = \frac{p_1}{p_2}$$

(3) $p_1x_1 + p_2x_2 = m$

$$\frac{mu_1}{p_1} = \frac{mu_2}{p_2} = \lambda$$

λ: Shadow price of wealth (If I increase wealth 1 TL, λ gives the change in my utility due to this change.)

Ex: $u(x_1, x_2) = x_1 \cdot x_2$

$p_1 = 4$, $p_2 = 5$, $m = 40$

$$\mathcal{L}(x_1, x_2, \lambda) = x_1 \cdot x_2 - \lambda(4x_1 + 5x_2 - 40)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = x_2 - 4\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = x_1 - 5\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 4x_1 + 5x_2 = 40$$

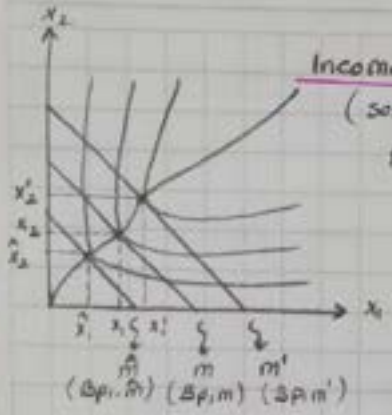
$x_1 = 5\lambda$, $x_2 = 4\lambda$, $4x_1 + 5x_2 = 40$

$\frac{x_2}{x_1} = \frac{4}{5}$

$\Rightarrow x_2 = \frac{4}{5}x_1$

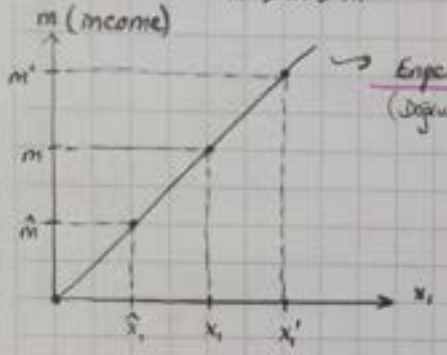
$4x_1 + 5\left(\frac{4}{5}x_1\right) = 40$

$8x_1 = 40$, $x_1 = 5$, $x_2 = 4$



Income offer curve
(some optimal bundles for all possible income levels)

$m' > m > \hat{m}$



Engel curve for good 1
(Bijusal olmak zorunda değil!)

- Good 1 is normal

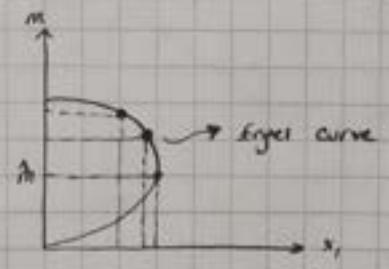
normal good (wealth ↑ demand ↑)

inferior good (wealth ↑ demand ↓)

$\frac{dx_1}{dm} > 0$ → normal

$\frac{dx_1}{dm} < 0$ → inferior

Ex:



Engel curve

- Good 1 is normal when $m < \hat{m}$
  " " " inferior when $m > \hat{m}$

3) Equivalent Variation (EV) :

How much should I compensate (take away from) before the price change get to same utility as B?

Ex : $u(x_1, x_2) = x_1 \cdot x_2$

$P_1 = P_2 = 1$, $m = 100$

$P_1' = 2$

when $P_1 = 1$ $A = (50, 50)$

$u(A) = 2500$

when $P_1' = 2$ $B = (25, 50)$

$u(B) = 1250$

CV : Find m'

$$\frac{m'}{4} \times \frac{m'}{2} = 2500$$

$m' \cong 141.4$

$CV = 141.4 - 100 = 41.4$

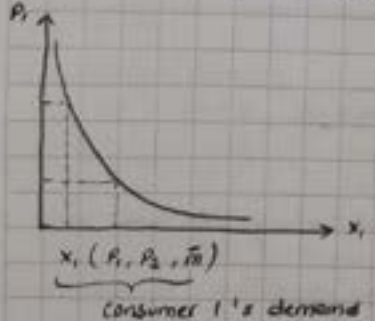
EV : Find m'

$$\frac{m'}{2} \times \frac{m'}{2} = 1250$$

$m' \cong 70.7$

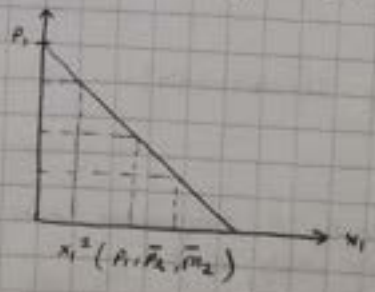
Individual Demand :

Consumer 1 : given $\bar{P}_2$, $\bar{m}_1$:



consumer 1's demand

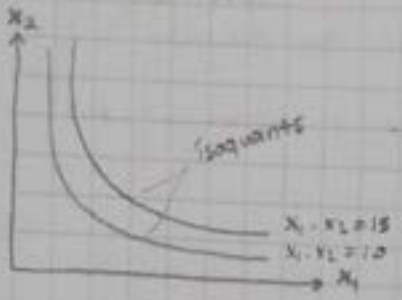
Consumer 2 : given $\bar{P}_2$, $\bar{m}_2$



Ex : Cobb-Douglas Production Technology

$f(x_1, x_2) = A \cdot x_1^a \cdot x_2^b \quad , \quad a, b, A > 0$

$f(x_1, x_2) = x_1 \cdot x_2$



Marginal Product (of an Input) :

- Suppose we operate at (x_1, x_2). If we change the amount of input 1 by Δx_1, how much does production change?

$\underbrace{\Delta y}_{\text{change in production}} = f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)$

Per unit change *

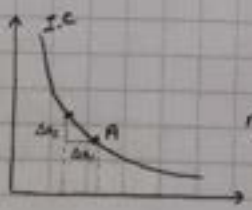
$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

if $\Delta x_1 \to 0$,

$$\frac{\Delta y}{\Delta x_1} \approx \frac{df}{dx_1}(x_1, x_2)$$

$$MP_1(x_1, x_2) = \frac{df}{dx_1}(x_1, x_2)$$

*/



$MRS = \frac{\Delta x_2}{\Delta x_1}$

Income and Substitution Effects :

- Suppose price of good 1 increases ;

Two effects :

(1) Income effect (I'm relatively poor.) → Purchasing power ↓
(2) Substitution effect : Good 1 is relatively more expensive.
∴ I can purchase more of good 2 when I give up 1 unit of good 1. → $\frac{p_1}{p_2}$ ↑

Q : Can we separate these two effects ?

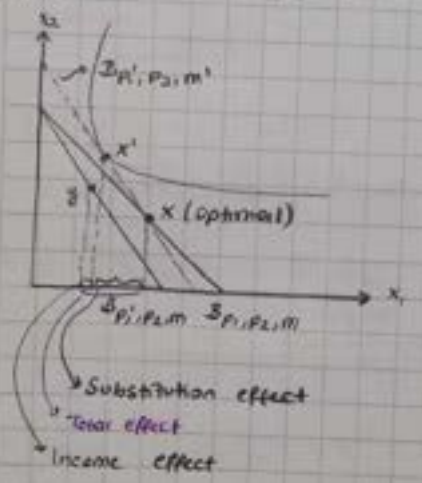
→ Yes , in two steps :

Step 1 : Adjust the income so that the consumer can Just afford the original optimal bundle.

$(p_1, p_2, m) \rightarrow (p_1', p_2, m)$

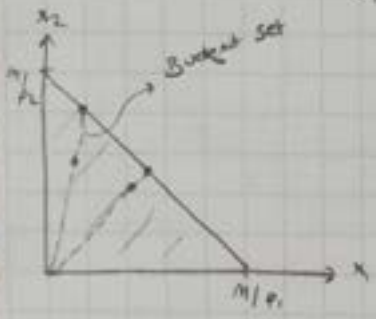
optimal bundle ⇒ $x = (x_1, x_2)$ $\quad m' = p_1' x_1 + p_2 x_2$

⇒ we can obtain substitution effect



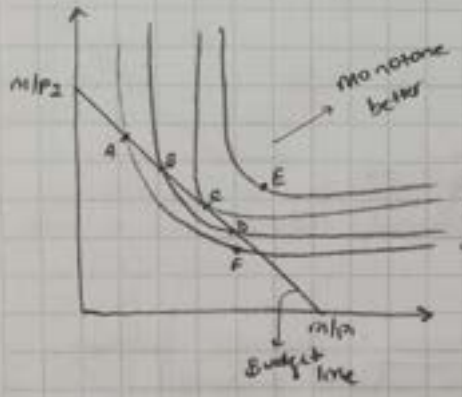
Step 2 : Compare the hypothetical budget and "new budget" which will identify "income effect".

$\max_{x_1, x_2} u(x_1, x_2)$ subject to $p_1 x_1 + p_2 x_2 \leq m$, $x_1, x_2 \geq 0$ } consumer's problem



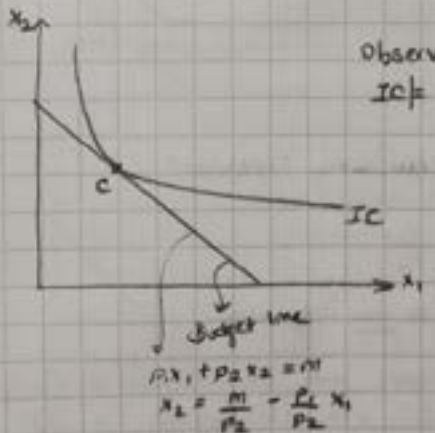
① If preferences are monotone, optimal bundle(s) must be on the budget line: $p_1 x_1^* + p_2 x_2^* = m$

②



C – optimal bundle

(IC 곡선: 같은 만족!)



First-order condition

Observation: At optimal point C, |slope of IC| = |slope of budget line|

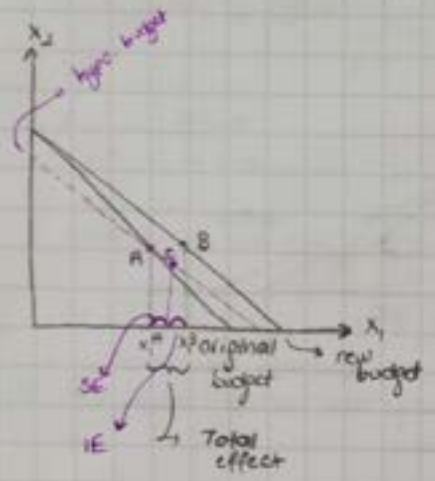
$$|MRS(C)| = \frac{p_1}{p_2}$$

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

$$\Rightarrow \frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

↳ optimal 조건

- Suppose $p_1 \downarrow$ p_1' :



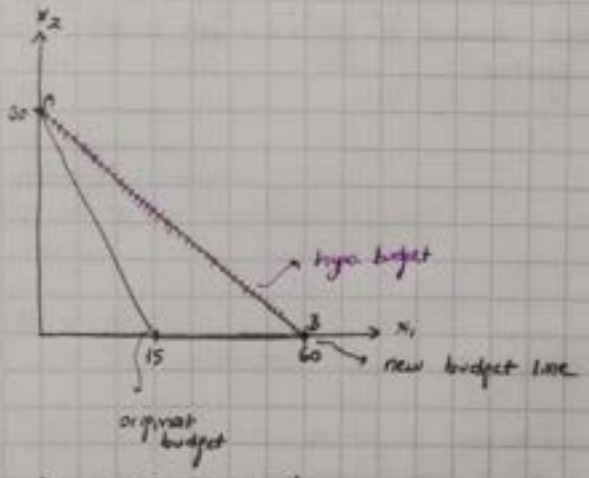
- Consider "hypothetical income" such that PP is equal to PP in orginal bundle.

Ex: $u(x_1, x_2) = x_1 + x_2$

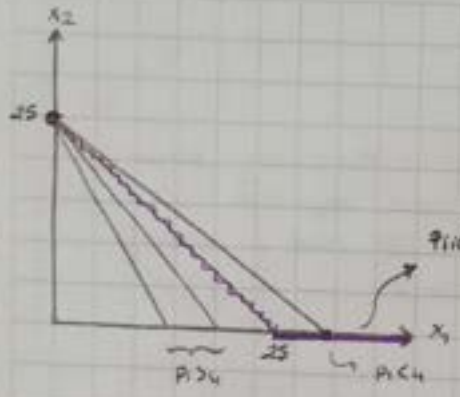
$p_1 = 4$, $p_2 = 2$, $m = 60$

$\downarrow$

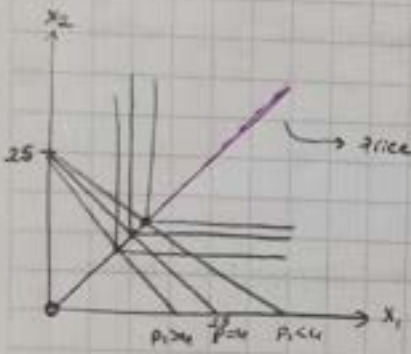
$p_1' = 1$



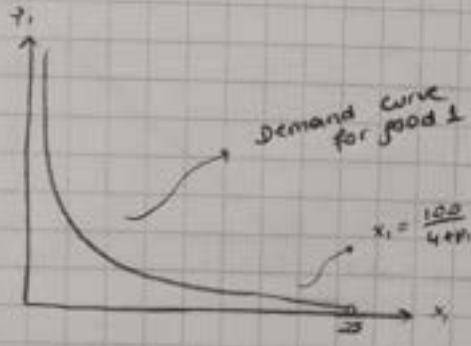
Total Effect = Substitution Effect



Ex: $u(x_1, x_2) = \min\{x_1, x_2\}$



$\bar{p}_2 = 4, \quad \bar{m} = 100$



$$x(p_1, p_2, m) = \left(\frac{m}{p_1+p_2}, \frac{m}{p_1+p_2} \right)$$

$$x_1 = \frac{100}{4+p_1}$$

Ex: $u(x_1, x_2) = x_1 \cdot x_2$

$MU_1(x_1, x_2) = \frac{\partial u}{\partial x_1} = x_2$

$MU_2(x_1, x_2) = \frac{\partial u}{\partial x_2} = x_1$

$\frac{x_2^*}{x_1^*} = \frac{p_1}{p_2}$

$\Rightarrow \boxed{x_2^* = \frac{p_1}{p_2} x_1^*}$

$p_1 x_1^* + p_2 \cdot \left(\frac{p_1}{p_2} \cdot x_1^*\right) = m$

$\Rightarrow x_1^* = \frac{m}{2p_1}, \quad x_2^* = \frac{m}{2p_2}$

- what does $|MRS| = \frac{p_1}{p_2}$ mean economically?

→ Suppose $|MRS(A)| = \frac{\Delta x_2}{\Delta x_1} = \frac{1}{2}$, $\frac{p_1}{p_2} = 1$

we claim A is not optimal

→ Suppose I decrease consumption of good 1 by 1 unit
Increase " " 2 by 1 unit

↳ I am indifferent between A and $A + (-1, \frac{1}{2})$

↳ $A + (-1, 1) > A$

affordable since $\frac{p_1}{p_2} = 1$

Alternative interpretation:

$|MRS| = \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$

$\Rightarrow \underbrace{\frac{MU_1}{p_1}}_{\text{Marginal utility per \$ spent on good 1}} = \underbrace{\frac{MU_2}{p_2}}_{\text{Marginal utility per \$ spent on good 2}}$

- Suppose at A: $\underbrace{\frac{MU_1}{p_1}}_{10} > \underbrace{\frac{MU_2}{p_2}}_{5}$

- I can spend 1\$ less on good 2 and 1\$ more on good 1, I will become better off.

Ex : $u(x_1, x_2) = x_1 \cdot x_2$

$$\frac{mu_1}{mu_2} = \frac{p_1}{p_2} \Rightarrow \frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$\Rightarrow x_2 = x_1 \cdot \frac{p_1}{p_2}$$

$$p_1 x_1 + p_2 x_2 = m$$

$$\Rightarrow p_1 x_1 + p_2 \left(x_1 \cdot \frac{p_1}{p_2} \right) = m$$

$$\Rightarrow x_1 = \frac{m}{2p_1} \quad , \quad x_2 = \frac{m}{2p_2}$$

Ex : $u(x_1, x_2) = x_1 + x_2$

$$x(p_1, p_2, m) = \begin{cases} \left(0, \frac{m}{p_2}\right) & \text{if } p_1 > p_2 \\ \left(\frac{m}{p_1}, 0\right) & \text{if } p_1 < p_2 \\ \{(x_1, x_2) : p_1 x_1 + p_2 x_2 \leq m, \; x_1, x_2 \geq 0\} & \text{if } p_1 = p_2 \end{cases}$$

① Wealth Changes :

How does a consumer's demand change with wealth?

If quantity demanded of a good increases as wealth increases :
normal good ★

If " " " " decreases as wealth increases :
" inferior good " ★

Ex : Low quality products
" Walmart shopping "

(It is possible that a good is normal until some income level, afterwards inferior.)

Ex: $u(x_1, x_2) = x_1 + x_2$

$p_1 = 2$, $p_2 = 4$, $m = 20$

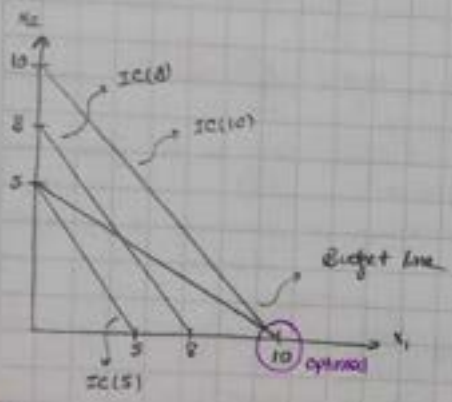
$\mathcal{L}(x_1, x_2, \lambda) = (x_1 + x_2) - \lambda(2x_1 + 4x_2 - 20)$

① $\frac{\partial \mathcal{L}}{\partial x_1} = -2\lambda + 1 = 0 \longrightarrow \lambda = 1/2$

② $\frac{\partial \mathcal{L}}{\partial x_2} = 1 - 4\lambda = 0 \longrightarrow \lambda = 1/4$

③ $\frac{\partial \mathcal{L}}{\partial \lambda} = 2x_1 + 4x_2 = 20$

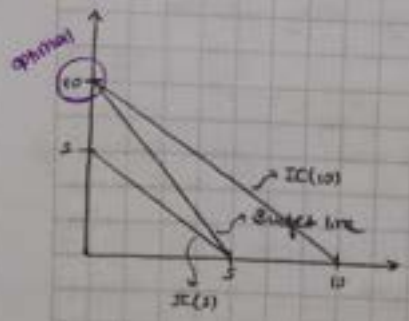
$\Rightarrow$ NO SOLUTION TO ①, ② & ③



We can apply Lagrange method only when solution is not a boundary point.

Ex: $u(x_1, x_2) = x_1 + x_2$

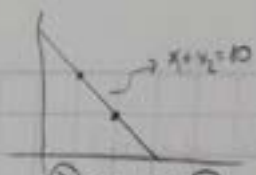
$p_1 = 4$, $p_2 = 2$, $m = 20$

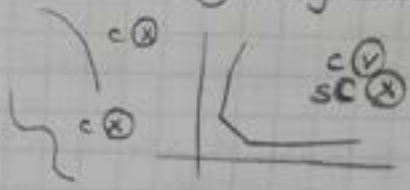


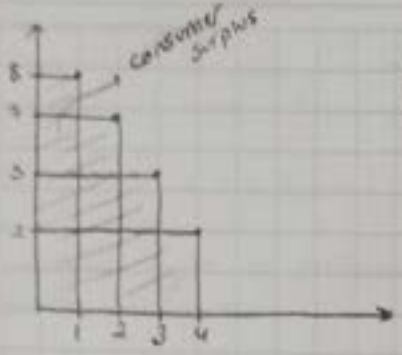
$4x_1 + 2x_2 = 20$

$2x_2 = 20 - 4x_1$

$\Rightarrow \boxed{x_2 = 10 - 2x_1}$

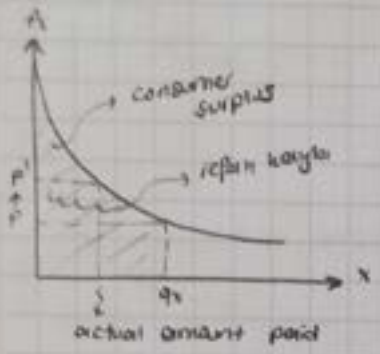


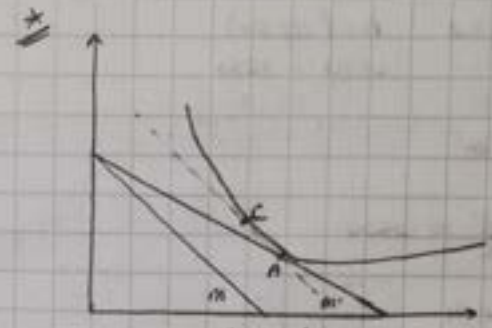




what is the difference between willingness to pay for 4 units and " actual amount paid for 4 units ?

$2 \times 4 = 8$ "

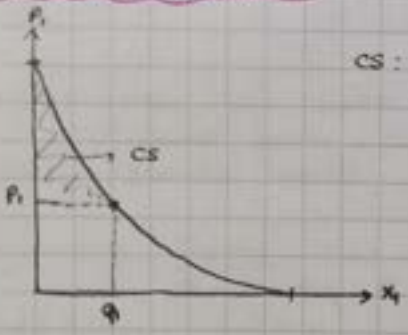




$CV = m' - m$

(after price change)

1) Consumer Surplus :



CS : Maximum willingness to pay - "actual amount to pay"

2) Compensating Variation (CV) :

$P_1 \nearrow P_1'$

A B

How much should I compensate after the price change so that she gets the same utility as A ?

$$\left\{ (x_1, x_2, y) : f(x_1, x_2) \geq y \right\}$$

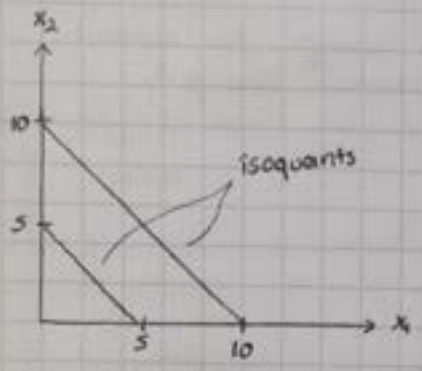
inputs ↳ output

Q: what is the "best" production plan for the producer among feasible production plans?

Isoquant : For a given output level $\bar{y}$,
$f(x_1, x_2) = \bar{y}$

Ex : (Linear Production Technology)

$f(x_1, x_2) = x_1 + x_2$

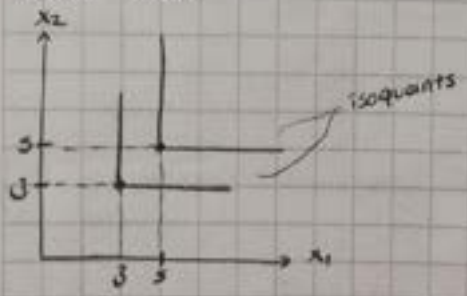


Ex : (Fixed Proportions Technology)

- we will produce "golf holes", I need "1 worker" & "1 shovel" to produce "1 golf hole".

$f(x_1, x_2) = \min\{x_1, x_2\}$

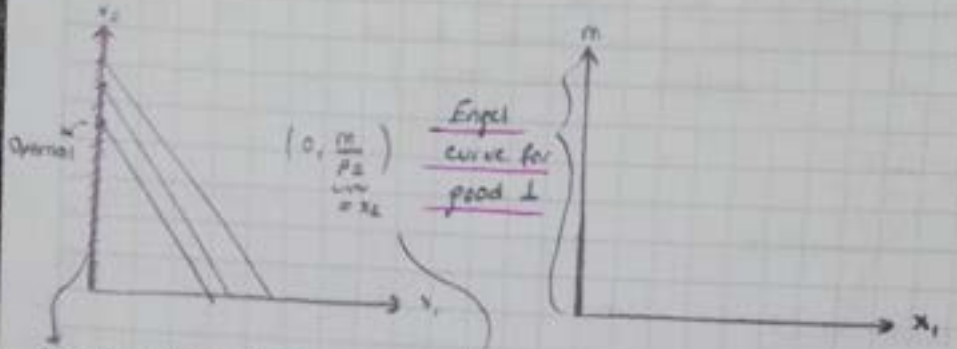
worker ↳ shovel



Ex : Perfect substitutes

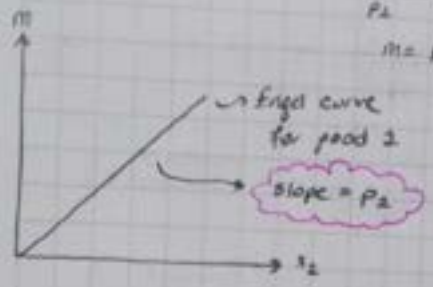
$u(x_1, x_2) = x_1 + x_2$

$p_1 > p_2$

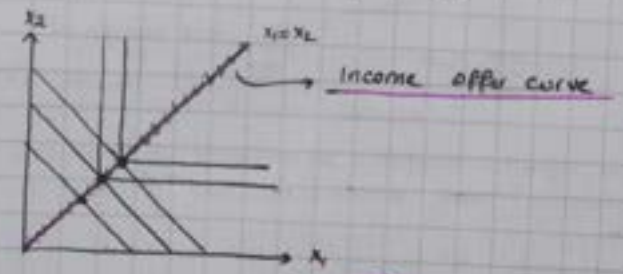


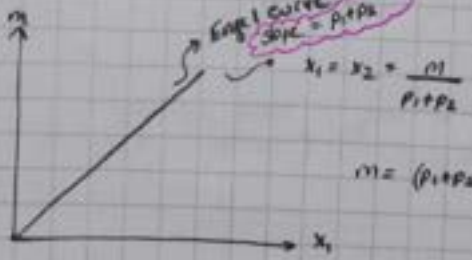
$\frac{m}{p_2} = x_2$

$m = p_2 x_2$



Ex : $u(x_1, x_2) = \min\{x_1, x_2\}$



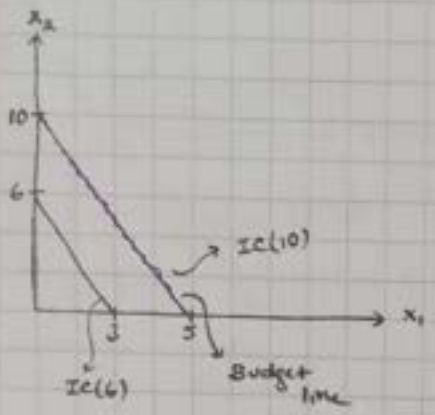


$m = (p_1+p_2) \cdot x_1$

Ex : $u(x_1, x_2) = 2x_1 + x_2$

$p_1 = 4$, $p_2 = 2$, $m = 20$

$4x_1 + 2x_2 = 20$



$2x_1 + x_2 = 6$

$2x_1 + x_2 = 10$

→ All the points on the budget line are optimal

A general method :

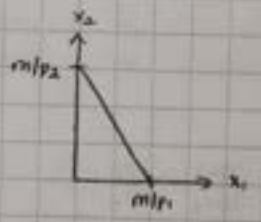
$\max \; u(x_1, x_2)$

subject to $p_1x_1 + p_2x_2 = m$

$p_2x_2 = m - p_1x_1$

$x_2 = \frac{m}{p_2} - \frac{p_1x_1}{p_2}$

$\max \; u\left(x_1, \frac{m}{p_2} - \frac{p_1}{p_2}x_1\right)$

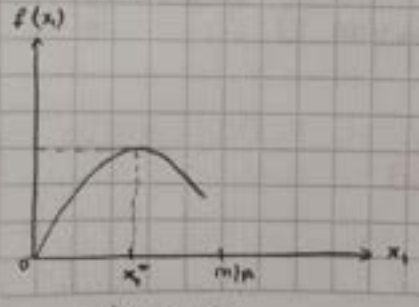


$0 \le x_1 \le m/p_1$

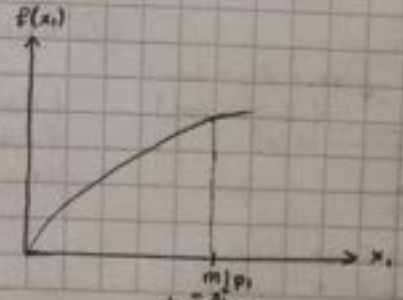
$f(x_1) = u\left(x_1, \frac{m}{p_2} - \frac{p_1}{p_2}x_1\right)$

$\max f(x_1)$

$0 \le x_1 \le \frac{m}{p_1}$



$f'(x_1^*) = 0$



$f'(x_1^*) > 0$

Similarly $MU_2(x_1, x_2) = \frac{dv(x_1, x_2)}{dx_2}$

Ex: $v(x_1, x_2) = 2x_1 + x_2$

$MU_1(x_1, x_2) = 2$
$MU_2(x_1, x_2) = 1$

$v(x_1, x_2) = x_1 \cdot x_2$
$MU_1(x_1, x_2) = x_2$
$MU_2(x_1, x_2) = x_1$

Relation between MRS and MU :

MRS : rate at which consumer is willing to substitute good 2 for good 1 to keep the same utility.

Suppose I change consumption of good 1 : Δx_1
" " " good 2 = Δx_2

ΔU = change in utility $= MU_1 \cdot \Delta x_1 + MU_2 \cdot \Delta x_2$

$MRS = \frac{\Delta x_2}{\Delta x_1}$ such that :

$0 = MU_1 \cdot \Delta x_1 + MU_2 \cdot \Delta x_2$

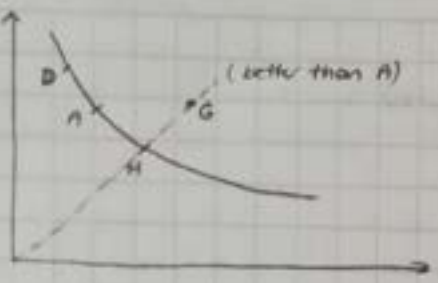
$\frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$

Remark : MRS is independent of the choice of $v()$: if we take any monotonic transformation of v, MRS is the same

Ex : v, $V = 2v$ $\quad \frac{dv/dx_1}{dv/dx_2} = \frac{2\, dv/dx_1}{2\, dv/dx_2} = \frac{2x_2}{2x_1} = \frac{x_2}{x_1}$ //

$v(x_1, x_2) = x_1 \cdot x_2$
$V(x_1, x_2) = 2x_1 \cdot x_2$

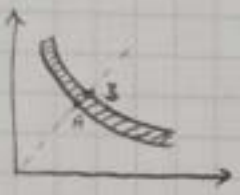
*



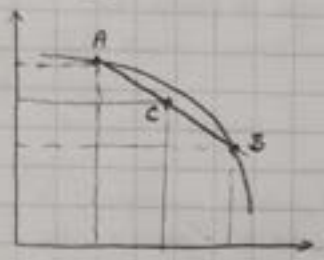
$G \succ H$, $H \sim D$ $\Rightarrow$ $G \succ D$

Q: Can indifference curves be "thick"?

→ If preferences are monotonic, the answer is No (X)



• If my preference is convex, this is not possible.



convex : $C \succ A$, $C \succ B$

$C = \frac{1}{2}A + \frac{1}{2}B$

Two "extreme" cases :

Def : 2 goods are "perfect substitutes" if the consumer is willing to substitute one good for the other good at a constant rate.

ex : There are 2 types of pens : blue & black

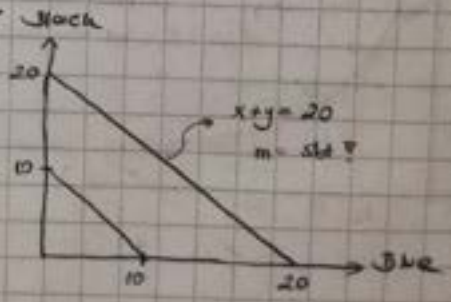
- I'd like to have as many pens as possible but I don't care about the color.

(20,0) is indifferent to (10,10)

(10,10) is indifferent to (5,15)

(30,0) $\succ$ (10,10)

IC



Is there an easier way to represent preferences?

Answer : Utility function

Defn : A utility function is a function from the set of consumption bundles to real numbers : Assigns a number to each bundle.

$u((5,2)) = 2$ (A)
$u((4,2)) = 1$ (B)

} A is strictly preferred to B. (since $2 > 1$) (not x2 relation)

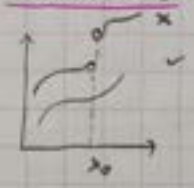
Defn : $u : \underbrace{\mathbb{R}^2_+}_{\| \{(x_1, x_2) : x_1, x_2 \geq 0\}} \to \mathbb{R}$ represents $\succsim$ if

(i) If $(x_1, x_2) > (y_1, y_2)$, $u(x_1, x_2) > u(y_1, y_2)$
(ii) If $(y_1, y_2) > (x_1, x_2)$, $u(y_1, y_2) > u(x_1, x_2)$
(iii) If $(x_1, x_2) \sim (y_1, y_2)$: $u(x_1, x_2) = u(y_1, y_2)$

Q : Can every preference relation be represented by a utility function?

A : Almost always. If preferences are rational and "continuous" we can find.

rational = complete + transitive



Q : Can there be more than one utility function that represent the same preferences?

A : $(2,3) > (4,1) \sim (2,2)$

$u_1(2,3) = 6$
$u_1(4,1) = u_1(2,2) = 4$

} $u_2(2,3) = 36$
$u_2(4,1) = u_2(2,2) = 16$

Both represent the same preferences

Ex 2: Preferences over "left shoes" & "right shoes"
"Perfect Complements"

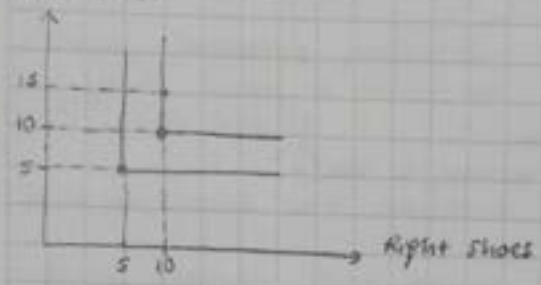
↳ I would like to have as many pair shoes as possible

$\underset{\uparrow}{L}\ \underset{\uparrow}{R}$ $\quad$ $\underset{\uparrow}{L}\ \underset{\uparrow}{R}$

$(15,10) \sim (10,10) \sim (12,10) \sim (10,15) \sim \cdots$

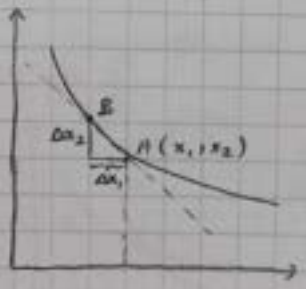
$(8,8) > (7,6)$

Left shoes



Marginal Rate of Substitution

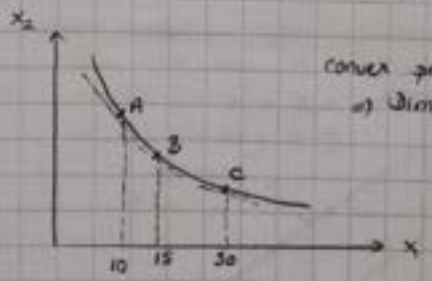
- Suppose consumer has bundle $A = (x_1, x_2)$
Suppose we take away Δx_1 of good 1
let's give consumer Δx_2 of good 2 just enough to keep her indifferent to A.



$\frac{\Delta x_2}{\Delta x_1}$ = The rate at which consumer is willing to substitute good 2 for good 1.

↓ MA

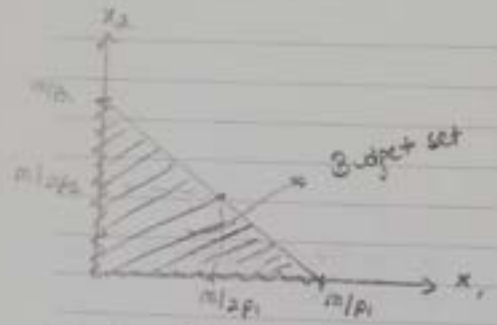
Suppose $\Delta x_1 \to 0$, then $\frac{\Delta x_2}{\Delta x_1}$: Marginal rate of substitution (MRS)



convex preferences
⇒ Diminishing MRS

m=2

__/__/__



$p_1 x_1 + p_2 x_2 = m$

$p_2 x_2 = m - p_1 x_1$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

$p_1, p_2, m > 0$ $\quad x_1, x_2 \geq 0$

$p_1 x_1 + p_2 x_2 \leq m$

Goods are perfectly divisible.

"Preference" of a consumer over bundles :
If a consumer reports that "A is preferred to B" this means all other things are equal he/she feels better off under A than under B.

$(x_1, x_2) > (y_1, y_2)$: (x_1, x_2) is strictly preferred to (y_1, y_2)

: (y_1, y_2) " " " " (x_1, x_2)

$(x_1, x_2) \sim (y_1, y_2)$: (x_1, x_2) is indifferent to (y_1, y_2)

$(x_1, x_2) \geq (y_1, y_2)$: (x_1, x_2) is weakly preferred to (y_1, y_2)

__/__/__

Consumer Theory

- Consider a consumer with limited resources.
- Question: What is the "best" consumption bundle" among the ones that are "affordable" for the consumer?

Assume that there are n goods: good 1, good 2, -- good n

Defn. A consumption bundle that consists of x_1 units of good 1, x_2 units of good 2, ---, x_n units of good n is denoted by $(x_1, x_2, -- x_n)$

Ex: $n=2$ $(3,1)$ ⇝ 3 units of good$_1$
1 unit of good$_2$

Let prices of goods be: $p_1, p_2, -- p_n > 0$ and income / wealth of consumer: $m > 0$

Defn: $(x_1, x_2 -- x_n)$ is affordable if

$$\underbrace{p_1 x_1 + p_2 x_2 + \cdots + p_n x_n}_{\text{total spending on } (x_1, x_2, -- x_n)} \leq m$$

We will call the set of affordable bundles as "budget set"

Suppose I only track 1 blue pen (x_1) with 2 black pens (x_2)

$(10,10) \sim (9,12) \sim (8,14) \sim \cdots$

$u(x_1, x_2) = 2x_1 + x_2$

Ex 2 Perfect Complements :

→ # of pair of shoes = $\min\{$ # left sh, # right sh $\}$

$u(x_1,x_2) = (x_1+x_2) - |x_1 - x_2|$

$(10,10) \sim (15,10)$

$(10,10) > (8,14)$

$x_1 > x_2 \Rightarrow u(x_1,x_2) = 2x_2 = 2\min\{x_1, x_2\}$

$x_2 > x_1 \Rightarrow u(x_1,x_2) = 2x_1 = 2\min\{x_1, x_2\}$

$x_1 = x_2 \Rightarrow u(x_1,x_2) = 2x_1 = 2x_2 = 2\min\{x_1, x_2\}$

$u(x_1,x_2) = \min\{x_1, x_2\}$

$v(x_1,x_2) = 2\min\{x_1, x_2\}$ ✓

$w(x_1,x_2) = (\min\{x_1, x_2\})^2 + 8$ ✓

$\vdots$

$x_1 = x_2$: $x_1 = x_2 = 8$

$x_1 > x_2$: $x_2 = 8$, $x_1 > x_2 = 8$

$x_1 < x_2$: $x_1 = 8$, $x_2 > 8$

Marginal Utility :

- Suppose I increase the consumption of good 1 by Δx_1, utility difference :

$\Delta u = u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)$

↳ change in utility if $x_1 \nearrow x_1 + \Delta x_1$

→ per unit utility change : $\frac{\Delta u}{\Delta x_1} = \frac{u(x_1+\Delta x_1, x_2) - u(x_1,x_2)}{\Delta x_1}$

$\Delta x_1 \to 0$: $MU_1(x_1,x_2) = \lim_{\Delta x_1 \to 0} \frac{u(x_1+\Delta x_1, x_2) - u(x_1,x_2)}{\Delta x_1}$

$= \frac{du(x_1,x_2)}{dx_1}$

Defn: Let $f: \mathbb{R} \to \mathbb{R}$ be a strictly increasing function
Given a function $u: \mathbb{R}^2_+ \to \mathbb{R}$, the function

$V: \mathbb{R}^2_+ \to \mathbb{R}$
$V(x_1, x_2) = f(u(x_1, x_2))$

} is called a (positive) monotonic transformation of u.

Ex: $u(x_1, x_2) = x_1 \cdot x_2$ $\quad f(x) = 2x - 3$
$f(x) = x^2$ $\quad w(x_1, x_2) = f(u(x_1, x_2)) = 2x_1 x_2 - 3$

e) $v(x_1, x_2) = x_1^2 x_2^2$

Result (Theorem): If $u()$ represents $\succsim$, then any monotonic transformation of u represents $\succsim$

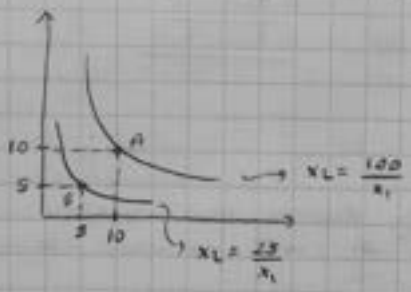
$$V(x_1, x_2) = f(u(x_1, x_2))$$

If $(x_1, x_2) > (y_1, y_2) \Rightarrow u(x_1, x_2) > u(y_1, y_2)$
$\Rightarrow f(u(x_1, x_2)) > f(u(y_1, y_2))$
$\because f$ is increasing $\forall$

Same indifference curve
$\Rightarrow A \sim B$.
$\Rightarrow u(A) = u(B)$

Ex: $u(x_1, x_2) = x_1 x_2$ $\longrightarrow$ $x_1 \cdot x_2 = 100$
$x_2 = \frac{100}{x_1}$

"Level curve" of utility function



Ex: Perfect Substitutes:
- Is there a utility function that represents such preferences?
Den example: Blue pen, black pen
(I only care how many pens I have in total and more is better.)

$(20,0) \sim (5,15) \sim (8,12) \sim \cdots$
$(20,0) \succ (5,13)$

$u(x_1, x_2) = x_1 + x_2$ ✓

$v(x_1, x_2) = x_1 + x_2 + 1$ ✓
$w(x_1, x_2) = (x_1 + x_2)^2$ ✓
$\vdots$

__/__/__

Defn : $\succsim$ is complete if for any (x_1, x_2), (y_1, y_2) one of the following holds :

(i) $(x_1, x_2) > (y_1, y_2)$
, or, (i) $(y_1, y_2) > (x_1, x_2)$
, or, (i) $(x_1, x_2) \sim (y_1, y_2)$

Defn : $\succsim$ is **transitive** if for any (x_1, x_2), (y_1, y_2), (z_1, z_2) such that $(x_1, x_2) \succsim (y_1, y_2)$ and $(y_1, y_2) \succsim (z_1, z_2)$ then $(x_1, x_2) \succsim (z_1, z_2)$.

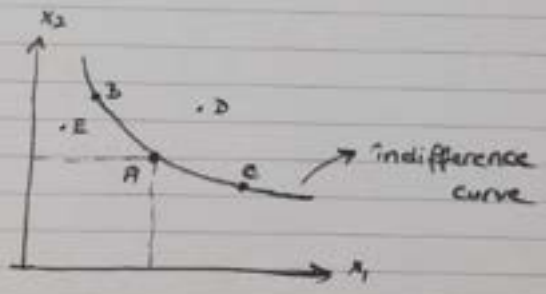
Ex : "Intransitivity of indifferences"

$0 \leq 0,0001$
$\vdots$
$0,9999 \leq 1$

$1 \geq 0$ (super in fact)

Indifference Curve :

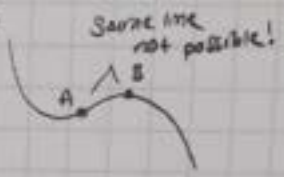
The indifference curve that passes through (x_1, x_2) consists of all bundles that are indifferent to (x_1, x_2)



Q: How does an indifference curve look like?

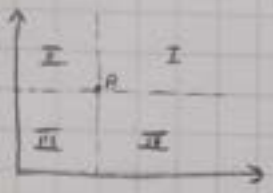
* Monotonicity of Preferences : We will assume "more is better".
- Monotone preferences : If a consumer consumes more of each good, he is better off.
$(5,3) > (4,1)$

- Strictly monotonic preferences : If a consumer consumes more of any good, he/she is better off.
$(5,3) > (4,3)$



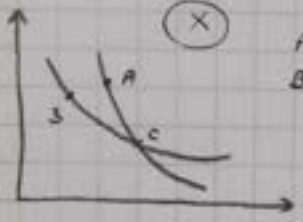
* If preferences are monotonic ;

-Q Is difference curve always downward sloping?
→ Yes, if preferences are monotonic



• point in regions I and III cannot be on the same I.C with A.

-Q. Can indifference curves intersect?
→ No (X)



A ~ C
B ~ C
A > B

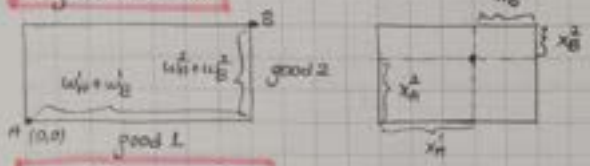
• Convex preferences : "Balanced bundles are better."
$x = (x_1, x_2)$, $y = (y_1, y_2)$
then for any $\alpha \in (0,1)$
$\alpha x + (1-\alpha) y \gtrsim x$, and
$\alpha x + (1-\alpha) y \gtrsim y$

pro teknik
Proje İnşaa ve Soğ. Sis. İnş. Müh. Tesis. İç ve Dış Tic. Ltd. Şti.

General Equilibrium

Two people : A, B
Two goods : 1, 2
Initial endowment : $w_A = (w_A^1, w_A^2)$
$w_B = (w_B^1, w_B^2)$
An allocation $(x_A, x_B) = \left(\underbrace{(x_A^1, x_A^2)}_{x_A}, \underbrace{(x_B^1, x_B^2)}_{x_B} \right)$
is feasible if
demand = supply
$x_A^1 + x_B^1 = w_A^1 + w_B^1$
$x_A^2 + x_B^2 = w_A^2 + w_B^2$

Edgeworth Box



Pure Exchange Econ

Ex: $w_A = (2,2)$
$w_B = (3,3)$

$u_A = 5x_1^A + x_2^A$
$u_B = x_1^B + 5x_2^B$

No trade $\Rightarrow$ $u_A(w_A) = u_A(2,2) = 12$
$u_B(w_B) = u_B(3,3) = 18$

$C_A(3,0)$ } feasible all. $u_A(3,0) = 25$
$C_B(0,5)$ } $u_B(0,5) = 25$

Ex: $w^A = (2,0)$
$w^B = (0,3)$
$u^A = u^B = x_1^{1/2} \cdot x_2^{1/2}$
What's the set of eff all?

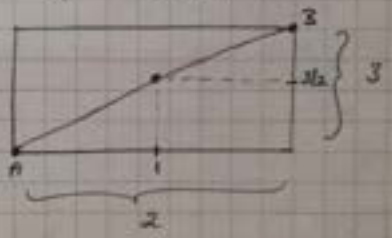
$MRS^A = -\frac{du/dx_1}{du/dx_2} = -\frac{x_2^A}{x_1^A}$

$IRS^A = MRS^B$

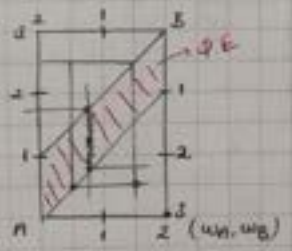
$-\frac{x_2^A}{x_1^A} = -\frac{x_2^B}{x_1^B}$

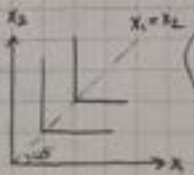
$x_1^A + x_1^B = 2 \Rightarrow x_1^B = 2 - x_1^A$
$x_2^A + x_2^B = 3 \Rightarrow x_2^B = 3 - x_2^A$

$\frac{x_2^A}{x_1^A} = \frac{3 - x_2^A}{2 - x_1^A}$



Ex: $w^A = (2,0)$
$w^B = (0,3)$
$u^A = u^B = \min\{x_1, x_2\}$



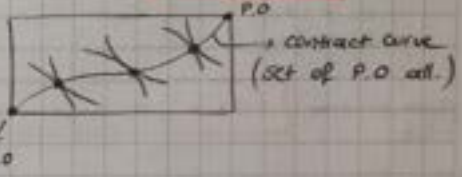


★ A result of a trade (final all.)
① be feasible
② be efficient
③ be better than the initial endowments (core all.)

★ Pareto Efficiency : An allocation is Pareto eff. if it's impossible to make one person better off without making another person worse off.

★ If an all. is P.O then slope of the I.C of agent A and B must be the same.

$MRS^A = MRS^B$

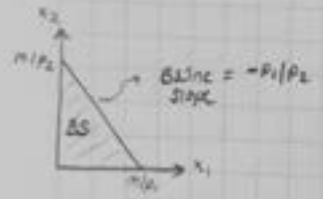


info@pro-teknik.com.tr | www.pro-teknik.com.tr

pro teknik
Proje İstanı ve Sağ. Sis. İnş. Müh. Teah. İç ve Dış Tic. Ltd. Şti.

Q: What is the best consumption bundle among the ones that are affordable for the consumer?

$p_1x_1 + p_2x_2 + \dots + p_nx_n \leq m$ } budget set



Preferences :

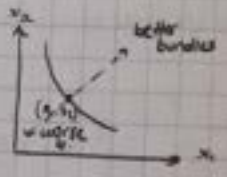
- Completeness → $(x_1, x_2) \succ (y_1, y_2)$
  $(y_1, y_2) \succ (x_1, x_2)$
  $(x_1, x_2) \sim (y_1, y_2)$

- Transitivity → $(x_1, x_2) \succsim (y_1, y_2)$ and $(y_1, y_2) \succsim (z_1, z_2)$
  ⇓
  $(x_1, x_2) \succsim (z_1, z_2)$ ✓

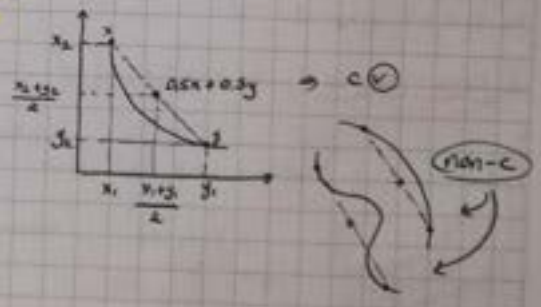
★ An IC that goes through a bundle (x_1, x_2) is the set of all bundles to which (x_1, x_2) is indifferent for the consumer.

★ Monotonicity "more is better" (more of both goods)
+ Strict " (more of any good)

★ If pref. are mon. → IC can't intersect
→ IC are negatively sloped
→ IC can't be thick
→ MRS < 0

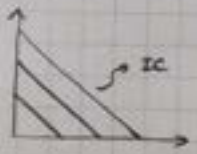


★ Convexity "balanced bundles are better"



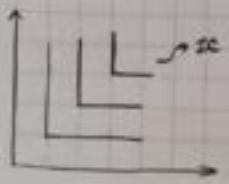
★ Perfect Substitutes (constant rate)
(total numbers ∀)

(10,10) ∼ (15,5)



★ Perfect Complements (fixed proportions)

(10r, 10l) ∪ (15r, 10l)

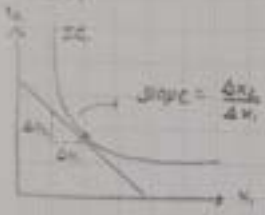


info@pro-teknik.com.tr | www.pro-teknik.com.tr

pro teknik
Proje Inşaat ve Soğ. Sis. İnş. Mük. Taah. İç ve Dış Tic. Ith. İhr. Ltd. Şti.

* Marginal rate of substitution (MRS) (slope of IC)

$\frac{\Delta x_2}{\Delta x_1}$ → rate at which the consumer is willing to subs. good 2 for good 1.



* Strictly convex → MRS ↓ x_1 ↑
* PS → MRS ↔ (constant)

2.2) Utility func. → assigning a number to every poss. con. bundle (order ↑)

$u: \mathbb{R}_+^2 \to \mathbb{R}$

$u(x_1, x_2) > u(y_1, y_2)$ if $(x_1, x_2) \succ (y_1, y_2)$
" < " = " " ~ "

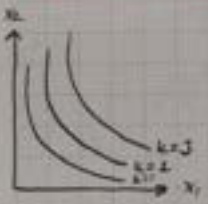
* Complete, Transitive, Continuous) pref.
* no unique utility repr.

$f: \mathbb{R} \to \mathbb{R}$ a strictly increasing f.
$u: \mathbb{R}_+^2 \to \mathbb{R}$ $v: \mathbb{R}_+^2 \to \mathbb{R}$

$v(x_1, x_2) = f(u(x_1, x_2))$ → (positive) mon. transformation of u
→ if u repr. ≽, then any pos. mon. tr. of u repr. ≽
↳ $u_1 > u_2$, $f(u_1) > f(u_2)$

$u(x_1, x_2) = x_1 \cdot x_2$
$v(x_1, x_2) = x_1^2 \cdot x_2^2$
$w(x_1, x_2) = 2x_1 x_2 + 7$
$h(x_1, x_2) = x_1 x_2 - 3$
} ✓ same pref.

* Cobb-Douglas $u(x_1, x_2) = x_1^c x_2^d$, $c, d > 0$



$u(x_1, x_2) = x_1 \cdot x_2$, $k > 0$ → u. level

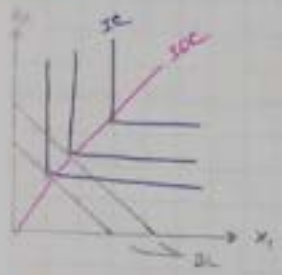
$k = x_1 \cdot x_2$
$x_2 = \frac{k}{x_1}$

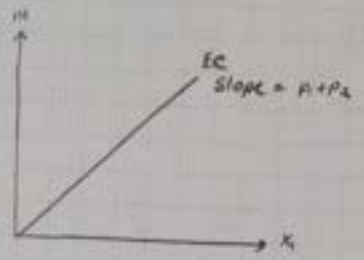
* PS ⇒ $u(x_1, x_2) = x_1 + x_2$
$u(x_1, x_2) = x_1^2 + 2x_1 x_2 + x_2^2$
$v(x_1, x_2) = x_1 + x_2 + 7$
} ✓ same pref.

* Substitute b units of good 1 for a units of good 2
↳ $u(x_1, x_2) = ax_1 + bx_2$

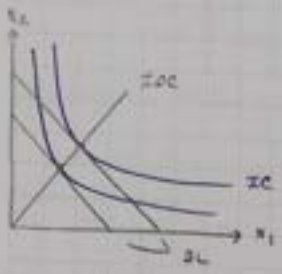
pro teknik
Proje İnşaat ve Soğ. Sis. İnş. Mük. Tesah. İç ve Dış Tic. Ltd. Şti. Ltd. Şti.

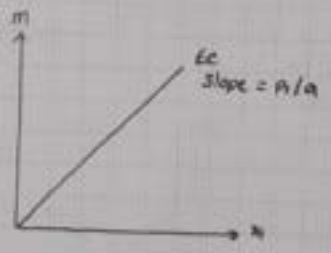
* PCC





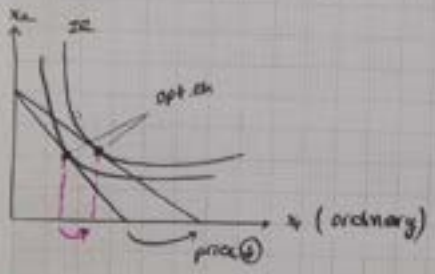
* Cobb - Douglas $u(x_1, x_2) = x_1^{a} x_2^{1-a}$

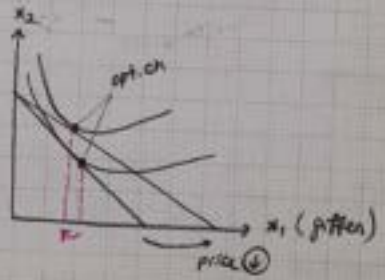




Ordinary Goods & Giffen G.

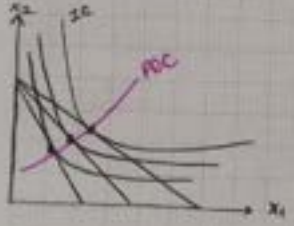
* demand ↑ price ↑ → Giffen
demand ↑ price ↓ → ordinary

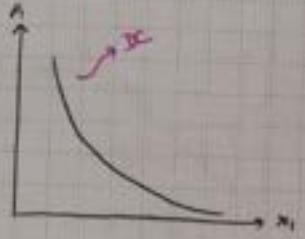




* Price Offer C. & Demand C.

combines opt. ch. for each poss. price of good 1, P_1 ($\bar{P}_2, \bar{m}$)





★ $\frac{\Delta x_2}{\Delta x_1} = \frac{-MP_1}{MP_2}$ $\quad TRS = \frac{-MP_1}{MP_2}$

pro teknik
Proje İstma ve Soğ. Sis. İnş. Mük. Tesh. İç ve Dış Tic. Ith. İhr. Ltd. Şti.

★ Isoprofit line : $\bar{\pi} = P \cdot y - w_1 x_1 - w_2 x_2$

$y = \frac{w_1}{P} \cdot x_1 + \left(\frac{\bar{\pi}}{P} + \frac{w_2}{P} \cdot x_2 \right)$

Profit max ⇒ slope of isoprofit line = slope of production func.

Short-run ($\bar{x}_2$) $\quad \frac{w_1}{P} = \frac{df}{dx_1}$

★ Long-run

↳ $\max_{x_1, x_2} \; P \cdot f(x_1, x_2) - (w_1 x_1 + w_2 x_2) \Rightarrow \frac{df/dx_1}{df/dx_2} = \frac{w_1}{w_2} \Rightarrow \boxed{\frac{MP_1}{MP_2} = \frac{w_1}{w_2}}$

★ Isocost line ⇒ $\bar{C} = w_1 x_1 + w_2 x_2$

$x_2 = \frac{\bar{C}}{w_2} - \frac{w_1 x_1}{w_2}$

At optimal point (cost min) ↳ slope of isoquant = slope of isocost

$\frac{-df/dx_1}{df/dx_2} = \frac{-w_1}{w_2}$

TRS

★ Lagrange

↳ $\mathcal{L}(x_1, x_2, \lambda) = \overbrace{(w_1 x_1 + w_2 x_2)}^{\min} - \lambda \overbrace{(f(x_1, x_2) - \bar{y})}^{s.t.o}$

⇒ $\frac{w_1}{w_2} = \frac{df/dx_1}{df/dx_2}$

At x^* :

★ Equilibrium all. ⇒ $MRS_A(x^*) = MRS_B(x^*) = \frac{-P_1}{P_2}$ ✓

★ $\pi \geq AVC(y)$

★ $P = MC(y)$

★ P.E. all. ⇒ $MRS_A = MRS_B$ + feasibility

↳ oran bul (interior) $\quad$ ör. $\underbrace{y^A = 2x^A}_{A \text{ için}} \Rightarrow \{(x, 2x), (\omega_x - x, \omega_y - 2x)\}$

★ Tangency ∇ // P.E ② için nokta(lar) yerine koyup tangency mi bak ∇

$\min\{x_B, y_B\} \Rightarrow$ P.E all. $\underbrace{x_B = y_B}_{B \text{ için}} \Rightarrow \{(\omega_x - x, \omega_y - x), (x, x)\}$

★ E. all. (interior)

↳ $MRS_A = \frac{-P_x}{P_y}$

↳ $MRS_B = \frac{-P_x}{P_y}$

} x_A, y_A, x_B, y_B cek ∇ (demand)

⊖ max $u()$ s.t.o $P_x \cdot x_A + P_y \cdot y_A = \underbrace{w_A^0 \cdot P_x + w_A^0 \cdot P_y}_{m_A}$

↳ $x_A + x_B = w_A^0 + w_B^0$

↳ $y_A + y_B = w_A^0 + w_B^0$

} $\frac{P_x^*}{P_y^*}$ bul ∇

Sınırlar değilse ⇒ her duruma uygun demand bul ∇ Sonra feasibility ∇

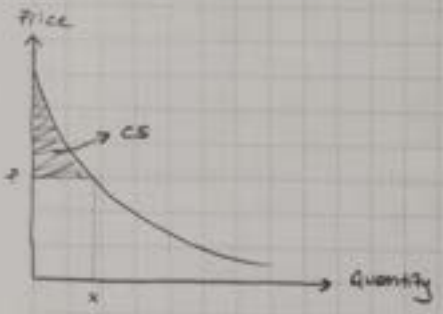
↳ demand ≠ supply

info@pro-teknik.com.tr | www.pro-teknik.com.tr

pro teknik
Proje İstma ve Soğ. Sis. İnş. Müh. Tesis. İç ve Dış Tic. Ltd. Şti.

Measuring welfare Changes

* Consumer Surplus :



- Compensating variation (CV) : how much money an individual needs to take away from (give to) a consumer after a price decrease (increase) such that she is as well off as before the price change.

- Equivalent variation (EV) : how much money one needs to give to (take away from) a consumer before a price decrease (increase) such that she is as well off as after the price change.

* $u(x_1, x_2) = x_1 \cdot x_2$ $\quad p_1 = p_2 = 1$, $m = 100$ $\quad p_1$ ② $\quad$ CV & EV?

demand of the consumer = (50,50) $\quad$ utility = 2500 // at or. prices
// // // = (25,50) $\quad$ util.ty = 1250 // at new prices

CV → to keep her at u. level 2500 after price change, m should be :

$\frac{m}{4} \times \frac{m}{2} = 2500$ //

$m \cong 141.4$

$141.4 - 100 =$ 41.4 → CV

EV → to keep her at u. level 1250 before price change, m should be :

$\frac{m}{2} \times \frac{m}{2} = 1250$ //

$m \cong 70.7$

$100 - 70.7 =$ 29.3 → EV

info@pro-teknik.com.tr | www.pro-teknik.com.tr

pro teknik
Proje İnşaat ve Sağ. Sis. İnş. Müh. Tesis. İş ve Dış Tic. İth. İhr. Ltd. Şti.

★ PC

$u(x_1, x_2) = \min\{x_1, x_2\}$

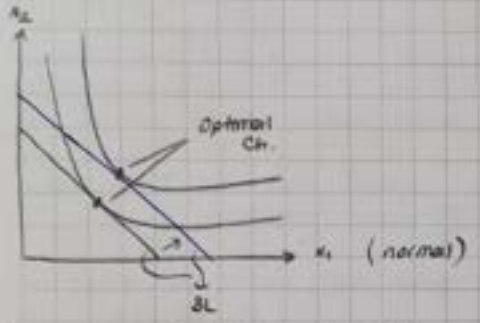
$x(p_1, p_2, m) = \left(\frac{m}{p_1+p_2}, \frac{m}{p_1+p_2}\right)$

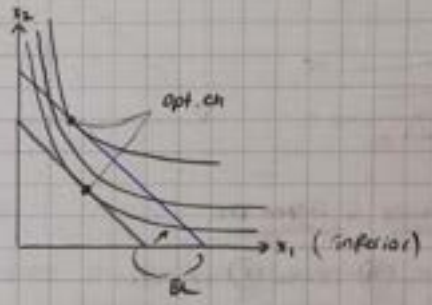
Ex: Cobb-Douglas $u(x_1, x_2) = x_1^c x_2^d$

$\frac{mu_1}{mu_2} = \frac{p_1}{p_2}$ $\quad p_1 x_1 + p_2 x_2 = m$

$x(p_1, p_2, m) = \left(\frac{c}{c+d}\cdot\frac{m}{p_1}, \frac{d}{c+d}\cdot\frac{m}{p_2}\right)$

(A) m ↑ demand ↑ → normal g.
m ↑ demand ↓ → inferior g.

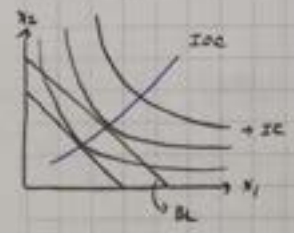


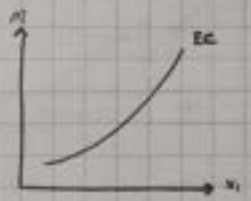


★ Income Offer C & Engel C

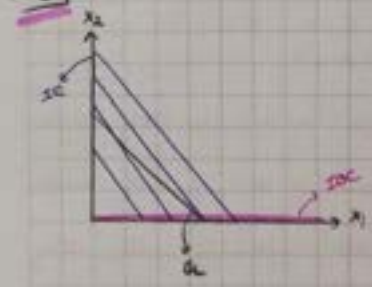
Income Offer C: combines opt. ch. for each poss. income (prices fixed)

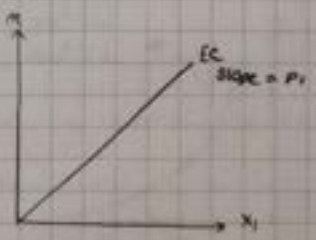
Engel C: graph of the demand





★ PS





info@pro-teknik.com.tr | www.pro-teknik.com.tr

proteknik
Proje İstanu ve Sağ. Sis. İnş. Müh. Teah. İş ve Dış Tic. Bth. Bir. Ltd. Şti.

Slutsky Equation

$\Delta x_1^m = -\Delta x_1^m$

$\hookrightarrow \Delta x_1^m = x_1(p_1', p_2, m') - x_1(p_1', p_2, m)$

$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$

$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1} \longrightarrow \frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} \cdot x_1$

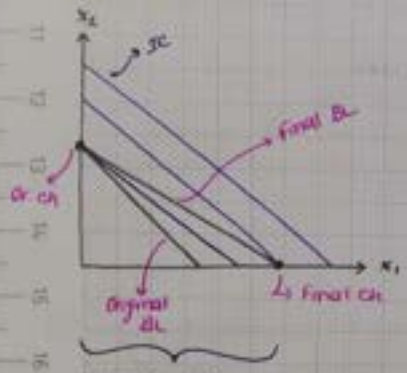
$\Delta m = m' - m$
$= (p_1' - p_1)\, x_1(p_1, p_2, m)$
$= \Delta p_1 x_1(p_1, p_2, m)$

$\frac{dx_1}{dp_1} = \frac{dx_1^s}{dp_1} - \frac{dx_1}{dm} \cdot x_1$

x_1 : demand func.
x_1^s : compensated demand f.

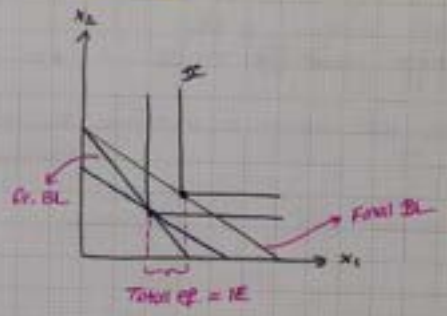
$\Delta p_1 \to 0$

* HS

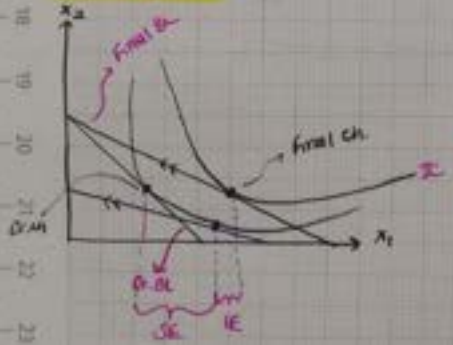


Total ef. = SE

* PC

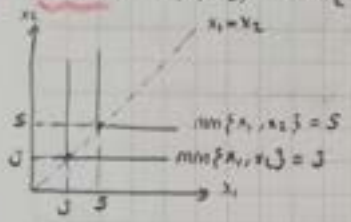


* Hicks SE : (at the same utility level)(not the original bundle)



pro teknik
Proje İnşaa ve Soğ. Sis. İnş. Mük. Taah. İş ve Dış Tic. Ltd. Şti.

★ PC ⇒ $u(x_1, x_2) = \min\{x_1, x_2\}$



Marginal Utility (utility change as we give someone a little more of good 1)

$$mu_1(x_1, x_2) = \lim_{\Delta x_1 \to 0} \frac{\Delta u}{\Delta x_1} = \lim_{\Delta x_1 \to 0} \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

$$= \frac{du(x_1, x_2)}{dx_1} \quad //$$

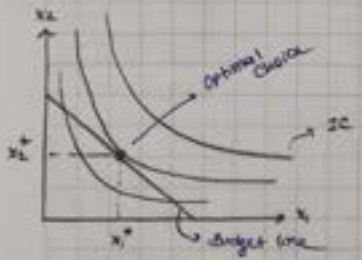
$$mu_2(x_1, x_2) = \frac{du(x_1, x_2)}{dx_2}$$

MRS ⇒ $0 = \Delta u$ ⇒ $\Delta u = mu_1 \Delta x_1 + mu_2 \Delta x_2$

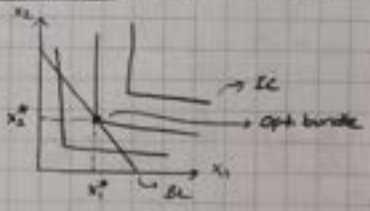
$$MRS = \frac{\Delta x_2}{\Delta x_1} = -\frac{mu_1}{mu_2}$$

★ mu is dependent on u.f. (u and 2u repr. the same pref. but mu are different)
★ MRS is independent of any mon. trans.

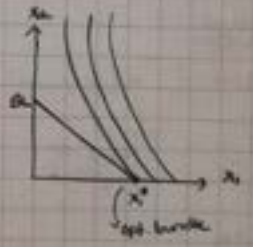
5.1 Choice



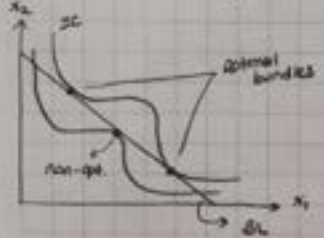
Exception : - kinked curves



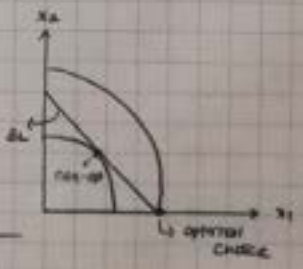
- Boundary solutions



★ Optimal choice → interior ★
If IC are smooth ★



non-convex

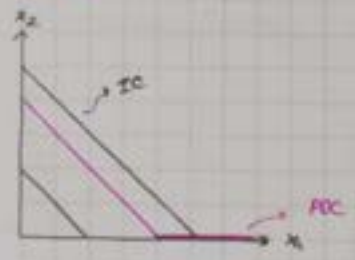


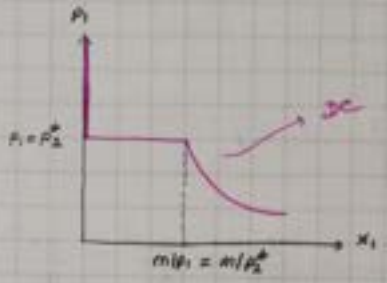
★ Interior (both goods are consumed in positive amounts)

info@pro-teknik.com.tr | www.pro-teknik.com.tr

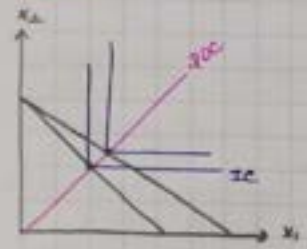
pro teknik
Proje İnşaat ve Sağ. Sis. İnş. Müh. Tesis. İş ve Dış Tic. Ltd. Şti.

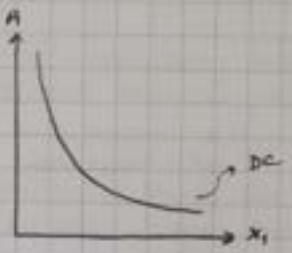
* PS





* PC



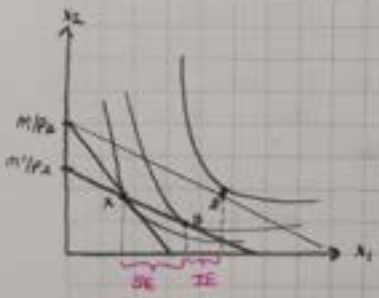


Income & Substitution Effects

p_1 (↓) ⟶ Subs. rate changes → SE
⟶ purchasing power (↑) → IE

* Adjust the income just enough to purchase the original bundle ⇒ pp (⟺)
↳ Slutsky SE

* Relative prices constant (⟺) ⇒ pp changes
↳ IE



$$* \underbrace{\left[x_1(p_1', p_2, m') - x_1(p_1, p_2, m)\right]}_{\Delta x_1^S} + \underbrace{\left[x_1(p_1', p_2, m) - x_1(p_1', p_2, m')\right]}_{\Delta x_1^n}$$

$$\Delta x_1 = \underbrace{\Delta x_1^S}_{SE} + \underbrace{\Delta x_1^n}_{IE}$$

$$m' = p_1' x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m)$$

↳ $(m' - m)(p_1' - p_1) \geq 0$

* $(p_1 - p_1')\,\Delta x_1^S \geq 0$ ⇔ $p_1 > p_1'$, $\Delta x_1^S \geq 0$
⇔ $p_1 < p_1'$, $\Delta x_1^S \leq 0$

normal good ⇒ $(p_1 - p_1')\,\Delta x_1^n \geq 0$ → $(p_1 - p_1')\,\Delta x_1 \geq 0$

inferior good ⇒ $(p_1 - p_1')\,\Delta x_1^n < 0$

giffen good ⇒ $(p_1 - p_1')\,\Delta x_1 < 0$ ⟶ if good 1 is giffen, it is inferior.

info@pro-teknik.com.tr | www.pro-teknik.com.tr

proteknik
Proje İnşaat ve Soğ. Sis. İnş. Mük. Taah. İç ve Dış Tic. ltd. Şti. Ltd. Şti.

* If pref. are convex a point that satisfies the tangency condition must be an optimal choice.
* If pref. are strictly convex (no flat parts) ⇒ only one optimal bundle

* At an interior optimum point ⇒ $MRS = -\frac{p_1}{p_2}$ //

* $MRS = -\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$

$$\frac{mu_1}{p_1} = \frac{mu_2}{p_2}$$

* pref. → convex
  > opt. bundle is interior > $\frac{mu_1}{mu_2} = \frac{p_1}{p_2}$ //

Lagrangian Method:

$\mathcal{L}$. multiplier

$$\mathcal{L}(x_1, x_2, \lambda) = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

$$\frac{mu_1(x_1^*, x_2^*)}{p_1} = \frac{mu_2(x_1^*, x_2^*)}{p_2} = \lambda^* \rightarrow \text{Shadow price of wealth}$$

Note: If u.f is differentiable
pref. are convex
opt. bundle (x_1^*, x_2^*) is interior (not a boundary p.)

General

$$\max u(x_1, x_2)$$
$$p_1 x_1 + p_2 x_2 = m \rightarrow x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$
$$\max u\left(x_1, \frac{m}{p_2} - \frac{p_1}{p_2} x_1\right) \overset{FOC}{\Rightarrow} \frac{mu_1}{mu_2} = \frac{p_1}{p_2}$$ //

13.2

Consumer Demand

$$x(p_1, p_2, m) = \left(x_1(p_1, p_2, m), x_2(p_1, p_2, m)\right)$$

↓ optimal cons. of good 1

* 75

$u(x_1, x_2) = x_1 + x_2$

$$v(p_1, p_2, m) = \begin{cases} (0, m/p_2), & p_1 > p_2 \\ (x_1, x_2), & p_1 = p_2 \\ (m/p_1, 0), & p_1 < p_2 \end{cases}$$

pro teknik
Proje Isıtma ve Soğ. Sis. İnş. Müh. Taah. İç ve Dış Tic. İth. İhr. Ltd. Şti.

Elasticity of demand = $\frac{\%\text{ change in quantity}}{\%\text{ change in price}}$

$|E_d| > 1$ → elastic (sonsuz olmayan)
$|E_d| = 1$ → unit e.
$|E_d| < 1$ → inelastic (su)

Cross-price el. → $\frac{\Delta x_1 / x_1}{\Delta p_2 / p_2}$ => subs. ≥ 0, comp. ≤ 0

pro teknik
Proje İnşaa ve Soğ. Sis. İnş. Müh. Tasar. İç ve Dış Tic. Ltd. Şti.

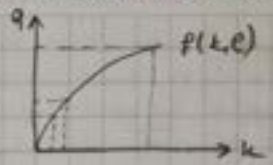
Production functions

$q = f(k, \ell, m \dots)$. $q = f(k, \ell)$

Marginal Productivity

$MP_k = \frac{df}{dk} > 0$ $\quad \frac{d^2 f}{dk^2} < 0$ → Diminishing MP

$MP_\ell = \frac{df}{d\ell} > 0$ $\quad \frac{d^2 f}{d\ell^2} < 0$



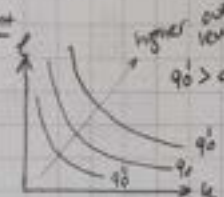
Average productivity of labor $= \frac{f(k,\ell)}{\ell}$

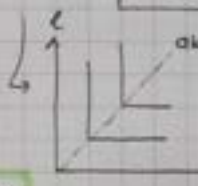
of capital $= \frac{f(k,\ell)}{k}$

→ $f(k,\ell) = q_0 \rightarrow$ Isoquant I_{q_0}

$f(k,\ell) = k^{1/2} \ell^{1/2}$ →

$f(k,\ell) = \min\{ak, b\ell\}$





Slope of an isoquant

$-\frac{df/dk}{df/d\ell} = -\frac{MP_k}{MP_\ell} = MRTS$ (Marginal rate of technical subs.)

Returns to Scale

$t > 1$

$f(tk, t\ell) = t f(k,\ell)$ → constant

$<$ → decreasing

$>$ → increasing

Cost functions

$C(k,\ell) = r.k + w\ell$

$C(q)$ technology $f(k,\ell)$

* min cost = ? → Lagr. ✓

if $q = rk + \ell$, $r=1$, $w=1$

from 1. → $k = \ell$

→ $k = \left(\frac{q}{2}\right)^2$, $\ell = \left(\frac{q}{2}\right)^2$

cost $= \frac{q^2}{4} \cdot 2$

$= \boxed{\frac{q^2}{2}}$

Profit = Revenue − Cost

$= p.q - \text{Cost}$

$\pi(k,\ell) = p.f(k,\ell) - r.k - w.\ell$

Cost minimizing input choices

if $q = f(k,\ell)$

r, w

$\min_{k,\ell} (r.k + w.\ell)$

s.to $q = f(k,\ell)$

$\mathcal{L}(k,\ell,\lambda) = rk + w\ell + \lambda[q - f(k,\ell)]$

$(-\lambda[f(k,\ell) - q])$

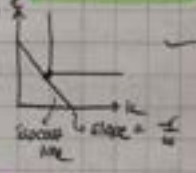
$\frac{df/dk}{df/d\ell} = \frac{r}{w}$

$-MRTS = \frac{r}{w}$ → input price ratio

→ factors of substitution

① $MRTS = \frac{-r}{w}$ $\left(\frac{MP_k}{MP_\ell} = \frac{df/dk}{df/d\ell} = \frac{r}{w}\right)$

② $q = f(k,\ell)$



$rk + w\ell = C_0$

→ ① $\min\{ak, b\ell\} = q_0$

② $ak = b\ell$

* cost $= C(q) = r.k(q) + w.\ell(q)$

* min cost = ?

if $q = \min\{2k, 3\ell\}$, $r=1$, $w=1$

↳ $2k = 3\ell$ ... ①

$q = f(k,\ell) = \min\{2k, 3\ell\}$ ... ②

$= \min\{3\ell, 3\ell\} = 3\ell$

$\ell(q) \leftarrow \boxed{\ell = q/3}$

$\boxed{k(q) = q/2}$

cost $= r.k(q) + w.\ell(q)$

$= 1.q/2 + 1.q/3$

$\boxed{C(q) = 5q/6}$

info@pro-teknik.com.tr | www.pro-teknik.com.tr

proteknik
Proje İnşaat ve Soğ. Sis. İnş. Müh. Tasar. İç ve Dış Tic. Ltd. Şti. Ltd. Şti.

Solving Wal. Eq. All. and Prices with Production

Ex: Two goods : x, y
cons. : A, B
$u_A(x,y) = x^A y^A$ & owner of the firm
$u_B(x,y) = \ln x^B + \ln y^B$

$w_A = (0,4)$
$w_B = (2,0)$

Firm produces x by using y
$f(y) = \sqrt{y}$

$[(x^A, y^A), (x^B, y^B), (x^{Firm}, y^{Firm})] = ?$

$f(y) = \sqrt{y^F} = x^F$

$\max_{x^A, y^A} u^A$ s.to $P_x \cdot x^A + P_y \cdot y^A = 4P_y$

MRS $= -\frac{P_x}{P_y} \Rightarrow -\frac{y^A}{x^A} = -\frac{P_x}{P_y} \Rightarrow \boxed{x^A P_x = y^A P_y}$

$2P_x \cdot x^A = 4P_y \Rightarrow \boxed{x^A = 2P_y/P_x}$, $\boxed{y^A = 2}$

$\max_{x^B, y^B} u^B$ s.to $x^B P_x + y^B P_y = 2P_x + \pi$

MRS $= -\frac{P_x}{P_y} \Rightarrow -\frac{y^B}{x^B} = -\frac{P_x}{P_y} \Rightarrow \boxed{x_B P_x = y_B P_y}$

$2P_x \cdot x^B = 2P_x + \pi$

$\boxed{x^B = 1 + \pi/2P_x}$, $\boxed{y^B = \frac{P_x}{P_y} + \frac{\pi}{2P_y}}$

$\max_{x^F, y^F} (\pi^F = P_x \cdot x^F - P_y \cdot y^F)$ s.to $\sqrt{y^F} = x^F$

$\pi^F = P_x \cdot \sqrt{y^F} - P_y \cdot y^F$

$\frac{d\pi}{dy^F} = 0 \Rightarrow \frac{1}{2} P_x \cdot (y^F)^{-1/2} - P_y = 0$

$\Rightarrow \frac{1}{2} \frac{P_x}{\sqrt{y^F}} = P_y$

$\Rightarrow \boxed{y^F = \frac{P_x^2}{4P_y^2}} \Rightarrow \boxed{x^F = \frac{P_x}{2P_y}}$

- Demand for x : $x^A + x^B$
- " " y : $y^A + y^B + y^F$
- Supply for x : $0 + 2 + x^F$
- " " y : $4 + 0$

$\frac{2P_y}{P_x} + 1 + \frac{\pi}{2P_x} = 2 + \frac{P_x}{2P_y}$ (x)

$1 + \frac{P_x}{P_y} + \frac{\pi}{2P_y} + \frac{P_x^2}{4P_y^2} = 4$ (y)

$\hookrightarrow 3P_x = 4P_y$

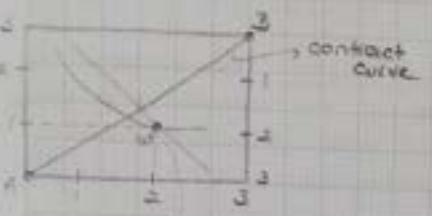
$\boxed{P_x/P_y = 4/3}$

pro teknik
Proje İstma ve Sog. Sis. İnş. Müh. Tasoh. İç ve Dış Tic. İth. İhr. Ltd. Şti.

Ex: $U_A = x_1 - x_2$
$U_B = x_1 + x_2$
$w_A = (2,1)$
$w_B = (1,2)$

$MRS_A = -\frac{x_2^A}{x_1^A}$, $MRS_B = -1$

Pareto eff. $\Rightarrow$ $\frac{x_2^A}{x_1^A} = 1 \Rightarrow \boxed{x_1^A = x_2^A}$



Ex: $U_A = x^A \cdot y^A$
$U_B = \ln x^B + \ln y^B$
$w_B = (2,0)$
$w_A = (0,4)$
Walrasian equ. $P_x/P_y = ?$

$\max_{x,y} U$ s.to $\boxed{x \cdot P_x + y \cdot P_y = P_x \cdot w_x + P_y \cdot w_y}$

A)
$MRS = -\frac{P_x}{P_y} \Rightarrow -\frac{y^A}{x^A} = -\frac{P_x}{P_y} \Rightarrow \boxed{x^A P_x = y^A \cdot P_y}$

$2 \cdot x^A \cdot P_x = 4P_y \Rightarrow \boxed{x^A = \frac{2P_y}{P_x}} \Rightarrow \boxed{y^A = 2}$ → Demand for A ∀

B)
$MRS = -\frac{P_x}{P_y} \Rightarrow \frac{1/x^B}{1/y^B} = -\frac{P_x}{P_y} \Rightarrow \boxed{x^B \cdot P_x = y^B \cdot P_y}$

$2 \cdot x^B \cdot P_x = 2P_x \Rightarrow \boxed{x^B = 1} \Rightarrow \boxed{y^B = P_x/P_y}$

$\left[(x^A, y^A), (x^B, y^B), \frac{P_x}{P_y}\right]$

$= \underbrace{\left[(1,2), (1,2), 2\right]}_{\text{W.E ∀}}$

− Market demand
good $x = x^A + x^B = \frac{2P_y}{P_x} + 1$
good $y = y^A + y^B = 2 + \frac{P_x}{P_y}$

− Market supply
good $x = w_x^A + w_x^B = 2$
good $y = w_y^A + w_y^B = 4$

$1 + \frac{2P_y}{P_x} = 2 \Rightarrow \boxed{P_y/P_x = 1/2}$

$2 + \frac{P_x}{P_y} = 4 \Rightarrow \boxed{P_x/P_y = 2}$

proteknik

Proje Isleme ve Sog. Sis. Ins. Mak. Tesh. Is ve Dis Tic. Ltd. Sti. Ltd. Sti.

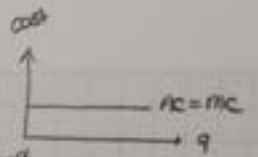
Cost functions

Total cost = $C(q)$

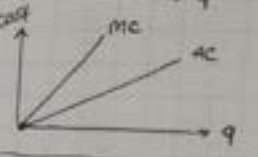
Average cost = $\frac{C(q)}{q}$

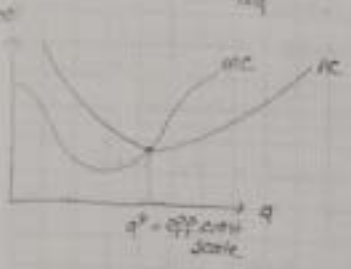
Marginal cost = $\frac{dC(q)}{dq}$ → (additional cost)

$C(q) = \frac{5q}{4}$ AC = 5/4, MC = 5/4



$C(q) = \frac{q^2}{2}$ AC = q/2, MC = q





- AC > MC ⇒ AC → falling
- MC > AC ⇒ AC → increasing
- AC = MC

Profit maximization

I, Revenue − Cost (R−C) = π

Maximize profit by choosing output q

$\frac{d\pi}{dq} = 0$ solve for q

$= \frac{dR}{dq} - \frac{dC}{dq} = 0$

(marginal revenue, m. cost) ⇒ MR = MC

If firm is a price taker

$R = p \cdot q$

$R(q) = p \cdot q$

$MR = p$

$P = MC$

Profit = $p \cdot q$ − cost

$= p \cdot f(k,\ell) - r \cdot k - w \cdot \ell$

Profit max problem ⇒ $\max_{k,\ell} \left(p \cdot f(k,\ell) - rk - w\ell \right)$

$L(k,\ell) = p \cdot f(k,\ell) - rk - w\ell$

$\frac{dL}{dk} = 0 \Rightarrow p \cdot df/dk - r = 0 \rightarrow p = \frac{r}{df/dk}$

$\frac{dL}{d\ell} = 0 \Rightarrow p \cdot df/d\ell - w = 0 \rightarrow p = \frac{w}{df/d\ell}$

Ex: $f(x_1, x_2) = \left(\sqrt{x_1} + 3\sqrt{x_2} \right)^2$, $p_1 = p_2 = 1$

a) MRTS?

b) optimal cost?

c) optimal q? (Assuming that market price is 7)

optimal cost ($C(q)$) = $x_1 + x_2$

$= 9q/(19)^2 + 36q/(19)^2$

$= 17q/(19)^2$ ✓

$\max_q \pi$

$\pi = p \cdot q - \frac{37q}{(19)^2}$

alternative opt.

$\max_{q, x_1, x_2} (p \cdot q - p_1 x_1 - p_2 x_2)$ s.to $f(x_1, x_2) = q$

$\frac{d\pi}{dq} = 0 \rightarrow p - \frac{37}{(19)^2} = 0$

$p = \frac{37}{(19)^2}$

$p = \ldots$

⇒ Suppose $p < \frac{37}{(19)^2} \rightarrow q = 0$

$p > \ldots \Rightarrow$

$MRTS = -\frac{MP_{x_1}}{MP_{x_2}} = -\frac{df/dx_1}{df/dx_2} = -\frac{2\left(\sqrt{x_1} + 3\sqrt{x_2}\right)\left(\frac{1}{2} \cdot x_1^{-1/2}\right)}{2\left(\sqrt{x_1} + 3\sqrt{x_2}\right)\left(\frac{3}{2} \cdot x_2^{-1/2}\right)} = -\frac{\sqrt{x_2}}{3\sqrt{x_1}}$

correct sol?

- we did a mistake by writing $\frac{-\sqrt{x_1}}{6\sqrt{x_1}}$ but the approach is the same?

$\min_{x_1, x_2} \underbrace{p_1 x_1 + p_2 x_2}_{(x_1 + x_2)}$ s.to $f(x_1, x_2) = q$

$MRTS = -\frac{p_1}{p_2} \Rightarrow \frac{-\sqrt{x_2}}{6\sqrt{x_1}} = -1$

$\sqrt{x_2} = 6\sqrt{x_1}$

$x_2 = 36 x_1$

$\left(\sqrt{x_1} + 3\sqrt{x_2}\right)^2 = q$

$x_2 = 36 x_1$

$\left(\sqrt{x_1} + 18\sqrt{x_1}\right)^2 = q$

$19\sqrt{x_1} = \sqrt{q}$

$x_1 = 9q/(19)^2$, $x_2 = 36q/(19)^2$