Phys-111
Final
-2022-

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1. (15 pts) A ball with mass $m = 0.2\,kg$ and radius $R = 0.1\,m$ is thrown into the air from an initial height of $y_0 = 1.25\,m$ at an angle of $\theta = 37^0$ with a speed of $v_0 = 20\,m/s$ and an angular velocity of $\omega = 30\,rad/s$. Ignore aerodynamic effects.

(Take $I_{cm} = \frac{2}{3}mR^2$; $g = 10\,m/s^2$ ; $sin37 = cos53 = 0.6$ ; $sin53 = cos37 = 0.8$)

a) (8 pts) How many revolutions does the ball make before it returns to the ground? Treat the ball as a point mass for its projectile motion.

Let's find the time for the ball to fall to ground.

$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$

$0 = 1.25 + (20)(\sin 37^0)t - \frac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 12t - 1.25 = 0$

$t = \frac{12 \pm \sqrt{144 + 25}}{10} = \frac{12 + 13}{10} = 2.5\,s$

During 2.5s, ball rotates through an angle of

$\Delta\theta = \omega_0 t + \frac{1}{2}\cancel{\alpha}_0 t^2$

$= (30)(2.5)$

$= 75\text{ rad} \Rightarrow N = \frac{75\text{ rad}}{2\pi\text{ rad/rev}} = \frac{35}{\pi}\text{ rev}$

$\boxed{N = 35/\pi}$

b) (7 pts) What is the total energy of the ball at the heighest point of its flight?

$E_{top} = E_i$

$= \frac{1}{2}mv_0^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{1}{2}(0.2)(20)^2 + \frac{1}{2}\left[\frac{2}{3}(0.2)(0.1)^2\right](30)^2$

$= 40 + 0.6$

$= 40.6\text{ J}$

| E = 40.6 | unit: J |
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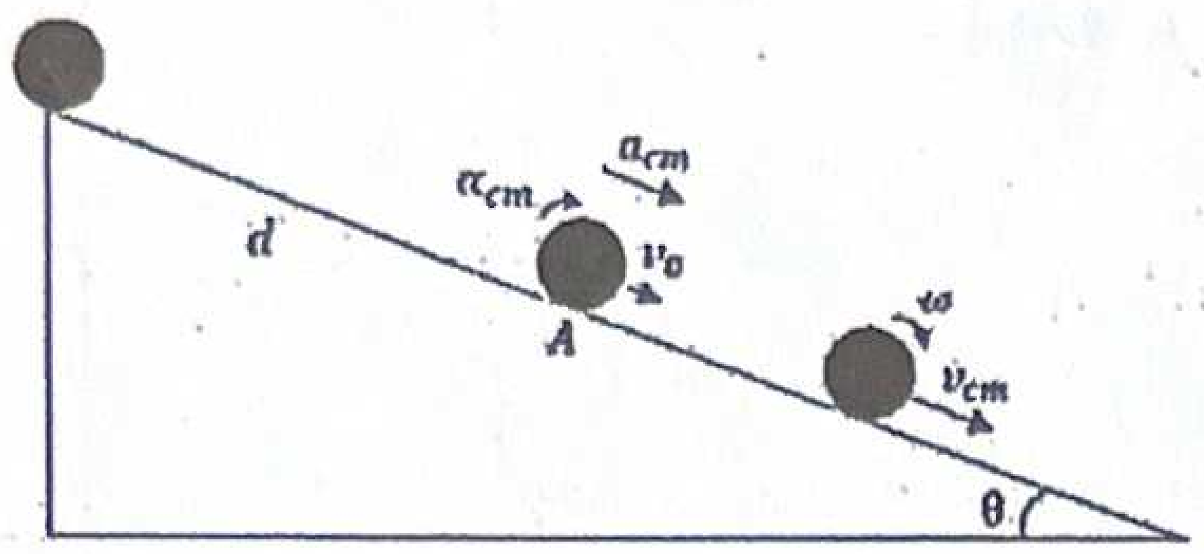
2. (15 pts) A solid ball with mass $m = 8\ kg$ and radius $R = 0.04\ m$ is placed at the top of a long incline with $\theta = 37^0$. When the ball is released from rest, it first slides for a distance of $d = 3\ m$ on the frictionless part of the incline until it reaches point A. The lower part of the incline has $\mu_k = 0.25$. At point A, the ball starts to slip and gain rotational speed.

(Take $I_{cm} = \frac{2}{5}mR^2$; $g = 10\ m/s^2$; $sin37 = cos53 = 0.6$; $sin53 = cos37 = 0.8$)

a) (6 pts) Write the equations (apply Newton's second law) for the translational and rotational motion of the ball. Don't solve them here!




From top to A :

$mg\sin\theta = ma_{cm}$ $\Rightarrow$ $a_{cm} = g\sin 37°$

From A to bottom :

$mg\sin\theta - F_k = ma \Rightarrow a_{cm} = g(\sin 37° - \mu_k \cos 37°)$

Rotational Motion :

$\Sigma \tau = I_{cm}\alpha_{cm}$

$F_k R = I_{cm}\alpha_{cm}$

$(\mu_k \cancel{m} g \cos 37°) R = (\frac{2}{5}\cancel{m}R^2)\alpha_{cm}$

$\Rightarrow \alpha_{cm} = 5\mu_k g \cos 37° / 2R$

b) (9 pts) b) Using the condition for rolling without slipping, find the time it takes for the rolling motion to start if t=0 at A.

From A to the instant rolling starts

$v_{cm} = v_0 + a_{cm}t$

$= v_0 + g(\sin 37° - \mu_k \cos 37°)t$

Condition for rolling without sliding is

$v_{cm} = R\omega$

But $\omega = \underbrace{\cancel{\omega_0}}_{\text{at A}} + \alpha_{cm}t$

$= \alpha_{cm}t$.

$= \frac{5\mu_k g \cos 37°}{2R} t$

$\Rightarrow v_0 + g(\sin 37° - \mu_k \cos 37°)t = \cancel{R}\left(\frac{5\mu_k g \cos 37°}{2\cancel{R}}\right)t$

$v_0 = g\left(\frac{7}{2}\mu_k \cos 37° - \sin 37°\right)t$

$\Rightarrow t = \frac{v_0}{g\left(\frac{7}{2}\mu_k \cos 37° - \sin 37°\right)}$

But $v_0 = \sqrt{2gh} = \sqrt{2gd\sin 37°} = 6\ m/s$ $\Rightarrow t = \frac{3}{4} s$

| | unit |
|---|---|
| $t = 3/4$ | s |

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3. (15 pts) The position of a simple harmonic oscillator (SHO) as a function of time is given by $x = 2\cos(\frac{\pi}{3}t + \frac{3\pi}{4})$ where $x$ is in meters and $t$ is in seconds. Give your answers in terms of $\pi$ when necessary.

a) (7 pts) Find the period $T$ and $v_{max}$.

$x = 2\cos\left(\frac{\pi}{3}t + \frac{3\pi}{4}\right)$

$\Rightarrow f = \frac{w}{2\pi} = \frac{\pi/3}{2\pi} = \frac{1}{6}$ Hz $\Rightarrow F = \frac{1}{f} = 6$ s

$v_{max} = \sqrt{\frac{k}{m}}\, A = wA$

$= \frac{\pi}{3}(2)$

$= \frac{2\pi}{3}$ m/s

| | unit |
|---|---|
| $T = 6$ | s |

| | unit |
|---|---|
| $v_{max} = \frac{2\pi}{3}$ | m/s |

b) (8 pts) Find the position and velocity at $t = 0$.

$x(t=0) = 2\cos\left(0 + \frac{3\pi}{4}\right)$

$= 2\cos\frac{3\pi}{4}$

$= (2)\left(-\frac{\sqrt{2}}{2}\right)$

$= -\sqrt{2}$ m

$v = \frac{dx}{dt} = -2\,\frac{\pi}{3}\sin\left(\frac{\pi}{3}t + \frac{3\pi}{4}\right)$

$\Rightarrow v(t=0) = -2\,\frac{\pi}{3}\sin\frac{3\pi}{4}$

$= -(2)\left(\frac{\pi}{3}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\pi\frac{\sqrt{2}}{3}$ m/s in $-x$ dir

| | unit |
|---|---|
| $x = -\sqrt{2}$ | m |

| | unit |
|---|---|
| $v = -\pi\frac{\sqrt{2}}{3}$ | m/s |

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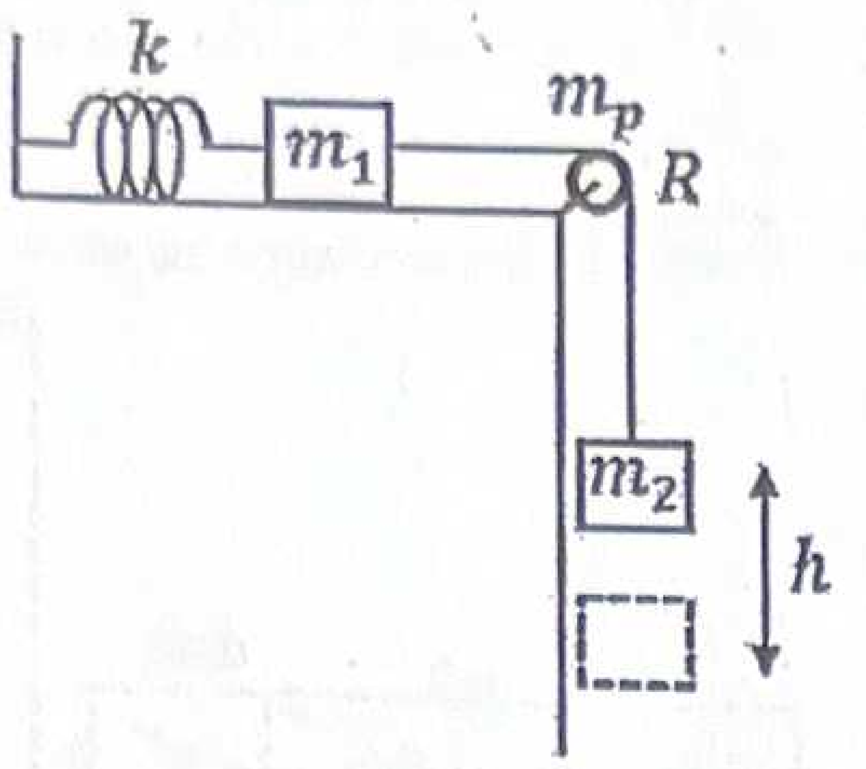
4. (20 pts) The system shown is released from rest when the spring ($k = 50\ N/m$) neither compressed nor stretched. The blocks have masses $m_1 = 10\ kg$ and $m_2 = 3\ kg$. The horizontal surface has friction with $\mu_k = 0.1$. The pulley is a solid cylinder ($I_{cm} = \frac{1}{2}mR^2$) with mass $m_p = 6\ kg$ and radius $R = 0.05\ m$. (Take $g = 10\ m/s^2$)




a) (15 pts) Write down the energy equation $\Delta K + \Delta U = W_{nc}$ needed to find the speed of the blocks at the instant block $m_2$ has fallen a vertical height of $h$. Don't solve your equation yet!

$$\Delta K + \Delta U = W_{nc}$$

$$\cancel{K_1}_{0} + U_1 = K_2 + U_2 - W_{F_k}$$

$$m_2 g h = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} I_{cm} w^2 + \frac{1}{2} k h^2 + F_k h$$

where $I_{cm} = \frac{1}{2} m_p R^2$, $w = \frac{v}{R}$ and $F_k = \mu_k m_1 g$

b) (5 pts) Find the speed of the blocks at the instant block $m_2$ has fallen a vertical height of $h = 0.4\ m$.

$$m_2 g h = v^2\left(\frac{m_1 + m_2}{2} + \frac{m_p}{4}\right) + \frac{1}{2} k h^2 + \mu_k m_1 g h$$

$$\Rightarrow \quad v = \left(\frac{m_2 g h - \frac{1}{2} k h^2 - \mu_k m_1 g h}{\frac{m_1 + m_2}{2} + \frac{m_p}{4}}\right)^{1/2}$$

$$= \sqrt{0.5}$$

| $v =$ $\sqrt{0.5}$ | unit: m/s |
|---|---|

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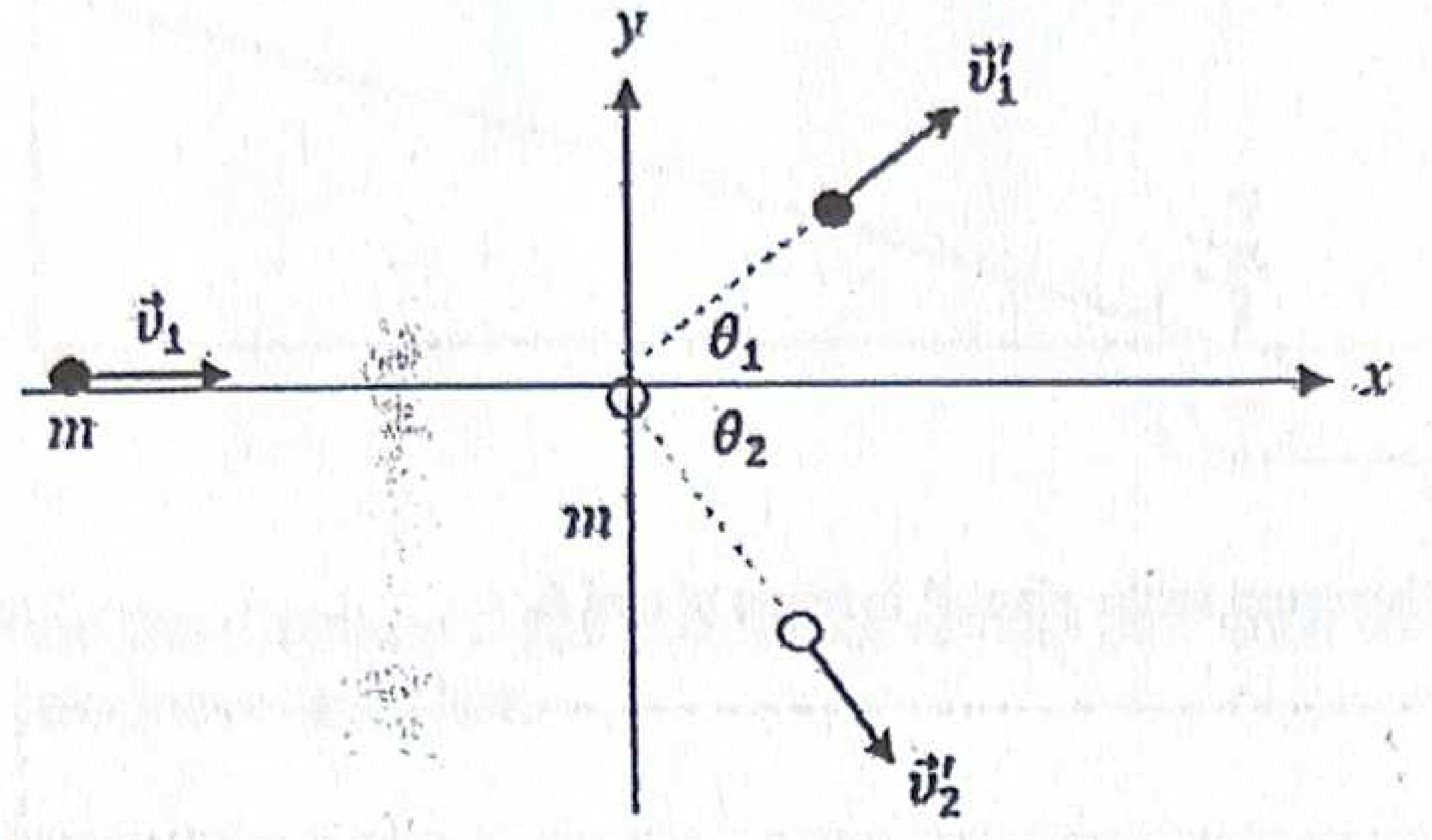
5. (15 pts) Figure shows the collision between two particles having the same mass $m$. The incoming particle has velocity $v_1\hat{i}$, and the target particle is initially at rest. After the non-head-on elastic collision, the incoming particle moves with speed $v_1' = 12\ m/s$ at $\theta_1 = 37^0$ above the horizontal, and the target particle moves with speed $v_2'$ at $\theta_2$ below the horizontal. The center-of-mass velocity after the collision is $\vec{v}'_{cm} = (15\hat{i})\ m/s$. (Take $sin37 = cos53 = 0.6; sin53 = cos37 = 0.8$)

7.5




Find $v_1$, $v_2'$ and $\theta_2$. You must present clear and detailed calculations!

$$\vec{v}'_{cm} = \frac{m\vec{v}_1' + m\vec{v}_2'}{2m} = \frac{\vec{v}_1' + \vec{v}_2'}{2}$$

$$\vec{v}_{cm} = \frac{m\vec{v}_1 + m\cancel{\vec{v}_2}}{2m} = \frac{\vec{v}_1}{2}$$

$$\vec{v}_{cm} = \vec{v}'_{cm} \Rightarrow \frac{\vec{v}_1}{2} = 7.5\hat{i} \Rightarrow v_1 = 15\ m/s$$

$$\Rightarrow \vec{v}_1 = \vec{v}_1' + \vec{v}_2'$$

$x:\ 15 = 12\cos37^\circ + v_2'\cos\theta_2 \qquad \Rightarrow v_2'\cos\theta_2 = 5.4$

$y:\ 0 = 12\sin37^\circ + v_2'\sin\theta_2 \qquad \Rightarrow v_2'\sin\theta_2 = -7.2$

$$\Rightarrow v_2' = \sqrt{(5.4)^2 + (-7.2)^2} = 9\ m/s$$

And $\theta_2 = 53^\circ$

$\Rightarrow \theta_1 + \theta_2 = 90^\circ$ exactly

| | unit |
|---|---|
| $v_1 =$ 15 | m/s |

OR

$v_2' = v_1^2 - v_1'^2$

$= (15)^2 - (12)^2$

$= 81$

$\Rightarrow v_2' = 9\ m/s \qquad \Rightarrow \cos\theta_2 = \frac{5.4}{9}$

$\theta_2 = 53^\circ$

| | unit |
|---|---|
| $v_2' =$ 9 | m/s |

$\theta_2 =$ 53°

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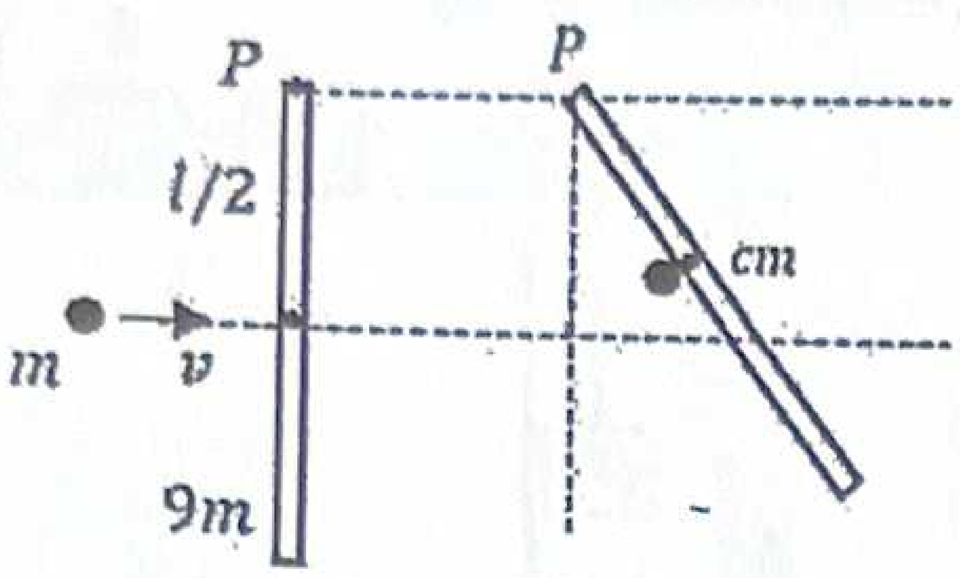
6. (20 pts) A small clay ball of mass $m$ and velocity $v$ strikes and sticks to a thin uniform rod of mass $9m$ and length $l$ where masses are in kilograms, velocity is in meter/second, and length is in meters. The center of mass of the clay+rod system rises to a height of $h = 0.06\,m$. (Take $I_P = \frac{1}{3}ml^2$ ; $g = 10\,m/s^2$)




a) (5 pts) Find the moment of inertia of the clay+rod system with respect to the pivot $P$ in terms $m$ and $l$.

$I_{sys} = I_{rod} + I_{clay}$

$= \frac{1}{3}(9m)l^2 + m\left(\frac{l}{2}\right)^2$

$= 3ml^2 + m\frac{l^2}{4}$

$= \frac{13ml^2}{4}$

| | unit |
| --- | --- |
| $I_P = \frac{13\,m\,l^2}{4}$ | $kg.m^2$ |

b) (7 pts) Find the angular velocity of the clay+rod system in terms of $v$ and $l$ immediately after the collision.

$L_i = L_f$

$L_{clay} = L_{sys}$

$\cancel{m} v \frac{\cancel{l}}{2} = \frac{13\cancel{m}\cancel{l^2}}{4}\,\omega$

$\Rightarrow \omega = \frac{2v}{13l}$

| | unit |
| --- | --- |
| $\omega = \frac{2v}{13l}$ | rad/s |

c) (8 pts) Find, approximately, the initial velocity of the clay.

After the collision

$\Delta K + \Delta U = 0$

$\frac{1}{2} I_{sys}\,\omega^2 = (m + 9m)gh$

$\frac{1}{2}\left(\frac{13ml^2}{4}\right)\left(\frac{2v}{13l}\right)^2 = 10\,mgh$

$\frac{1}{2}\left(\frac{\cancel{13}\cancel{m}\cancel{l^2}}{\cancel{4}}\right)\left(\frac{\cancel{4}v^2}{13^{\cancel{2}}\cancel{l^2}}\right) = 10\,\cancel{m}gh \Rightarrow v^2 = 2600h = 156$

$v \cong 12.5\ m/s$

| | unit |
| --- | --- |
| $v \cong 12.5$ | m/s |

**PHYS 111 – FINAL EXAM**

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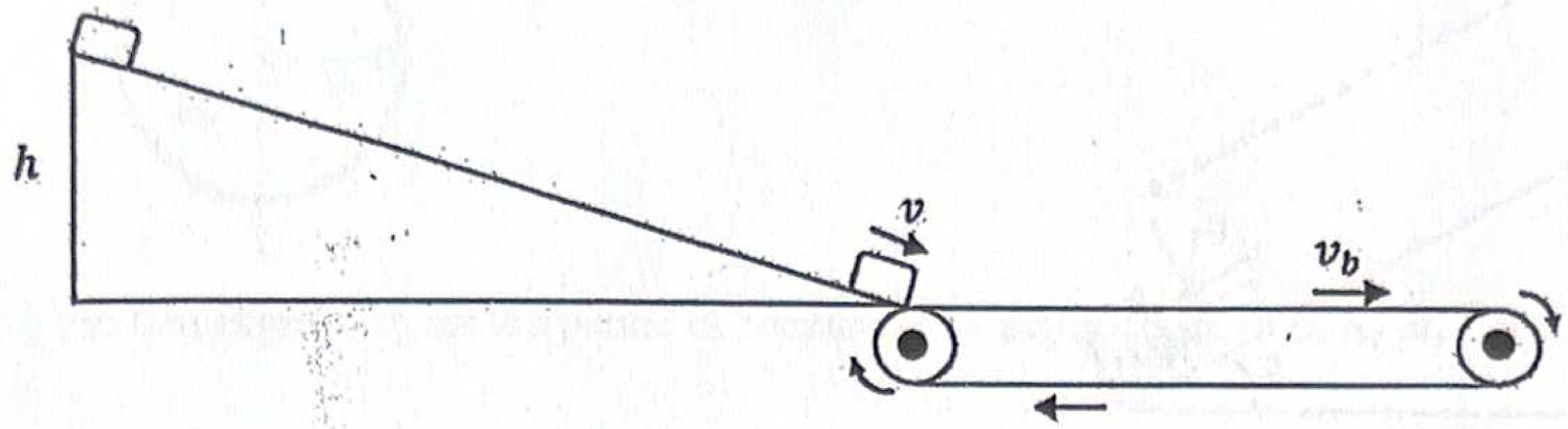
Pr 1. (20 pts) A box of mass $m = 20$ kg is released from rest and slides down a frictionless ramp with height $h = 1.8$ m. At the bottom of the ramp, the box slides onto a conveyor belt moving at a constant speed of $v_b$. The coefficient of kinetic friction between the box and the belt is $\mu_k = 0.5$. ($g = 10$ m/s$^2$)




a) Find the distance $d$ the box slides on the belt until its velocity matches that of the belt $v_b = 4$ m/s and the two move together.

Ramp is frictionless

$\Rightarrow \Delta K + \Delta U = 0$

$\cancel{m} g h = \frac{1}{2} \cancel{m} v^2$

$v = \sqrt{2gh}$

$= \sqrt{2(10)(1.8)}$

$= 6$ m/s at the bottom

Now, apply

$W_{net} = \Delta K$ for motion on belt

Since $v > v_b$, kinetic friction acts opposite to motion and slows the box down until $v_f = v_b$.

$\Rightarrow -F_k d = \frac{1}{2} m v_b^2 - \frac{1}{2} m v^2$

$-\mu_k \cancel{m} g d = \frac{1}{2} \cancel{m} (v_b^2 - v^2)$

$-0.5(\cancel{10}) d = \frac{1}{2}(4^2 - 6^2) = -\cancel{10}$

$\Rightarrow d = 2$ m

$\boxed{d = 2 \text{ m}}$

b) Find the distance $d$ the box slides on the belt if $v_b = 8$ m/s.

In this case, $v < v_b$ and kinetic friction acts in the direction of motion to increase the box's velocity.

$\Rightarrow$ kinetic friction does positive work!

$F_k d = \frac{1}{2} m (v_b^2 - v^2)$

$\mu_k \cancel{m} g d = \frac{1}{2} \cancel{m} (v_b^2 - v^2)$

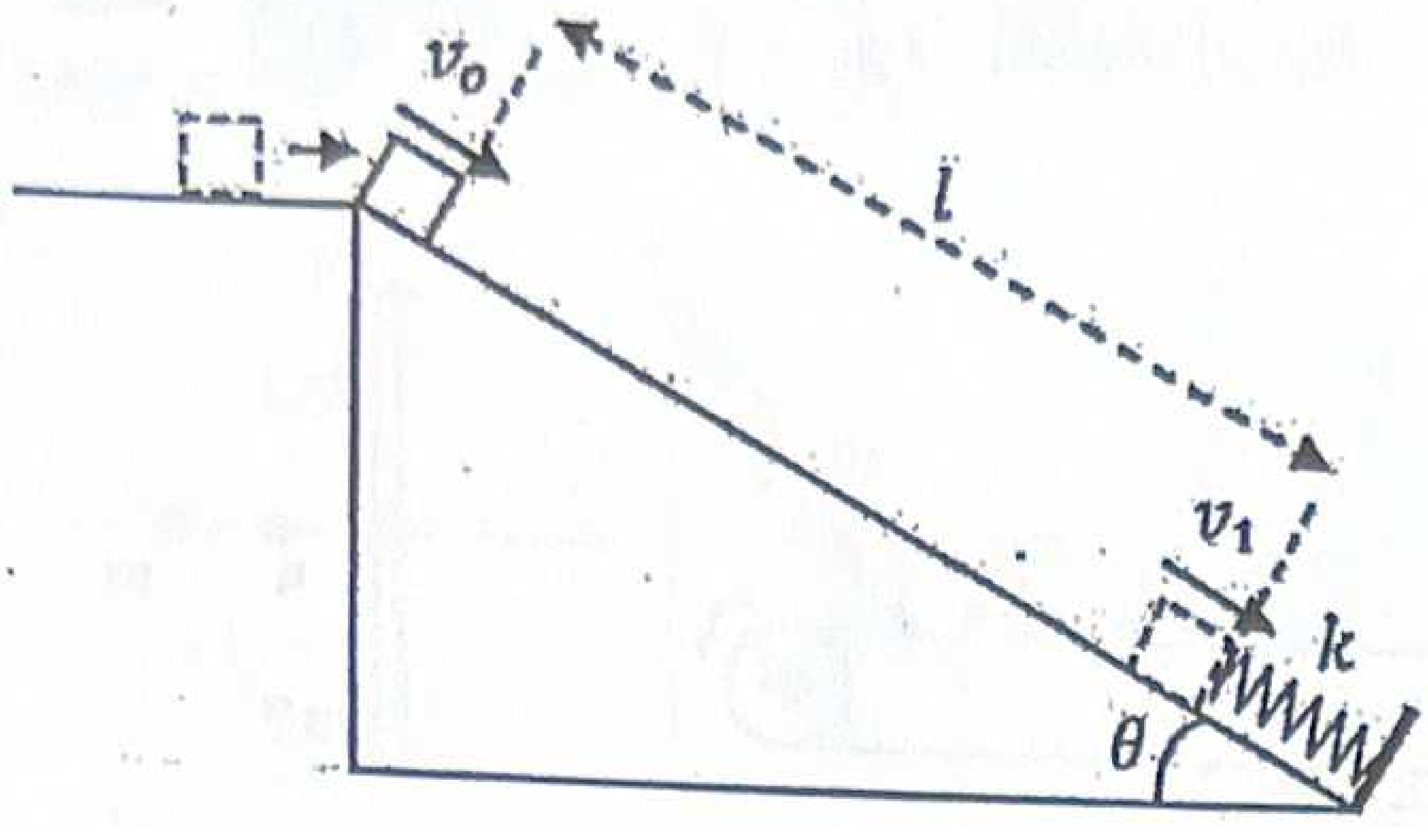
$0.5(10) d = \frac{1}{2}(8^2 - 6^2) = 28$

$\Rightarrow d = \frac{28}{10}$

$= 2.8$ m

$\boxed{d = 2.8 \text{ m}}$

Pr 2. (20 pts) A box of mass $m = 15$ kg at the packaging section of a factory comes to the top of a ramp ($\theta = 37^o$) with speed $v_o$ and slides down where it is picked up for shipment. In order to avoid damage to the box, a spring bumper is used with spring constant $k = 100$ N/m and the maximum spring force $F_s(max) = 100$ N. The box slides a distance of $l = 4$ m down the incline before it hits the spring as shown. The coefficient of kinetic friction between the box and the entire ramp is $\mu_k = 0.75$. ($g = 10$ m/s$^2$; $\sin 37^o = \cos 53^o = 3/5$ ; $\cos 37^o = \sin 53^o = 4/5$)




a) Find the work done by the gravity, normal force and kinetic friction on the box until it hits the spring.

$W_G = \vec{F}_G \cdot \vec{l} = (mg\sin\theta)\, l$

$= (15)(10)(3/5)(4) = 360$ J

$W_{FN} = \vec{F}_N \cdot \vec{l} = F_N\, l \cos 90^\circ = 0$

$W_{Fk} = \vec{F}_k \cdot \vec{l} = -(\mu_k mg\cos\theta)\, l$

$= -(0.75)(15)(10)(4/5)(4)$

$W_{Fk} = -360$ J

$\boxed{W_G = 360 \text{ J}}$

$\boxed{W_{FN} = 0}$

$\boxed{W_{FK} = -360 \text{ J}}$

b) Find the maximum speed of the box at the top of the ramp if the box is to be picked up when the spring is in maximum compression.

Because of kinetic friction, the system is nonconservative.

$\Delta K + \Delta U = W_{Fk}$

$\Rightarrow \frac{1}{2} m v_o^2 + mg(l + \Delta l)\sin\theta = \frac{1}{2} k \Delta l^2 + (\mu_k mg\cos\theta)(l + \Delta l)$

In part (a), we showed

$W_G = |W_{Fk}|$

which is true when the distance travelled by the box is $l + \Delta l$.

Thus, the energy equation simplifies to

$\cancel{\frac{1}{2}} m v_o^2 = \cancel{\frac{1}{2}} k \Delta l^2$

But $F_s(max) = k \Delta l \Rightarrow \Delta l = \frac{100}{100} = 1$ m

$\Rightarrow v_o = \sqrt{\frac{(100)(1)^2}{15}}$

$= \sqrt{\frac{100}{15}} = \sqrt{\frac{20}{3}}$ m/s

$\boxed{v_o = \sqrt{20/3} \text{ m/s}}$

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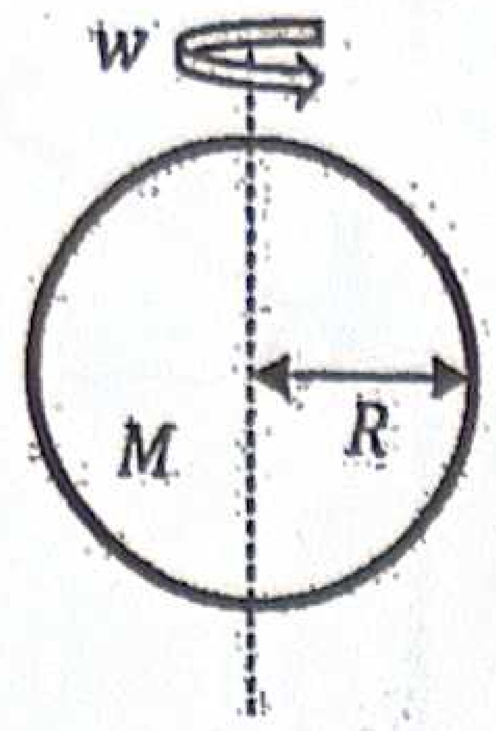
Pr 3. (10 pts) Suppose that the Earth was a perfectly uniform sphere of radius $R$ and mass $M$, rotating about its North–South axis with an angular velocity $w$. ($I_{cm} = 2/5MR^2$)




a) Find an expression for the period of rotation, $T = 2\pi/w$, in terms of $R$, $M$, and the angular momentum $L$.

$T = 2\pi/w$

Using $L = I_{cm} w \Rightarrow L = (\frac{2}{5}MR^2)w$

$\Rightarrow w = \frac{5L}{2MR^2}$

Thus, $T = \frac{2\pi}{\frac{5L}{2MR^2}} = \frac{4\pi MR^2}{5L}$

$\boxed{T = \frac{4\pi M}{5L} R^2}$

b) Suppose the radius increases by a small amount $\Delta R$, but the mass $M$ is kept constant. What will be the fractional change $\Delta T/T$ in the period in terms of $\Delta R$ and $R$? (Hint: $\frac{\Delta T}{\Delta R} \approx \frac{dT}{dR}$)

$\frac{\Delta T}{\Delta R} = \frac{dT}{dR} = \frac{d}{dR}\left(\frac{4\pi M}{5L}\right)R^2$

$= \frac{4\pi M}{5L} 2R$

Using $T = \frac{4\pi M}{5L} R^2$

$\frac{\Delta T}{\Delta R} = \frac{4\pi M}{5L} 2R \left(\frac{R}{R}\right) = \left(\frac{4\pi M}{5L} R^2\right)\frac{2}{R}$

$= T \cdot \frac{2}{R}$

$\Rightarrow \frac{\Delta T}{T} = \frac{2}{R}\Delta R$

$\boxed{\Delta T/T = \frac{2}{R}\Delta R}$

c) Note that a fractional increase of $\Delta T/T = 0.25$ day / 365 days means that leap years (29 days in February) will not be needed. If $R = 6370$ km, what is $\Delta R$, approximately, in km?

$\frac{\Delta T}{T} = \frac{2}{R}\Delta R$

$\Rightarrow \Delta R = \frac{R}{2}\frac{\Delta T}{T}$

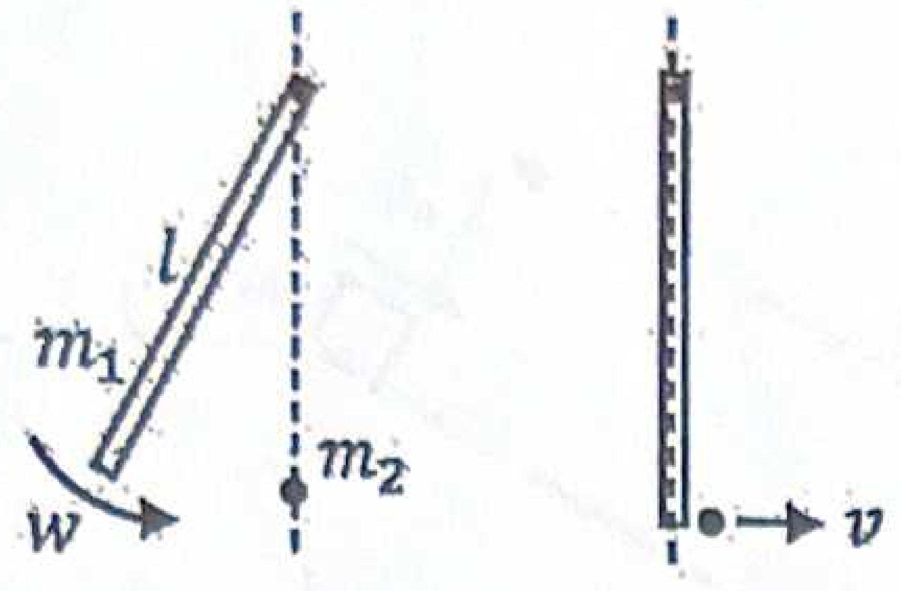
$= \frac{6370}{2} \cdot \frac{0.25}{365}$

$\Delta R = \frac{6370}{2920}$

$\cong 2.2$ km

$\boxed{\Delta R \cong 2.2 \text{ km}}$

**Pr 4.** (20 pts) A uniform rod of length $l$ and mass $m_1$ initially rotates at constant angular speed $w$ on a frictionless horizontal table around a fixed pivot at one end, as shown below. The rod strikes a small object of mass $m_2$, which is initially at rest. Assume that the collision is elastic. As a result, the rod stops and the small object moves, as shown.
(Use $I = m_1 l^2/3$ for the rod pivoted at one end)




before collision after collision

Top views

a) Find $v$ in terms of the given quantities.

Angular momentum is conserved

$L_i = L_f$

where

$L_i = I w$ for the rod

$L_f = m_2 v l$ for the object

$$\Rightarrow \left(\frac{1}{3} m_1 l^{\cancel{2}}\right) w = m_2 v \cancel{l}$$

$$v = \frac{m_1 l}{3 m_2} w$$

$$\boxed{v = \frac{m_1 l w}{3 m_2}}$$

b) Use energy considerations to find the ratio of the masses $m_1/m_2$.

Collision is elastic

$\Rightarrow K_i = K_f$

where

$K_i = \frac{1}{2} I w^2$

$K_f = \frac{1}{2} m_2 v^2$

$$\Rightarrow \cancel{\tfrac{1}{2}}\left(\frac{1}{3} m_1 l^2\right) w^2 = \cancel{\tfrac{1}{2}} m_2 \left(\frac{m_1 l w}{3 m_2}\right)^2$$

$$\Rightarrow \frac{1}{\cancel{3}} \cancel{m_1} \cancel{l^2} \cancel{w^2} = \cancel{m_2} \frac{m_1^{\cancel{2}} \cancel{l^2} \cancel{w^2}}{3 \cancel{9} m_2^{\cancel{2}}}$$

$$\Rightarrow \frac{m_1}{m_2} = 3$$

$$\boxed{m_1/m_2 = 3}$$

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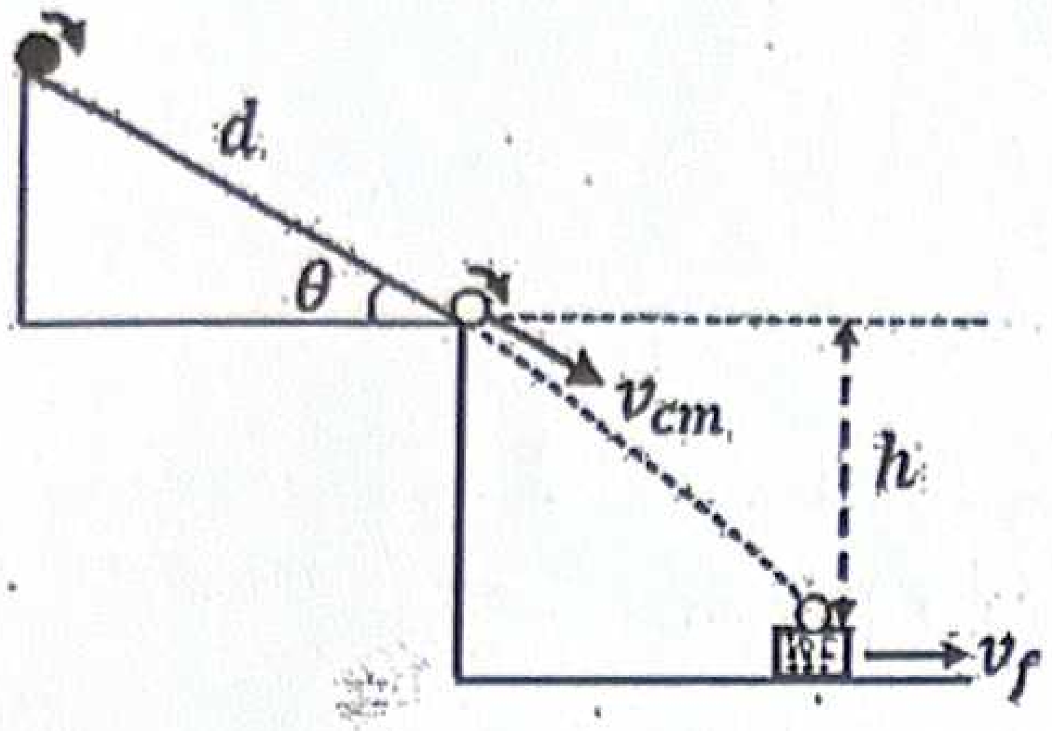
Pr 5. (20 pts) A bowling ball of radius $R$ and mass $m_1 = 7$ kg is released from rest from the top of a roof with $\theta = 37°$, and rolls down a distance of $d = 4.2$ m without slipping.
($I_{cm} = 2/5 m_1 R^2$ ; g = 10 m/s$^2$; sin37° = cos53° = 3/5 ; cos37° = sin53° = 4/5)




a) Find the magnitude of the friction force acting on the ball as it rolls down the roof.

Apply N2$^{nd}$L to translational and rotational motions:

$m_1 g \sin\theta - F_s = m_1 a_{cm}$ trans.

$F_s R = I_{cm}\alpha = I_{cm}\frac{a_{cm}}{R}$

$\Rightarrow m_1 g \sin\theta - F_s = m_1 a_{cm}$ ①

$F_s \cancel{R} = \frac{2}{5} m_1 R\cancel{^2}\left(\frac{a_{cm}}{\cancel{R}}\right)$ ②

Solving ① and ② for $F_s$ gives

$F_s = \frac{2}{7} m_1 g \sin\theta$

$= \frac{2}{\cancel{7}}(\cancel{7})(10)(3/5) = 12$ N

$\boxed{F_s = 12 \text{ N}}$

b) Find the center-of-mass speed $v_{cm}$ of the ball at the instant it reaches the edge of the roof and starts its flight (projectile motion).

Static friction only..

$\Rightarrow \Delta K + \Delta U = 0$

$m g d \sin\theta = \frac{1}{2} m_1 v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2$

But $\omega = \frac{v_{cm}}{R}$

$\cancel{m_1} g d \sin\theta = \frac{1}{2}\cancel{m_1} v_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m_1}\cancel{R^2}\right)\left(\frac{v_{cm}}{\cancel{R}}\right)^2$

$g d \sin\theta = \frac{7}{10} v_{cm}^2$

$\Rightarrow v_{cm} = \sqrt{\frac{10}{7}(10)(4.2)(3/5)} = 6$ m/s

OR Use $\Sigma F = m a_{cm} \Rightarrow m_1 g \sin\theta - F_s = m_1 a_{cm}$

$\Rightarrow a_{cm} = 30/7$ m/s$^2$

And $v_{cm}^2 = \cancel{v_0^2} + 2 a_{cm} d$

$\boxed{v_{cm} = 6 \text{ m/s}}$

c) Suppose that the ball falls a vertical distance of $h = 6$ m from the roof and lands in a sandbox of mass $m_2 = 14$ kg that brings the ball to a sudden stop. The sandbox+bowling ball system then moves horizontally. Find the speed of the sandbox, with the ball in it, immediately after the ball lands in it.

In projectile motion, the horizontal component of velocity does not change. The collision is completely inelastic.

$\Rightarrow m_1 v_{cm}\cos\theta = (m_1 + m_2) v_f$

$\Rightarrow (7)(6)(4/5) = (7+14) v_f$

$v_f = 8/5$ m/s

$\boxed{v_f = 8/5 \text{ m/s}}$

Pr 6. (10 pts) A small block of mass $m = 0.5$ kg is attached to the end of a horizontal spring with $k = 2$ N/m. It is pulled and released from rest at $t = 0$ s from an extended position $A = x_{max}$. Thereafter the motion of the block is due entirely to the spring force. After 0.1 s, the speed of the block is measured to be $v = 0.2$ m/s. Assume that the motion takes place on a frictionless surface.
($\sin 0.2 \approx 0.2$ ; $\cos 0.2 \approx 0.98$ ; $\sin 0.4 \approx 0.39$ ; $\cos 0.4 \approx 0.92$)

a) Find the amplitude of the motion $x_{max}$.

$x(t) = A\cos(\omega t)$

$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t)$

$= -\omega x_{max} \sin(\omega t)$

But $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{0.5}} = 2$ rad/s

$\Rightarrow x_{max} = \frac{v}{\omega \sin(\omega t)}$ dropped (-) !

Given : $v = 0.2$ m/s at $t = 0.1$ s

$\Rightarrow x_{max} = \frac{0.2}{2\sin[(2)(0.1)]}$

$= \frac{0.2}{2\sin 0.2}$

$= \frac{0.2}{(2)(0.2)}$

$= 0.5$ m

$\boxed{x_{max} = 0.5 \text{ m}}$

b) Maximum speed $v_{max}$ of the block.

Max speed is when $\sin(\omega t)$ is maximum.

$\Rightarrow v_{max} = \omega x_{max}$

$= (2)(0.5)$

$= 1$ m/s

$\boxed{v_{max} = 1 \text{ m/s}}$

c) Total energy $E$ of the system.

$E = K + U_{el} = \frac{1}{2} k x_{max}^2$

OR $= \frac{1}{2} m v_{max}^2$

$= \frac{1}{2}(0.5)(1)^2$

$= 0.25$ J

$\boxed{E = 0.25 \text{ J}}$